

Author: Riccati, Giordano

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[-678-] The laws of counterpoint deduced from phenomena and confirmed on the basis of reason by Count Giordano Riccati [[Nobleman from Treviso], Fourth Book

[-679-] Fourth Book

First Chapter

On temperaments

[signum] 1. The musical temperament consists in distributing among several intervals the difference of the comma $81/80$, by which certain consonant intervals are altered too much, so that it may become less audible. Such distribution is also called participation, because many intervals participate of the distributed difference of the comma thanks to it. Many theorists, when they discuss with the temperaments, begin by exaggerating the imperfections of the diatonic system. In order to remove any confusion, I note that the diatonic can be considered in three ways, namely, as the tone C with the major third, as the tone a with the minor third or as a combination of said two tones. If it is considered in the first two ways, it is absolutely perfect, since it originates from the application of an exact system of the melody to the consonant accompaniment with the major third or with the minor third. The intervals that are out of tune by the measure of a comma in one of the two tones when they are considered as fundamental harmonies or melodies do not occur in the aforementioned tones in that role. Our ratios are employed as derivative movements of the melody, and under this point of view they are not altered, since they lead from a perfect consonance to another perfect consonance, as their function requires. For instance, in the tone C with the major third the fifth D a, which, considered as fundamental harmony and melody, would be a comma lower, it is employed as a perfect passage leading from A, major tenth of F to D fifth of G. This is an important observation and I am not aware that anybody else made it. If the diatonic is considered in the third way, one notices immediately the need for the temperament. We achieve through it that the difference of the comma, through which the tones C with the major third and A with the minor third do not agree on the value of D, is removed, so that the consonances of both the tones prove suitably perfect.

[signum] 2. Moreover, different intellectual processes have led music scholars to the theory of the temperaments. [-680-] If I am not mistaken, the method that I propose leads directly to our aim and is based on demonstration. First of all, I lay down some principles.

When discussing the temperaments, one must only consider the consonances. The dissonances, which are harsh on the ear by their own nature, tolerate great alterations. The difference of an enharmonic diesis does not bother the ear in the minor seventh, in the eleventh and in the thirteenth.

The simpler the consonances are the less they need to be altered, which means they need to be altered in inverse proportion to their simplicity. However, as we have observed elsewhere (book 1., chapter 1., [signum] 13.) the largest odd numbers that occur in the consonant chords are inversely proportional to the simplicity of the consonances. Therefore, they need to be altered according to the proportion of the largest odd numbers that occur in their ratios. If we compare two consonances, one perfect and the other one imperfect, because the largest odd number in the perfect consonances is the number three and in the imperfect ones is the number five, their reciprocal alterations shall correspond to each other according to the ratio 3:5.

The aequisonances, namely, apart from the unison, the simple octave and the compound

intervals of the octave must not be altered at all. Said law originates from the nature of the aequisonances, which requires that the simple octave, the double octave, the triple octave must be altered by the same difference. This cannot be done in general and advantageously with any advantage except if one keeps all the octaves perfect. I said that it cannot be done in general and advantageously, because one could realise it in such a way that the first in a series of octave is altered while all the others are perfect, so that the simple, double and triple octave etcetera would be tempered by the same amount. However, the alteration of a single octave would prove useless for the whole participation of an instrument, and, besides, we are looking for the best temperament which treats the consonances that are similar in the same way. If one modifies appropriately the simple octave, the imperfection of the compound octaves is too great, while, if one

leavens the difference of the furthest compound octave, the alteration of the [-681-] simple octave is so minute that, in practice, it is as if it was not there. The consequence of the fact that the aequisonances must be preserved intact is that the portion of the comma by which an interval is too narrow or too wide must be the same by which, conversely, the interval that added to it forms the octave is too wide or too narrow. For instance, if one reduces the fifth by a sixth of a comma, the fourth, which added to the fifth forms the octave, shall have to be widened by the same amount.

[signum] 3. I move on now to illustrate three origins of reasonable participations, of which the middle one is the best. First of all, if we take as base the letter E and we place three fifths in sequence and a minor third one after the other, I observe that in the tone C with the major third as well as in the tone A with the minor third the two octaves C c consist of three fifths and a minor third, and one of these fifths is smaller by a comma $80/81$, which I mark with the letter c.

[Riccati, The laws of counterpoint, 681; text: Tuono C per Terza maggiore, C, G, d, a, 1, $3/2$, $5/4$, $10/3$, $6/5$, 4, minore, $20/9$]

The only difference consists in the fact that the fifth d a is narrower in the first tone, while in the second tone the fifth G d is narrower. One should consider that those lower fifths are not involved as fundamental harmonies and melodies in the origin of the tone that produces them as such. In other words, the fourth d a does not partake to the origin of the tone C with the major third, while the fifth G d does not partake to the origin of the tone A with the minor third. One shall provide equally to both tones if the comma $81/80$ is distributed among the four intervals constituting the double octave, so that the three fifths [-682-] C G, G and d a, and the minor third a c may sound suitably correct to the ear without it feeling repulsion towards them. The aforesaid comma shall be divided in the best way if each of the fifths receives a portion of it whose relationship with the portion allotted to the third is expressed by the ratio 3:5, in conformity with the second principle established above. Therefore, let us divide the comma into fourteen parts, three of whom are assigned to each fifth, while the remaining five parts are allotted to the minor third.

[Riccati, The laws of counterpoint, 682,1; text: Ottimo Temperamento della sottoscritta serie. C, G, d, a, c, 1, $3/2$ - $3/14$ c, $20/9$ + $8/14$ c; $9/4$ - $6/14$ c, $10/3$ + $5/14$ c, 4, $6/5$]

[signum] 4. We encounter the same temperament if we consider that two octaves consist of two fourths and by one major sixth, and one of those fourths is wider by a comma. I invite the Reader to consider the two sequences placed herewith based on the letter C

[Riccati, The laws of counterpoint, 682,2; text: Tuono C per Terza maggiore, C, A, d, g, c, $5/3$, $4/3$ + c, $9/4$, 3, 4, minore, $20/9$]

One of the fourths is a comma wide both in the tone C with the major third and in the tone A with the minor third. In the first tone the wider third is A d, while in the second one it is d g. In order to reconcile the two tones together, the three fourth, which are perfect consonances, and the major sixth, which is an imperfect consonance, whose sum amounts to two perfect octaves, must be

tempered taking into account their simplicity. [-683-] Thus one shall discover that the comma has to be divided into fourteen parts even in this case. Each fourth must be wider by three parts, while the major sixth must be wider by the remaining five.

[Riccati, The laws of counterpoint, 683,1; text: Ottimo Temperamento della sottoscritta seria. C, A, d, g, c, $1 \frac{5}{3} + \frac{5}{14} c$, $\frac{4}{3} + \frac{3}{14} c$, $\frac{20}{9} + \frac{8}{14} c$, $\frac{9}{4} + \frac{6}{14} c$, $3 - \frac{3}{14} c$, 4]

I said that this temperament is the same as the one explained above. In fact, if the fifth is reduced by $\frac{3}{14} c$ and the minor third by $\frac{5}{14} c$, the fourth and the major sixth, the intervals that added to the former ones complete the octave, must be wider by the same amount, as it occurs in the latter temperament. I noted elsewhere that, because of the aequisonance of the octave, a fifth taken upwards is equivalent to a fourth taken downwards in a certain way. I invite the Reader to consider the two series.

[Riccati, The laws of counterpoint, 683,2; text: C, G, d, a, C, A, d, g]

where one can see that the same sequence of letters that one encounters proceeding upwards from low C to high c through three fifths and a minor third is the same as the one that one encounters descending from high c to low C proceeding through three fourths and a major sixth. Therefore, since our two series consists of sounds that are the same or aequisonant to each other, and, since they receive the same alterations thanks to both the temperaments, it follows that two temperaments join together in a single one. This is a true fact that I wanted to illustrate in the following way as well.

Since the fifth in our temperament is narrow by $\frac{3}{14} c$ and the minor third by $\frac{5}{14} c$, it follows that the major third is wider by $\frac{2}{14} c$. In fact, since the fifth consists of the two thirds, major and minor, [-684-] its reduction is equivalent to the reduction of the minor third, if one subtracts the portion by which the major third is wider, namely, $\frac{5}{14} c - \frac{2}{14} c = \frac{3}{14} c$, which is the true reduction of the fifth in the present participation. One has to conclude that the minor sixth, which added to the major third constitutes the octave, is narrow by $\frac{2}{14} c$.

[signum] 5. I inform the Reader that, considering the property according to which two octaves consist of three fifths and a minor third or of three fourths and a major sixth with the difference of a comma, the participation that first springs to mind is to divide the comma equally into four intervals that constitute the double octave, so that the fifth and the minor third are reduced by one fourth of a comma, while the fourth and sixth are widened by the same amount, while the major third and the minor sixth remain in their exact proportions. This temperament is one of the ones mentioned by Giuseppe Zarlino, which comprehends adequately the Cristiano Ughenio's general circular system, which I shall describe in the appropriate place. However, said temperament lacks the requirement to consider the simplicity of the consonance, since it treats the imperfect consonances and the perfect ones in the same way.

[signum] 6. Progressing on our way, I note that each of the fifth of the series C G d a c can be divided into the two thirds, one major and the other one minor, so that the double octave C c consists of seven thirds, three major and four minor ones. I lay out here two sequences, one belonging to the tone C with the major third and the other one belonging to the tone A with the minor third.

[Riccati, The laws of counterpoint, 684; text: Tuono C per Terza maggiore, C, E, G, B, d, f, a, c, 1, $\frac{5}{4}$, $\frac{3}{2}$, $\frac{15}{8}$, $\frac{9}{4}$, $\frac{8}{5}$, $\frac{10}{3}$, 4, $\frac{6}{5}$, $\frac{6}{5} - c$, minore]

[-685-] In the tone C with the major third, I find that the minor third d f, which is not part of it, is reduced by a comma. However, it is involved in the origin of the tone A with the minor third, which, while it provides with the adjusted minor third d f, it passes on the lack of the comma on to the minor third B d, which does not belong to it, but which is involved in the origin of the tone C with

the major third.

It is clear that, in order for the two tones to agree with each other, it is necessary that the three major thirds and the four minor ones are all suitably perfect, which shall be achieved by dividing the comma into seven parts and reducing every major and minor third (which belong to the same degree of simplicity and need to be modified on the basis of the same principle) by a seventh of the comma.

[Riccati, The laws of counterpoint, 685,1; text: Ottimo Temperamento della sottoscritta serie, C E G B d f a c, $5/4 - 1/7c$, $3/2 - 2/7c$, $15/8 - 3/7c$, $20/9 - 3/7c$, $9/4 - 4/7c$, $8/3 + 2/7c$, $10/3 + 1/7c$, 4, $6/5 - 1/7c$]

[signum] 7. One encounters the same participation if one considers that, just as three fourth and a major sixth constitute two octaves, thus three fourths above the octave and a major sixth constitute five octaves. Now, a fourth above the octave, for instance A d is divided into two sixths, A f and f d, one of them minor and the other one major. Consequently, five octaves consist of seven sixths, four major and three minor.

[Riccati, The laws of counterpoint, 685,2; text: Tuono C per Terza maggiore, C, A, F, d, [sqb], g, e, c, 1, $5/3$, $8/3$, $15/2$, 12, 20, 32, $8/5$, $5/3 + c$, minore, 40/9]

[-686-] The tone C with the major third presents to us the perfect sixth, except only for the major one f d, which is wider by a comma. The tone A with a minor third reduces appropriately the mentioned major sixth f d, but contains the adjacent sixth d [sqb], which is also major, as wider by a comma.

If the same sounds have to be employed in both of our tones, it is necessary that all the sixths that constitute the five octave are as perfect as possible, and this shall be achieved in the most elegant manner by dividing the comma into seven portions and widening each of the seven sixths, four major and three minor, by one seventh of the comma.

[Riccati, The laws of counterpoint, 686; text: Ottimo Temperamento della sottoscritta serie. C, A, f, d, [sqb], g, e, c, $5/3 + 1/7c$, $8/3 + 2/7c$, $40/9 + 3/7c$, $9/2 - 4/7c$, $15/2 - 3/7c$, $12 - 2/7c$, $20 - 1/7c$, 32, $8/5 + 1/7c$]

The two octave consisting of three thirds, and the five octaves consisting of sixth sixths provide us with the same temperament. In fact, if the major and minor thirds are narrowed by a seventh of the comma, while the major and minor sixth, which added to the minor and major thirds constitute the octave, must be wider by the same amount in conformity to the fourth principle, as it occurs in reality in the last temperament.

As the major third and the minor third are each reduced by one seventh of the comma, it follows that the fifths consisting of those two thirds is narrower by two sevenths of a comma. Similarly, the fourth above the octave, and, consequently, the simple fourth as well are wider by two seventh of a comma. In fact, the fourth above the octave consists of the two sixth, major and minor, each of whom is wider by one seventh of a comma. The same widening by two seventh of a [-687-] comma in the fourth could have been deduced immediately from the fact that it constitutes a perfect fifth when added to the fifth, which is narrower by the same amount.

Giuseppe Zarlino mentions the participation that we are discussing. As we shall see in the appropriate place, the general and circular system that divides the octave into fifty equal parts proposed to the public by Signor Henfling in the *Miscellanea Berolinensia* of the year 1710 is in agreement with Zarlino's participation.

[signum] 8. It remains for me to mention the third and last source of the temperaments. Four fifths and a minor sixth constitute three octaves, as one can observe in the following sequences.

[Riccati, The laws of counterpoint, 687,1; text: Tuono C per Terza maggiore. C, G, d, a, e, c, 1, $3/2$, $9/4$, $10/3$, 5, 8, $8/5$, minore. 20/9]

This equivalence of four fifths and of a minor sixth to three octave is not accurate, since one notes the difference of a comma which the fifth d a lacks in the tone C with the major third and the fifth G d lacks in the tone A with the minor third. The union of the two aforementioned tones requires that the fourth fifths and the minor third be altered as little as possible, with the due consideration towards their simplicity. Such aim is achieved by dividing the comma into seventeen parts and reducing each fifth by three of these parts and the minor fifth by five of them.

[Riccati, The laws of counterpoint, 687,2; text: Ottimo Temperamento della sottoposta serie. C, G, d, a, e, c, $3/2 - 3/17c$, $20/9 + 11/17c$, $9/4 - 6/17c$, $10/3 + 8/17c$, $5 + 5/17c$, $8 \ 8/5 - 5/17c$]

[-688-] [signum] 9. The consideration that two octave consist of a major third and of four fourths leads us to the same consideration. I invite the Reader to consider the two sequences that belong to the usual tones C with the major third and A with the minor third.

[Riccati, The laws of counterpoint, 688,1; text: Tuono C per Terza maggiore. C. E. A. d. g. c. 1, $5/4$, $5/3$, $9/4$, 3, 4, $4/3$, $4/3 + c$, 20/9]

As one sees that the fourth A d is wider by one comma in the tone C with the major third, while it is perfect in the tone A, and that the fourth d g is wider by the same amount in the tone A, while it is perfect in the tone C, the union of our tones requires that the major third and all of the four fourths sound adequately perfect to the ear. Such requirement is accomplished by dividing the comma into seventeenth part in this case as well, and by increasing the major third by five seventeenth parts and the fourth by three seventeenth parts, so that, in conformity with the fourth principle, the alterations are proportional to the numbers 5. and 3., which are the largest odd numbers that occur in the ratios of the major third and of the fourth.

[Riccati, The laws of counterpoint, 688,2; text: Ottimo Temperamento della sottoposta serie. C, E, A, d, g, c, 1, $5/4 + 5/17c$, $5/3 + 8/17c$, $20/9 + 11/17c$, $9/4 - 6/17c$, $3 - 3/17c$, 4, $4/3 + 3/17c$]

I stated that the three octave consisting of four fifths and by a minor sixth, as well as the two octave consisting of four fourths and of a major third provide the same participation. In fact, since the fourth added to the fifth forms an octave, if [-689-] the latter is narrower by $3/17 c$, the former must be wider by the same amount. For the same reason, if the minor sixth is narrower by $5/17 c$, the major third, which added to the minor sixth constitute the octave, must be wider by the same amount, as it occurs in fact in the last temperament of the four fourths and of the major third that constitute two octaves.

Note that in the current third temperament the major sixth, for instance C A, is wide by $8/17 c$, or by just less than half of a comma, as it consists of the major third C E and by the fourth E A, the first one of whom is wider by $5/17 c$ and the second one by $3/17 c$, differences that added together constitute $8/17 c$, the total amount by which the major sixth C A is wider. Consequently, the minor third, which constitutes the octave when it is added to the major sixth, is narrower by $8/17 c$.

[signum] 10. The third participation is very similar to the one very well known to musicians according to which the fifth is reduced by $1/6 c$, the minor sixth by $1/3 c$ and the minor third by $1/2 c$, while, consequently, the intervals that added to them form the octave are widened by the same amounts. If we compare the corrections that the two temperaments assign to the same ratio, we shall discover that they are nearly the same. If we compare $1/6c - 3/18c$ with $3/17 c$, $1/3c - 6/18 c$ with $5/17 c$ and $1/2c - 9/18c$ with $8/17 c$, we shall discover that my statement is true.

The third temperament is embraced even more strictly, as I shall observe in the following second chapter, by the general circular system, which divides the octave into fifty-five equal parts.

Since these parts correspond very closely to the comma, they cause this system to be called usually system of fifty-five commas.

Who does not want to take into account the simplicity of the five intervals that constitute the two or three octaves, would fall into the participation that reduces the fifth and the minor sixth by $1/5$ c while increasing the fourth and the major third by the same amount. Consequently, it is necessary that the minor third is reduced by $2/5$ c and the major sixth is widened by the same amount. I invite the Reader to consult the contributions by Monsieur Sauveur to the *Acta Academiae Parisiensis* for the years 1700, 1701, 1707 and 1711 on such temperament considered by him as the best one. This was confirmed shortly after by the general and circular system invented by the aforementioned author, who divides the octave into forty-three called by him merides.

[signum] 11. I wrote earlier that the union of the two tones with the major third and with the minor third that share the same scale renders the temperament necessary. Nevertheless, I could have stated more generally that it is required by the union of the principal tone and of its subordinate ones. For instance, consider the distribution of two octaves C c into three fifths and a minor third, as required by the tone C with the major third and the tone A with the minor third, either of which is taken as principal, and by their subordinate tones G and F with the major third and E and D with the minor third.

[Riccati, The laws of counterpoint, 690; text: Tuono C per Terza maggiore. C, G, d, a, c, 1, $3/2$, $9/4$, $10/3$, $6/5$, 4, minore, $3/2 - c$, $20/9$, $6/5 -c$, $27/8$, $40/27$]

[-691-] It is easy to deduce from the study of the table placed above that the union of two octaves in three fifths and a minor third shall be appropriate for the complex of the six tones when, after one has divided the comma among the three fifths and the minor third, all of these four intervals sound almost perfect to the ear.

[signum] 12. The necessity of the temperament can be viewed from a different point of view, if we consider that it depends on the union and reciprocal adjustment of the numbers 3 and 5. If we want three fifths and a minor third to add up to two octaves, since a major sixth above the octave and a minor third add up to two octaves, one comes to presume tacitly that three fifths are equivalent to a major sixth above the octave. Now, the ratio of the fifth raised to the power of three, namely, the ratio $27/8 = 81/24$ representing the fifth C a, can never be equal to the ratio $10/3 = 80/24$ that belongs to the sixth above the octave C a, which contains a multiple of the prime number 5, which is not involved at all in the ratio of the fifth. The difference consists of the comma $81/80$, which can be eliminated by distributing it, so that, once the numbers 3 and 5 are harmonised with each other, the triple fifth is equivalent in practice to a major sixth above the octave. If one considered only the number tree, would find that the three fifths C G d a are correct, but the major sixth above the octave C a and the minor third a c would be out of tune by a comma, as it occurred in the diatonicum diatonum system of the [-692-] ancients, to which the imperfect consonances were not admitted, as the number five was excluded. The imperfect temperament that reduces the fifth by $1/11$ c and the minor third $8/11$ c, which was later accepted in the general circular system dividing the octave into twelve equal parts and commonly called system of twelve semitones, is not very different from such a system. If one concentrates on the sixth above the octave $10/3$ and on the minor third $6/5$, so that they are preserved intact, the complex of the three fifths is out of tune by a comma. If said comma is distributed equally among the three fifths, the result of it is the irregular temperament that reduces the fifth by $1/3$ c and that is embraced by the general circular system dividing the octave into nineteen equal parts. It is very reasonable that one should take into account the perfect and imperfect consonances and that, considering their simplicity, one should distribute the comma to the three fifths and to the minor third in the excellent way indicated above.

[signum] 13. The other two temperaments considered above, namely, the third one and the second one, derive from the premise that three fifths are equivalent to a major sixth above the octave.

Given the equivalence between three sixth and a major sixth above the octave, and since a

major sixth above the octave and a fifth constitute a major third above two octaves, we deduce that four fifths are equivalent to the major third above two octave. However, such interval together with a minor sixth adds up to three octaves. Therefore, four fifths and a minor sixth add up to three octaves.

Moreover, since three fifth are worth a major sixth above the octave, while four fifths are worth a major third above the double octave, it follows that twelve fifths are equivalent to four major sixth above the octave and also to three major thirds above the double octave. [[Moreover, if we subtract from each side the quadruple octave]] Therefore, the equivalence [-693-] between four major sixth above the octave and three major thirds above the double octave, and, if one subtracts the quadruple octave from each side, between four major sixth and three major thirds with the addition of two octaves, is proven. If I add four minor thirds to one side and to the other, I find that four minor thirds plus three major third plus two octaves are equivalent to four major sixth plus four minor sixth. However, four major sixths plus four minor thirds are equivalent to four octaves; therefore, four minor thirds plus three major ones with the addition of two octaves add up to four octaves. Hence, if one subtracts two octaves from each side, four minor thirds plus three major thirds add up to two octaves.

[signum] 14. Moreover, the three divisions of two octaves into three fourths and a major sixth and into four fourths and a major thirds, and those of five octaves into four major sixths and into three minor sixths, that lead us to the three participations already discovered, depend on three assumptions, namely, that three fourths add up to a minor third above the octave, that four fourths add up to a minor sixth above the octave and, finally, that four minor thirds with the addition of an octave are equivalent to three minor sixths. It is enough to mention this without repeating an argument entirely similar to the one employed in the preceding paragraph.

[signum] 15. The conclusion that I am at pains to draw consists in the fact that only the six described distributions are possible, which originate three different participations. I lay out the double series of the fifths and of the fourths based on the letter C.

[Riccati, The laws of counterpoint, 693; text: C, G, D, A, E, B, F #, C #, G #, A #, E #, B #, B b, E b, A b, G b, F b, B 2b, E 2b, D 2b et cetera]

Of the six consonances accepted in music, I find three, namely, the fifth C G, the major sixth C A and the major sixth C E in the series of the fifths, while the three remaining, namely, the fourth C F, the minor third C E b and the minor sixth C A b in the series of the fourths. A consonance that belongs to a series does not belong to the other one, because, in order for it to be found in both, it would be necessary for the [-694-] the same letter, either simple or altered by the same sign, to occur in both the series, which, except for the base C, never occurs. It follows from this that an exact equality cannot be found except between consonances, whether simple or compound, that belong to the same series. Therefore, it follows that one can only compare the fifth and the major sixth to that effect, so that one may deduce that three fifths are aequisonant to a major sixth, the fifth and the major third, so that one may deduce that fourth fifth are aequisonant to a major third, and the major sixth and the major third, so that one may deduce that four major sixth are aequisonant to three major sixth, namely, C E G # B #.

Similarly, if one compares the three consonances that occur in the series of the fourths, namely, the fourth, the minor third and the minor sixth, one shall discover the aequisonance between three fourths and a minor third, between four fourths and a minor sixth and between four minor thirds, C e b G b B 2b D 2b, and three minor sixth, C A b F b D 2b.

Since, as I demonstrated earlier, the same number of divisions of more octaves into two species of intervals originate from the six premised mentioned above, since only six premises are possible, there can be only six divisions and three participations that distribute the comma in the best way between the two species of consonances.

[signum] 16. Since the best tuning of a musical instrument must serve in the best possible way all three of the best participations of the comma between the two species of consonances, it is

clear that, of the three aforementioned participations, one shall have to choose the one that is between the two remaining ones, because, since it contains the most perfect distribution of the comma as to two species of intervals and to their completing the octave, it is equally well adapted to both of the other excellent [-695-] participations, from whom it is equidistant. The first temperament, which reduces the fifth by $3/14$ c is half-way between the second one, which reduces it by $2/3$ c and the third one that reduces it by $7/17$ c. I observe that the second temperament reduces excessively the fifth by $2/7$ c in order to cater exactly to the reciprocal correspondence between the two thirds, the major and the minor one. On the contrary, the third temperament reduces excessively the minor third by $8/17$ c in order to match the reciprocal tuning of the fifth and of the minor sixth. What I say about an interval has to be applied to the intervals that complement it to form the octave, which have to be altered by a similar degree, but in an opposite way. The excessive alterations of $2/7$ c in the fifth and of $8/17$ c in the minor third are proportionate to the character of said perfect and imperfect consonances, since their ratio is close to the ratio 3:5, according to the principle established above. In fact, the analogy $3:5$ c: $2/7$ c: $10/21$ c gives the fourth term $10/21$ c as very close to $8/17$ c, since both these quantities amount to just less than half a comma. One must deduce that, since there are three consonances that need to be tempered together with the intervals that complement them to form the octave, the second and their participations are faulty proportionally in modifying too much the remaining one. This does not occur in the case of the first participation, where, as the only aim is to distribute the comma well between the fifth and the minor third, the remaining consonance, namely, the major third together with the minor sixth, which added to it forms the octave, either benefits from it or does not suffer because of it. It benefits in relation to the third participation, because it reduces the widening of the major third from $5/17$ c to $1/7$ c. It is not suffer in relation to the second participation, because, as this one reduces the major third by $1/7$ c, our first and most excellent participation widens it by the same amount.

[signum] 17. Therefore, we can establish an easy rule to determine the best temperament. The best temperament is the one that, as it aims to adjust together the numbers three and five, it balances proportionally the difference [-696-] of the fifth and of the fourth with the greater difference by which the imperfect consonances are altered. Of the three participations of ours that distribute the comma perfectly in relation to two consonances and to the consonances that added to them form the octave, the mentioned differences are found to be proportional only in the first one. Therefore, the rank of absolutely best participation shall belong to the first one, as it balances proportionally the adjustments of the fifth and of the minor third, as well as those of the fourth and of the major sixth. In said temperament the greater adjustment is assigned to the imperfect consonances, namely the minor third and the major third. If the adjustments of the fifth and of the minor third are perfectly proportional, it does not bother the ear that the adjustment of the major third is smaller than theirs. The same applies to the intervals that added to them constitute the octave. In fact, the ear is affected by the great adjustments rather than by the small ones.

[signum] 18. Of the temperaments suggested by music theorists, the two temperaments that come more closely to the best one are the aforementioned ones by Sauveur, which reduces the fifth by $1/5$ c, and by Giuseppe Zarlino, which reduces the fifth by $1/4$ c. The best temperament, however, approaches more the former than the latter, because the difference between $3/14$ c and $1/5$ c = $3/15$ c is smaller than the other one between $3/14$ c and $1/4$ c = $3/12$ c. I do not want to fail to point out that the best temperament is more precisely half-way between the two general and circular systems, one of 43merides invented by Monsieur Sauveur, and the other one of 31. parts invented by Cristiano Ughenio which reduces the fifth less than $1/4$ c by a more noticeable difference. Since the tuning of the musical instruments do not consist of indivisible elements, one can establish the law according to which the most exquisite tunings must not reduce the fifth by less than one fifth of a comma or more than one fourth. If one wanted to widen somewhat such boundaries, one could determine our second and third participations, which reduce the fifth by $3/17$ c and $2/7$ c as boundaries of the good tunings. These participations, as I noted above, [-697-] are equally embraced by the general circular system that divide the octave into fifty-five commas and into fifty parts. The aforementioned limits can be considered equally good, if one considers the matter in abstract and

without referring to any instrument in particular. We shall see in the fourth chapter that the tuning of the ordinary harpsichords must necessarily err towards altering the fifths too little rather than too much.

In truth Monsieur Sauveur was not far from the truth when he stated that his temperament is the best one, as it reduces the fifth by $1/5$ c. However, since he was not aware of the principles from which I deduced the best tuning through demonstration, the most convincing reasons that he adduces consist in the fact that the minor third does not tolerate to be altered more than the fifth and that the tuning of an instrument according to his system pleases the ear more than any other. We shall observe further on (Chapter 4., [signum] 9.) how we can establish that an instrument is tuned according to the system of the forty-three merides.

Second Chapter

On the general circular tempered systems

[signum] 1. General system is called the one in which all the consonant and dissonant intervals needed in music correspond upwards and downwards to any letter simple or altered by the sharp or by the flat sign. If, after a series of similar ratios for instance of the fifth had been laid out, every term of the series required a new sound in order to ensure that a particular fifth corresponded to any letter, whether simple or altered, one would have to introduce an infinite number of sounds into the system. Such issue shall be resolved if one search for the compound fifth that is very close to a compound octave, so that, if their reciprocal difference is shared, one can reasonably suppose that a particular number of fifths is equivalent to a particular number of octaves. In this way not every term of the series shall require a new sound, since one returns in the end to find sounds that are aequisonant to the ones already introduced, so that the system deserves the name of circular, as the one that [-698-] goes back to itself after a particular number of fifths or of any other number chosen freely.

[signum] 2. Once the proportion between a given interval and the octave is established, one determines by necessary consequence the proportions of all the other intervals not only with the octave, but between themselves. I shall continue to use the example of the octave and of the fifth. In fact, if one subtracts the fifth from the octave, the remainder is the fourth, and, assigned the proportion between the fourth and the octave, one discovers that the ratios from the fourth to the fifth and from the fourth to the octave are equally ascribed. If I subtract the fourth from the fifth, the remainder is the tone, and I discover its proportions with the fourth, with the fifth and with the octave. If I know the tone, I also know the relative values of the major third, which equals two tones, of the augmented fourth, which equals three tones, of the augmented fifth, which equals four tones and of the augmented sixth, which equals five tones. If I subtract the tone from the fourth, I encounter the minor third; if I subtract the tone from it, and I encounter the major semitone. The difference between the tone and the major semitone is the minor semitone. The other intervals, all diatonic or chromatic, are either intervals that added to the ones already mentioned constitute the octave or intervals that differ from the ones already known by a minor semitone. Therefore, their value can be ascertained by adding the minor semitone to the known ratios or by subtracting it from them according to the need. Who wants to progress to ascertain the measure of the enharmonic elements must subtract the value of the minor semitone from the value of the major semitone. Thus, one shall encounter the diminished second. If then one subtracts the diminished second from the minor semitone, one shall encounter the second twice diminished. One would be able to know the size of the more complex harmonic elements by continuing with the subtractions and subtracting, according to what the different premises require, either the diminished second from the twice diminished one, or, vice versa, the latter from the former, and by continuing [-699-] then such operations with a similar method, should one want to do so.

[signum] 3. The matter that I explained in general needs to be illustrated with an example to the benefit of those who are less experienced. Twelve fifths do not exceed seven octaves, except by about $12/11$ of the comma. Before I go any further, I observe that one must choose those compound fifths that are somewhat wider than the corresponding compound octaves. In fact, in this way the fifths, as any reasonable participation requires, are to be narrowed a little, so that the compound

tempered equals the multiple perfect octave. Were the compound fifth narrower than the octave, in order to balance the ratios one would incur the paradox that one would have to enlarge the difference of the comma instead of distributing it, since the imperfect consonance would be found to be altered by more than a comma. Moreover, I inform the Reader that such the results of such calculations, for instance the result that twelve fifths exceed seven octaves by $12/11$ c, are easily obtained through the logarithms. Were I to employ ordinary numbers, I would have to raise the ratio $3/2$ from the power of the fifth to the power of twelve, the ratio $2/1$ of the octave to the power of seventh and then I would have to divide the twelfth power of the fifth by the seventh power of the octave, in order to obtain the proportion by which twelve fifths exceed seven octaves. If I resort to the logarithms, all I have to do is to multiply by twelve the logarithm of the fifth and by seven the logarithm of the octave and, after subtracting the last product from the first one, I have to compare the difference with the logarithm of the comma in order to understand easily that it is equivalent to $12/11$ c. The logarithm of a given ratio is found by subtracting the logarithm of the denominator from the one of the numerator. For instance, one shall discover the logarithm of the comma $81/80$ by subtracting the logarithm of 80 from the logarithm of 81. The method of performing mathematical operations through the logarithms is well suited to the expressions that we employ with regard to the musical intervals, to their mixtures and to their [-700-] temperaments. When I say that twelve fifths exceed seven octaves by $12/11$ of a comma, this manner of speaking comprehends within itself and indicates clearly the operations that I have to perform with the logarithms. Moreover, to say that the two thirds, major and minor constitute the fifth and that a particular participation reduces the fifth by $1/11$ of a comma involves the application of the logarithms, since, as a matter of fact, one has to add together the logarithms of the two thirds, major and minor, in order to obtain the logarithms of the fifth, while one has to subtract the fourth part of the logarithm of the comma from the logarithm of the fifth in order to discover the logarithm of the tempered fifth according to the proposed participation.

[signum] 4. After the previous digressions, which were most necessary, I resume the thread of my explanation. Since twelve fifths exceed seven octaves by $11/12$ c, if every fifth is reduced by $1/11$ c, twelve tempered fifths shall equal seven octaves. I mentioned such temperament by calling it imperfect, since, in order to produce too accurate fifths, it produces thirds that are out of tune by $8/11$ c. However, this does not pertain to the present overview. Since twelve fifths equal seven octave, it follows that the fifth contains seven of the parts of whom the octave contains twelve, because in this way the fifth 7. multiplied by 12. equals the octave 12. multiplied by 7., according to the previous supposition. If one subtracts the fifth 7. from the octave 12., I am presented with the fourth 5.; if the latter is subtracted from the fifth, I find the tone or major second 2., added to which the minor seventh 10. fulfils the octave 10. The major third consisting of two tones is equivalent to 4., to which corresponds as the interval that complements it to form the octave the minor sixth 8. The tritone, or major fourth, equivalent to three tones, occupies six parts. Six parts also belong to the minor fifth which complements it to form the octave. The augmented fifth, consisting of four tones, is equivalent to eight parts, while four are left to the diminished fourth as the interval that complements the augmented fifth to form the octave. The augmented fifth consisting of five tones contains ten parts, [-701-] while two are left to the diminished third as the interval that complements it to form the octave. After I determined the values of the intervals and those of the intervals that complement them to form the octave, I proceed with the subtractions. If I subtract the tone 2. from the fourth 5., I obtain the minor third 3. If I add to the latter the major sixth 9., they produce the octave. If again I subtract the tone 2. from the minor third 3., I encounter the minor second or major semitone 1., which, subtracted from the tone 2., demonstrates to me that the minor semitone is also equivalent to 1. Both the intervals that added to the two semitones complete the octave, namely, the major seventh and the diminished octave are allotted eleven parts. The only intervals that I have not mentioned, among those employed in the counterpoint, are the augmented second and the diminished seventh, which is the interval that constitutes the octave added to the augmented seventh. Now, one can discover easily that three parts are assigned to the sum of the augmented second 2. and of the minor semitone 1., and that, consequently, nine parts must be assigned to the

diminished seventh as the interval that combines with it to form the octave.

[signum] 5. Once established the proportion between fifth and octave as 7:12, very many other methods would have lead me to determine the proportion of all the other intervals, both with the octave and with each other. However, I have employed the one that came first to my mind. Among the other consequences that depend on the proportion 7:12 between the fifth and the octave, the fundamental one in relation to the current general temperament is the fact that presents us the two semitones, major and minor, as equivalent, being represented by the same unit, since one must presume that their difference of the diminished second can be overlooked and that it has to be participated. Therefore, the part on the basis of which our system measures the octave and the other intervals is a semitone half-way between the major and the minor that can perform the role of either of them according to the circumstances. Therefore, if one remembers the structure of the chromatic system, which, [-702-] as we have seen, constitutes all the diatonic and chromatic ratios of semitones, some of them major and some of them minor, it is easy to discover that the system that we are discussing assigns to a ratio number of parts or medium semitones corresponding to the number of semitones, some of them major and some of them minor, of which it consists in the structure of the chromatic system. Thus, as the octave consists of twelve semitones, seven major and five minor, the general temperament ascribes to it twelve medium semitones, whose value is found by dividing the octave, which in every participation must be preserved intact, twelve equal parts. Therefore, the system that divides the octave into twelve parts or medium semitones is based on the structure of the chromatic system and originates from the premise that all the chromatic elements of different sort are the same, namely, the minor and major semitones. The property, according to which our system assigns the same number of parts to the ratios, albeit different, that consist of an equal number of semitones in the chromatic, derive from mixing the two semitones, major and minor, in a medium one. Thus, two parts are allotted to the tone and to the diminished third, since they consists of two semitones in the chromatic, which semitones are one major and the other minor in the tone and both major in the diminished third.

[signum] 6. Finally, the general nature of the system derives from the fact that all the ratios employed in counterpoint correspond both upwards and downwards to any sound taken as their foundation. If I count one part above a given sound, I obtain one or the other of the semitones; if I count two parts, I obtain the tone and the diminished third; if I count three, the augmented second and the minor third; if four, the major third and the diminished fourth; if five, the augmented third, an interval that is not employed, and the fourth; if six, the major fourth and the minor fifth, and so on up to the octave and beyond, if it is required, by counting [-703-] as many parts as the semitones are, so that the interval is chromatically formed.

Were one to establish different proportions from the one employed earlier, namely 7:12, one would find different general system by employing a method similar to the one that I have employed. However, let what has been said be sufficient on this matter.

[signum] 7. I move on to explain a much easier method that leads us to discover all the possible general tempered systems in a less convoluted way. Since all the intervals employed in music consists of major and minor semitones, if one suppose that the aforementioned chromatic elements correspond to each other according to simple ratios, one shall establish easily into how many parts the octave and any other interval must be divided by substituting to one or the other of the semitones the number of the parts that are assigned to it by the given ratio.

I have considered the two semitones (the major one $16/15$, and the minor one $25/24$ that constitute the minor tone $10/9$) have been considered in detail in the appropriate place, and I did not fail to point out that they are the difference between two consonances. Now, since there are only the two mentioned semitones that constitute the difference between consonances, which semitones will ever constitute the major tone $9/8$ in the most elegant manner? The ratios that follow immediately the imperfect consonances in order of simplicity are the ones that not contain an odd number larger than the number seven. As these intervals are half-way between the consonances and the dissonances, they can be called semi-consonances or semi-dissonances. I demonstrated earlier that the ratio of the augmented sixth is $7/4$, to which the diminished third $8/7$ correspond as its

complementary interval to form the octave. I also proved that the major fourth is allotted the proportion $7/5$, while the fraction $10/7$ belongs to the minor fifth, which forms the octave when added to the former interval. Finally, I have shown that the augmented second and the [-704-] diminished seventh, which added together form the octave, accept the proportions $7/6$ and $12/7$. If I compare these intervals with the consonances closest to them, I find that the major fourth $7/5$ and the fifth, the fourth $4/3$ and the minor fifth $10/7$, the augmented second $7/6$ and the major third $5/4$ and the minor sixth $8/5$ and the diminished seventh $12/7$ differ by the major semitone $15/14$. Conversely, I find that the major sixth $5/4$ and the augmented sixth $7/4$, the minor fifth $10/7$ and the fifth $3/2$, the fourth $4/3$ and the major fourth $7/5$, the diminished third $8/7$ and the minor third $6/5$ all differ by the minor semitone $21/20$. The major tone $9/8$ consists, in its most elegant form, precisely of these semitones, namely, of the major semitone $15/14$ and of the minor one $21/20$. I added in its most elegant form, because there is no shortage of other distributions both of the major tone and of the minor tone. I overlook them as they lack the property according to which all the semitones that they contain are differences of consonances or, at least, of consonances and semi-consonances.

[signum] 8. The two couples of semitones one major, and the other minor, $16/15$ and $25/24$, which form the minor tone $10/9$, and $15/14$ and $21/20$, which form the major tone $9/8$, form a median ratio with each other that is contained between the simplest ones 1:1, 2:1. On one side, the ratio $25/24$ is smaller than the ratio $16/15$, and on the other it is larger than half of the ratio $16/15$, which is half-way between the two ratios $31/30$ and $32/31$. Similar conclusions must be drawn on the semitones $15/14$ and $21/20$.

If I employ as one limit the simplest ratio of all, namely, the one of equality 1:1, which presumes the major and minor semitones to be equal, as we have seen, I come to embrace the system of 12 semitones, which is based on the chromatic structure. If I employ as another limit the ratio 2:1, according to which the major semitone is twice the minor one, and, if I substitute the values 2 and 1 instead of the aforementioned semitones, one shall find that the octave, which equals [-705-] twelve semitones, seven major and five minor, must be divided into nineteen parts. If I subtract the minor semitone 1. from the major 1. we are left with the diminished second 1.; if we subtract the latter from the minor semitone, we are left with the second doubly diminished, which equals 0. Therefore, our system presupposes that the chromatico-enharmonic elements of different species are equal (the minor semitone and the diminished second) and it takes to distribute and erase their difference (the diminished second) so that it is equal to zero. Therefore, the chromatico-enharmonic system in which the octave consists of nineteen elements of two different species, namely, twelve minor semitones and seven diminished seconds provides the basis for the system that we are discussing, a part of which is half-way between the two chromatico-enharmonic elements and it is able to represent both of them equally.

[signum] 9. The opposite suppositions of the systems of twelve and nineteen parts in relation to the diminished and twice diminished second, which are enharmonic elements, deserve to be discussed. The first system supposes the diminished second as equal to zero, while the second twice diminished is equivalent to the minor semitone. On the contrary, the second system overlooks the second twice diminished as equal to zero and renders equivalent the diminished second and the minor seventh. These opposing supposition illustrate that our systems are based on limits. In fact, we observed above (chapter 1., [signum] 12.) that the participations that they embrace err in opposite directions, one in reducing too much the minor third and privileging the fifth excessively and the other one in reducing the fifth excessively and privileging too much the minor third. Those who would like to know how much is altered a particular interval in a general system should divide the logarithm of the octave by the number of intervals required by the system, so that one can have the value of a portion of the system in expressed by a logarithm. Moreover, one must consider the number of the parts that constitute a given interval. If one multiplies by such number [-706-] the part that was found earlier, their product shall equal to the logarithm of the tempered interval. If we compare this with the logarithm of the perfect interval and we subtract the smaller one from the larger one, the resulting difference shall express the quantity by which the proposed interval is reduced or widened the system. If we compare said difference with the logarithm of the comma, we

shall discover their mutual relationship, and one shall discover the way to measure said difference through the comma and its parts.

[signum] 10. One must not even think about exceeding the established limits of the system on either side. Should one do so, one would disturb both the participation and the simplicity of the system, and, if one proceeds further in it, one would encounter systems that disrupt rather than temper the consonances. For instance, the system of fifty-three parts is the one that is almost totally equivalent to the diatonicum diatonum of the ancients, which preserved the perfect consonances in their perfect measure but altered the imperfect ones by a whole comma. The system of fifty-three parts originates from changing the nature of the two semitones, assigning to the major semitone a smaller value than the one assigned to the minor semitone and making them relate to each other according to the proportion 4:5. The semitone that is the minor semitone in our system was major in the diatonicum diatonum and, vice versa, the one that is the major semitone in our system corresponds to the minor semitone of the diatonicum diatonum. Now, who shall ever follow the mentioned system which is more complex and less in tune than the one of twelve semitones? The two systems of five and of seven parts are more simple than the ones of twelve and nineteen parts that I established as limits. These systems originate from the chromatic scale, either by supposing that the seven major semitones or the five minor semitones are equivalent to zero, which semitones added together form the octave. However, [-707-] our systems are deduced from the diatonic constitution, which constitutes the octave from five tones and two major semitones. If establish the two major semitones as equivalent to zero, I find a system of five parts, while, if I deem the tone and the major semitone, two diatonic elements belonging to a different species, to be equal, I am presented with a system consisting of seven parts. In order to discover the absurd nature of the two systems of five and seven parts, it is sufficient to observe that, since the first one requires that the major semitone equals zero, the major third, the fourth, the fifth and the minor sixth become indistinguishable from each other, while, since the latter considers the minor semitone as equal to zero, the two thirds and the two sixths become indistinguishable from each other. This is not a way to temper, but to disrupt the consonances. The system of five parts, which exceeds not only the boundaries of the system of twelve semitones, but also those of the system of fifty-three parts widens the fifth by $\frac{5}{6} c$ and the major third by $\frac{13}{3} c$, while it reduces the minor fifth by $\frac{7}{3} c$. The system of seven parts falls into the opposite error, as it exceeds the established limit of the system of nineteen parts. In fact, it reduces the fifth by $\frac{7}{9} c$ and widens the third by $\frac{4}{3} c$. Therefore, it is established with certainty that the limits that contain all the reasonable participations are on one side the system of twelve semitones, and on the other side the system of nineteen parts.

[signum] 11. it is necessary to discover the simplest general system while also investigating the participations on which they are based, all of whom shall prove more or less good. I invite the reader to select the simplest proportions between these two, 1:1 and 2:1, in which, according to the systems that constitute the limits, the major and minor semitone are made to correspond one to the other. This proportion shall be achieved by adding together the first terms and the second terms of such proportions, so that it shall be expressed by the numbers 3:2. if one substitutes the established values 3 and 2 in the chromatic scale instead of the major and minor semitones, the seven major semitones shall be equivalent [-708-] to twenty-one parts and the five minor semitones to ten parts, while the octave shall be found to be distributed into thirty-one parts, according to the system invented by the famous Cristiano Ughenio. If I remove the minor semitone 2. from the major semitone 3., the remainder is the diminished second 1. The proportion 2:1 between the minor semitone and the diminished second could have been determined by adding together the first terms and the second terms of the ratios 1:0 and 1:1, in which the two chromatico-enharmonic relate to each other, according to the systems that constitute the limits. If we substitute the values 2 and 1 to the aforementioned elements in the chromatico-enharmonic scale, we are presented once again with the distribution of the octave into thirty-one parts. Finally, if I subtract the diminished second 1 from the minor semitone 2, I am presented with the twice diminished second 1, which the present systems considers equivalent to the diminished second. Such proportion of equality between the enharmonic elements of different species could have been discovered by adding that first terms and

the second terms of the proportions 1:1 and 1:0, which are ascribed to those elements by the systems that constitute the limits. We have seen earlier (book 1., chapter 5., [signum] 24.) that the enharmonic constitution divides the octave into thirty-one elements of two classes, namely, nineteen diminished seconds and twelve twice diminished seconds. If these elements are presumed to be equal, one finds the most natural origin of the general system that divides the octave into thirty-one parts. Therefore, the three systems of twelve, nineteen and thirty-one parts respectively that are the simplest among the reasonable ones, are based on the three constitutions, the chromatic, the chromatico-enharmonic and the enharmonic, which the aforementioned systems present to us when the elements of different species that constitute the octave are required to be equal. In the enharmonic itself, one can note easily also the two that constitute the limits, apart from the system of thirty-one parts. If I set the nineteen diminished seconds as equal to zero, I discover the system of twelve parts. [-709-] If a set as equal to zero the twelve twice diminished seconds, I am presented with the system of nineteen parts. The number thirty-one, therefore, is produced by the addition of the numbers twelve and nineteen. This observation suggests an easy method to discover the simplest system between two given ones. If one adds together the number of parts into which the proposed systems divide the octave, one shall obtain the number of the parts into which the new system is required to divide the octave. We shall find the participation embraced by the general system of thirty-one parts by performing the operations indicated above ([signum] 9.). Thanks to them we shall find that our system is very similar to the participation mentioned by Zarlino, which reduces the fifth and the minor third by $1/4$, while it leaves the major third untouched. The system of 31. parts reduces the fifth a little less and the minor third a little more than $1/4$ c, but the difference is so small that the widening of the major third, which is utterly inaudible, does not reach $1/27$. c.

[signum] 12. added together the two numbers twelve and thirty-one, we are presented with the system of forty-three merides invented by Monsieur Sauveur, whose participation is half-way between those of the systems of twelve semitones and of thirty-one parts. In this system the ratio between the major and minor semitones is expressed by the proportion 4:3, the one between the minor semitone and the diminished second is expressed as 3:1, while the one between the diminished second and the twice diminished second is expressed by the proportion 1:2. Said proportions derive from adding together the first and second terms of the proportions embraced by the system of twelve and thirty-one parts. If the logarithms ascribed to the consonances in the present system are compared with those of the perfect consonances, one shall discover that it reduces the fifth and widens the major third for a value close to $1/5$ c, while it reduces the minor third by $2/5$ c.

[signum] 13. If I add together the numbers thirty-one and nineteen, I find the system of 50 parts suggested to the public by Signor Henfling. This system accepts a [-710-] temperament that is half-way between the ones of the system of thirty-one and nineteen parts. The elements of various sort correspond to each other according to the following proportions, which can be obtained by following the usual method. The major semitone corresponds to the minor one according to the ratio 5:3, the minor semitone corresponds to the diminished second according to the ratio 3:2 and the diminished second to the twice diminished second according to the ratio 2:1. Performed the due calculations, we shall discover that the system of fifty parts reduces the fifth a little less than $2/7$ c, the major third by $1/9$ c and the minor third by $1/6$ c. This does not depart very much from my second participation, attested also by Giuseppe Zarlino, which reduces the fifth by $2/7$ c and the two thirds, major and minor, by half of that amount.

[signum] 14. I lay out the five systems that I found, respectively of twelve, forty-three, thirty-one, fifty and nineteen parts, and I observe that between one system and the one next to it one can find four new systems.

The first one that I find is the one of the comma $55 = 12 + 43$. Said system reduces the fifth by nearly $3/17$ c and the minor third by nearly $8/17$ c, while it widens the major third by $5/17$ c, thus resembling my third participation. The major semitone and the minor form the ratio 5:4, the minor semitone and the diminished second form the ratio 4:1, the diminished second and the twice

diminished second form the ratio 1:3.

Secondly, I consider the system of sixty-nine parts ($69 = 50 + 19$) which reduces the fifth by a little more than $2/7$ c, the major third by $1/6$ c and the minor third by $1/8$ c. We shall realise easily that our second participation is half-way between the two that are accepted by the systems of fifty and sixty-nine parts. In the first one the fifth is a little less narrow than $2/7$ c, while in the second one it is a little more narrow than $2/7$ c. The chromatic elements, those that are chromatico-enharmonic and the enharmonic ones of the system of sixty-nine parts correspond to each other in the following ratios. The ratio between the major semitone and the minor one is 7:4, the ratio between the minor semitone and the diminished second is 4:3, while the ratio between the diminished second and the twice diminished one is 3:1.

[-711-] Thirdly, I look for the simplest system between those of forty-three and thirty-one parts. It consists of seventy-four parts ($74 = 43 + 31$). The temperament embraced by it resembles very closely the best one, which reduces the fifth by $3/14$ c, the minor third by $5/14$ c and widens the major third by $1/7$ c. In our system the reduction of the fifth is a trifle more than $3/14$ c, while the alteration of the two thirds are slightly less than $5/14$ c and than $1/7$ c. The widening of the major third is contained almost exactly between $1/7$ c and $1/8$ c. The ratio between the minor semitone and the diminished second is 5:2, while the ratio between the diminished second and the twice diminished one is 2:3.

Finally, the simplest system between the one of thirty-one parts and the one of fifty parts is the one of eighty-one parts ($81 = 31 + 50$). The participation to which it belongs is very close to the one by Zarlino that reduces the fifth and the minor third by $1/4$ c, while leaves the major third untouched. Our system reduces the fifth a little more than one fourth of a comma, namely by $5/19$ c, the minor third a little less than $4/19$ c and the major third by the inaudible quantity $1/19$ c. If one compares together the two systems of thirty-one and eighty-one parts, one can discover easily that their participations are half-way between the one by Zarlino just mentioned, since the former reduces the fifth by a trifle less than one fourth of a comma and the latter by a trifle more than one fourth of a comma. I write here in succession the ratios that represent the reciprocal relationships between the chromatic elements, those that are chromatico-enharmonic and the enharmonic ones in the present system.

The major semitone and the minor semitone are expressed by the ratio 8:5,
the minor semitone and the diminished second are expressed by the ratio 5:3, and
the diminished second and the twice diminished second are expressed by the ratio 3:2.

[Riccati, The laws of counterpoint, 711; text: Progressione di Sistemi generali. 12. 55. 43. 74. 31. 81. 50. 69. 19.]

[signum] 15. The systems that lay at the same distance from the one of thirty-one parts have a particular property thanks to which the enharmonic elements correspond to each other in contrary ratios. I took care to create the systems in such a sequence [-712-] that would alert the Reader to this particular property. It was said clearly earlier on([signum] 9., [signum] 11.) that the systems of twelve semitones and of nineteen parts assign to the diminished second and to the twice diminished one the reciprocal ratios 0:1, 1:0. Similarly, the system of forty-three merides presumes that the ratio between the diminished second and the twice diminished second is 1:2, while the system of fifty parts presumes it to be 2:1, which is the opposite of the former.

We shall find that the same occurs in all the other systems that are equally distant from the one with thirty-one parts, if one compares with each other the aforementioned proportions of the enharmonic elements. The infinite distance between the ratios 0:1 and 1:0 accepted by the systems that represent the boundaries and also the very notable difference between the frequently reciprocal proportions ascribed by the systems, even the good ones, to the enharmonic elements illustrates how arbitrarily we can establish such proportions, without deviating from the best participations and in particular from the one that we have set ourselves to follow. We have seen that even the systems of thirty-one and eighty-one notes fall back appropriately on to Zarlino's temperament that reduces the

fifth and the minor third by $1/4$ c and preserves the major third perfect, although there is a considerable difference between the ratios 1:1 and 3:2 assigned by the aforementioned systems to the enharmonic elements. One has to deduce that any intermediate ratio between the two mentioned ones would lead us to the same participation.

[signum] 16. The same method that proved useful to discover the seven systems intermediate between the two extreme ones of twelve and nineteen parts can be applied to find an infinite number of other systems. The reason why I considered it appropriate not to go any further consists in the fact that, if one looks for the greatest ease without a high degree of perfection, we can rely on the extreme systems of twelve and nineteen notes; if I look for the best temperament, I find it with accuracy in the system of seventy-four parts, or, with greater simplicity, in the systems of forty-five and thirty-one notes with sufficient precision. After this, it proves useless and irrational to consider the other systems of fifty-five, eighty-one, fifty, and sixty-nine notes and all the ones [-713-] that are more and more complex that fall outside the two ones of forty-three and thirty-one notes, because such systems are lacking both simplicity and perfection. Nor it is worth to aim at the systems contained between forty-three and seventy-four parts and between seventy-four and thirty-one parts, because they shall never be physically exact, often they shall be less perfect and always more complex than the one of seventy-four parts. Therefore, the general systems that deserve separate and detailed consideration are five, namely the two extreme ones of twelve and nineteen parts, because to their extreme and not unreasonable simplicity, the one of seventy-four parts, because of its scrupulous exactness, and, finally, the two of forty-three and thirty-one parts invented by Signori Sauveur and Ughenio, because their closeness to perfection coupled with their convenient simplicity.

[signum] 17. After we have explained the method to discover all the general tempered systems moving gradually from the most simple to the most complex, let us discuss the matter a little in the opposite way, and, given a participation, let us search for the general system that is suited to it. Let us reduce the logarithm of the fifth by the portion of the logarithm of the comma required by the participation. Then, let us subtract the logarithm of the tempered fifth from the logarithm of the octave, in order to obtain the logarithm of the tempered fourth. Let us continue then these subtractions in the order that I followed earlier ([signum] 2. and [signum] 4.) until the two enharmonic elements of the diminished second and the twice diminished second are expressed as logarithms. Observe the proportion among the simplest said elements correspond to each other with sufficient approximation, and the system that derives from this shall embrace the proposed participation with physical precision.

An example shall clarify the matter entirely, as it is required. Let us set out to determine the general system that satisfies the best temperament that reduces the fifth by $3/14$ c and the minor third by $5/14$ c, and that widens the major third by $1/7$ c. I reduce the logarithm of the fifth 1760913 by $3/14$ of the logarithm of the comma 53950, namely, by the logarithm 11561, and I obtain the logarithm of the tempered fifth 1749352. If I subtract this from the logarithm of the [-714-] octave, the remainder 1260948

expresses the tempered fourth. If I remove the fourth from the fifth, I obtain the tone 488404, which, doubled and subtracted from the fourth, expresses the major semitone, which corresponds to the logarithm 284140. If I subtract the major semitone from the tone, and I obtain as a remainder the minor semitone 284140. If I subtract the major semitone from the tone, I obtain as a remainder the minor semitone 204264. The difference between the two semitones is the diminished second 79896, while the difference between this and the minor semitone is the twice diminished second 124388. One discovers at first sight that the proportion among the simplest one that is sufficiently close to the most complex one of our enharmonic elements (diminished second and twice diminished second) is the proportion 2:3, which leads us to the general system that divides the octave into seventy-four parts, which, as we have seen earlier at [signum] 14., matches the perfect temperament very accurately.

[signum] 18. I invite those who would like to know the reason why such great precision is necessary in substituting a simple ratio to the compound one in which the harmonic elements

correspond to each other according to a given participation, to make with me the following observations. Let us lay out a double series of fifths and fourths.

[Riccati, The laws of counterpoint, 714; text: C, G, D, A, E, B, F #, C #, D #, A #, E #, B #, F 2#, C 2#, G 2#, D 2#, A 2#, E 2#, B 2#, C, F, B b, # b, D b, G b, F b, B 2b, E 2b, A 2b, D 2b]

Observe that a diminished C D 2b becomes equivalent to twelve fourths over the distance of five octaves. If, according to the system of twelve semitones, we presume that the diminished seventh equals zero, five octaves are equivalent to twelve fourths, and, consequently, the proportion between the octave and the fourth is 12:5, which is the one accepted by the aforementioned system. Now, since the octaves are not affected by the temperament, the alteration that the diminished second receives moving from the compound proportion to the simple one, is the same that the diminished second receives above the fifth octave. However, our intervals is equivalent to twelve fourths, hence each tempered fourth [-715-] is allotted only one twelfth of the aforementioned alteration. Therefore, one can see clearly that, while it is considerable the alteration of the diminished second, the one of the fourth can be overlooked very easily.

I invite the Reader to note that nineteen fifths C B 2# are equivalent to a twice diminished second above the eleventh octave. If, as the system of nineteen parts requires, the second twice diminished is presumed equal to zero, nineteen fifths are equal to eleven octaves, and, consequently, the octave and the fifth constitute the proportion 19:11, as the aforementioned system presumes. Therefore, since the twice diminished second is aequisonant to nineteen fifths, each fifth receives only one nineteenth of the alteration by substituting the aforementioned simple ratio to the compound one. Moreover, since the fourth and the fifth have to be altered in the opposite direction by the same amount, it follows that the twelfth part of the alteration of the diminished second and of the nineteenth of the twice diminished second must equal to the same quantity. One deduces from this that the alterations of our harmonic elements form the ratio 12:19, so that nineteenth diminished seconds become wider or narrower by a measure is the equivalent but opposite to the measure by which twelve twice diminished seconds become narrower or wider, in order to preserve the integrity of the whole octave, which consists of the enharmonic system of nineteenth diminished seconds and twelve twice diminished seconds. The thirty-first part of the sum of the aforementioned alterations is equivalent to one twelfth of the difference that occurs in the diminished second, which is equal to one seventeenth of the difference of the twice diminished second, since the number thirty-one is equivalent to the sum of the numbers 12 and 19. [[and as I have just said in thirty-one elements of two different species.]]

The system of seventy-four parts, derived from establishing the proportion 2:3 between the diminished second and the twice diminished one, assigns the value 81359 to the diminished second and the value 122039 to the twice diminished one. [-716-] If one compares these values with the correspondent ones established by the best temperament, I find that the general system widens the diminished second by 1483, thirty-sixth part of the comma, and, conversely, the twice diminished second is reduced by 2349, twenty-third part of the comma. These alterations correspond to each other according to the ratio 12:19, as one can work out with ease, and, if they are added together, the result is the logarithm $3832 = 1/14 c$, which is the quantity by which the general system of seventy-four parts differs from the proportion assigned by the best temperament to the enharmonic elements. If I divide said logarithm by thirty-one, the quotient $124 = 1/435 c$ teaches me that the general system reduces the fifth and widens the fourth too much by such very small amount. If I compare the best temperament, which reduces the fifth by $3/14 c$ with the one by Zarlino, which reduces the fifth by $1/4 c$, the difference is $1/28 c$. Therefore, if I widen the reduction to the fifth by $1/28$, I move from the best temperament to the aforesaid one by Zarlino. Now, by embracing the general system of seventy-four parts, I do not depart from the most exquisite participation but for $2/31$ of the mentioned difference $1/28 c$, and, consequently, I adhere to that participation with the greatest physical accuracy that can be expected with reason in this sort of research.

[signum] 19. As I have shown that, if we divide the octave by thirty-one, which is the

number of the elements in which the enharmonic divides the octave, the quantity, by which the general system alters the proportion of the enharmonic elements of the participation (which quantity is equivalent to the sum of the alterations of those elements) turns out to be the alteration produced by the general system in the fifth, similarly, I could demonstrate by employing a similar method that, established a simple proportion between the elements of any other chromatico-enharmonic, chromatic or diatonic constitution, then I shall discover the measure of the enlargement or reduction caused [-717-] by the general system in the tempered fifth, when I divide the sum of the alterations of the two temperate elements by the number representing the quantity of the elements in which the proposed constitution distributes the octave. If the elements are chromatico-enharmonic, I shall have to divide the sum of the alterations of said elements by nineteen; if they are chromatic, I shall have to divide it by twelve; if they are diatonic, I shall have to divide them by seven, because the mentioned constitutions form the octave from those particular numbers of elements. Hence one easily understands the advantage that one derives from relying on the enharmonic system to identify successfully the general system that suits the proposed participation. Given that one is equally inaccurate in substituting the simple proportions to the complex ones in which the couple of the tempered elements belonging to the various systems correspond to each other, the general systems that originate from said simple proportions shall come closer to the given temperament, the greater is the number by which said error is distributed, so that the tempered fifth receives the smallest possible alteration. If I employ the enharmonic elements, I alter the tempered fifth by one thirty-first of the aforementioned error; if I employ the chromatico-enharmonic elements, I alter it by one nineteenth; if I employ the chromatic elements, the alteration ascends to the value of one twelfth, while if, finally, I employ the diatonic elements the alteration of the tempered fifth arrives to one seventh of the error, which is presumed constant.

One should add that, as the chromatico-enharmonic elements, the chromatic ones and the diatonic ones become gradually larger, it is easy to incur an more considerable error and one that is proportionate to the size of those elements when one attempts to reduce them from a complex proportion to a simple one. The reason is that the alteration of the fifth shall grow twice as much, since the error is larger and it has to be divided into a smaller number of [-718-] parts.

[signum] 20. In fact, should one decide to determine immediately the proportion, for instance, between the tempered fifth and the octave expressed in numbers as much as possible simple, one should know, if one has not been informed of this before, that it will be difficult to encounter the proportion that would lead to the general system that accepts with the appropriate precision the proposed participation. In fact, as they begin to consider the general extreme systems of twelve and nineteen notes, they assign the infinitely distant ratios 0:1 and 1:0 to the enharmonic elements of the diminished second and of the twice diminished second. In the extremely wide field that extends from one to the other, one can, as we have seen, choose the one that satisfies the temperament the most with ease. This does not occur in the case of the ratios in which the fifth and the octave correspond to each other in the same way as the extreme system. In the system of twelve semitones the fifth and the octave form the ratio 7:12 or 77:132, while in the one of nineteen parts they form the ratio 11:19 or 77:133. Therefore, it is sufficient to add the number one to the number 132 to move by leap from an extreme system to the other one. I invite the Reader to imagine the precision required in substituting a simple proportion to the complex one between the tempered fifth and the octave, and how difficult it is to perceive it. The need for the aforementioned exactness depends on the fact that the difference between the true proportion and the one that is substituted to it is ascribed all to the fifth, since the octave must remain untouched, as it is known. The mere error of $\frac{1}{28} c$ separates the best participation from the one by Zarlino, which reduces the fifth by $\frac{1}{4} c$, as I have observed not much earlier.

[signum] 21. Now I have to say something on the scales of the general systems. Every general system consists of as many non-aequisonant sounds as the parts into which the system itself divides the octave. These sounds are all contained within an orderly series of fifths, whose number is the same of the number of the parts that constitute the octave minus [-719-] one. Thus, a series of

eleven fifths shall contains the non-aequisonant sounds of the system of twelve semitones, while a series of eighteen fifths shall contain the non-aequisonant sounds of the system of nineteen parts. If one lengthens the series of the fifths, one shall never find new sounds, but different names shall be assigned to the sounds already introduced. In any system the same sounds represents several letters that are at the distance of a number of fifths equivalent to the parts that constitute one or more octaves. In the system of twelve semitones the same note is employed as C, as B # and as D 2b, because B # lays twelve fifths upwards from C, while D 2b lays twelve fifths downwards from C. The system by Ughenio would employ the same key for the altered letters C 2# and E 3b, which lay at the distance of thirty-one fifths. One of the different names of the same sound is the main one and it is employed for the most part, while the others are more or less subordinate according to whether they are employed more or less frequently. The more a letter is modified, particularly by the repeated sharp and flat signs, the less it is employed. I do not believe to have ever found an example of the double flat in a musical composition. The following method proves useful to ascribe the simplest and principal names to the non-aequisonant sounds of a system. I subtract from the number of the different sounds belonging to the given system the seven diatonic sounds contained in a series of sixth fifths beginning with the letter F and ending with the letter B. If the remainder is an even number, I divide it in half; if it is an odd number, I divide it by the number one. I add above the letter B the number of units contained in the half or in the larger part. These units produce the simple sharps and the compound ones. I add as many fifths as the number of unit contained in the half or in the smaller part below the letter F. These produce the simple flats and the compound ones. Thus, the whole sum of all the sounds, some of the altered by the flats, some [-720-] natural and some altered by the sharps is equal to the number of the parts through which the octave is measured by the general system. These sounds then are expressed in the most simple way because sharps and flats are introduced in the same way or almost in the same way. When the aforementioned remainder is off, I add an extra sound on the side of the sharps, because sharps are employed in music more than flats. If we indicate the sounds that are not natural partly with the sharp signs and partly with the flat signs, we escape the sharps and the flats too distant and too often repeated, which one would encounter if, taken the letter C as the base all the fifths were taken upwards or downwards. I invite the Reader to observe the different and non-aequisonant sounds expressed in the simplest form and contained in the general systems of twelve, fifty-five, forty-three, seventy-four, fifty and nineteen parts, whose scales I want to extend.

[-721-] [Riccati, The laws of counterpoint, 721; text: C, G, D, A, E, B, F #, C #, G #, D #, A #, E #, B #, F 2#, C 2#, G 2#, D 2#, A 2#, E 2#, B 2#, F 3#, C 3#, G 3#, D 3#, A 3#, E 3#, B 3#, F 4#, C 4#, G 4#, D 4#, A 4#, E 4#, B 4#, F 5#, C 5#, G 5#, D 5#, A 5#, E 5#, B 5#, F, B b, E b, A b, D b, G b, C b, F 2b, B 2b, E 2b, A 2b, D 2b, G 2b, C 2b, F 3b, B 3b, E 3b, A 3b, D 3b, G 3b, C 3b, F 4b, B 4b, E 4b, A 4b, D 4b, G 4b, C 4b, F 5b, B 5b, E 5b, A 5b, D 5b, G 5b, C 5b, F 5b, Sistema di 12 Semituoni, parti 19. 31. inventato dall'Ughenio. 43 Meridi, da Monsieur Sauveur. 50. Henfling, 55. Comma. 74. che accetta l'ottimo temperamento.]

[signum] 22. The last two sounds in any system, one on the side of the sharps and the other one on the side of the flats, form a fifth. Therefore, we have a circulation of as many fifths as many are the parts that constitute [-722-] the octave. For instance, in the system of twelve semitones, the sounds indicated by the letters G # and E b correspond to each other at the fifth, because the same sound corresponds to the letters E b and D #, and G # D # is a fifth. This observation allows us to see clearly that the general systems return onto themselves and that one retraces the same steps and hears again the same sounds after a precise number of fifths. If the series of the fifths is circular, it follows that the same occurs in the case of any other interval and element, which consist all of fifths taken upwards or downwards with the subtraction of the appropriate octaves. I noted elsewhere that the fifths taken downwards are equivalent to the fourths taken upwards. If one takes as the base the letter C, two fifths produce the major second C D, three fifths the major sixth C A, four fifths the major third C E etcetera. If we move on to the fifths taken downwards or to the fourths taken

upwards, two fourths correspond to the minor seventh C B b, three fourths correspond to the minor third C E b, four fifths to the minor sixth C A b etcetera. If one applies the circulation with twelve fifths to the system of twelve semitones, three major thirds and four minor thirds, which are aequisonant to twelve fifths, complete the circulation by returning to themselves.

[signum] 23. After these preliminary considerations it shall prove easy to dispose in order within the limits of the octave C c all the sounds of any general system by assigning to them a double value expressed both by the parts that measure the octave and by the elements of a particular constitution. I shall employ of the chromatic elements of the major and minor semitone indicating them with the letters (S) and (s). As all the elements and all the intervals consist of fifths taken upwards or downwards or of fifths and fourths all taken upwards with the subtraction of the appropriate octaves, as long as I know the values of the octave, of the fifth and of the fourth, I shall discover the values of all the other intervals. Once established the general system, I become aware of how many parts constitute the octave, which is expressed as $7(S) + 5(s)$ with [-723-] the aid of the chromatic elements. The general system also teaches us about the number of the parts that are allotted to the semitones (S), (s). Therefore, one discovers easily how many parts are assigned to the fifth, $4(S) + 3(s)$ and to the fourth $3(S) + 2(s)$. Let us presume that we want to determine the values of the sounds belonging to the system of Ughenio, which divides the octave into thirty-one parts and are contained within the octave C c. In our system (S) equals 3 and (s) equals 2, with the understanding that the numbers indicate the thirty-one parts of the octave, hence $7(S) + 5(s) = 31$. If we substitute the values 3. and 2. instead of the Semitones (S) and (s), one shall find that the fifth, $4(S) + 3(s)$, equals 18. and the fourth, $3(S) + 2(s)$ equals 13. This data are sufficient to assign its appropriate value to every sound. If one subtracts the octave, $7(S) + 6(s) = 31$, from two fifths, $8(S) + 6(s) = 36$, one obtains the major second, C D = (S) + (s) = 5. I also add a new fifth D A = $4(S) + 3(s) = 18$, and I encounter the major sixth C A = $5(S) + 4(s) = 23$. If I add again the fifth A E = $4(S) + 3(s) = 18$, I obtain the interval $9(S) + 7(s) = 41$; if I subtract the octave $7(S) + 5(S) = 31$ from it, I am left with the major third C E = $2(S) + 2(s) = 10$. One should proceed with the same method by adding fifths and subtracting octaves when it is needed, until one discovers at last the value of the sound A 2# on the side of the ascending fifths. One will find the [[sounds]] values of the sounds contained in the series of the descending fifths by adding one fourth to another one and by subtracting the octave appropriately. Two fourths form the minor seventh C B b, three fourths minus the octave form the minor third C E b etcetera up to the last interval C G 2b. Once we have discovered all non-aequisonant sounds that belong to the system of Ughenio or to any other system within the terms of the octave C c, said values shall teach us to order the scales of the aforementioned system starting from the base C and moving on to those ratios that contain one, two, three parts of the system etcetera.

The tables containing the scales of the general [-724-] systems of twelve, fifty-five, forty-three, seventy-four, thirty-one, fifty and nineteen parts have been distributed by me within seven columns. The first one contains the natural letters and those modified by the simple sharps and flats, simple and repeated. One shall often find several letters in the same place. These letters indicate the simplest forms in which the same sound can appear. For instance, in the system of twelve sounds the same sound is employed as C # and as D b. In the second column I have registered the names of the elements and of the intervals employed in music, whose ratios one can see laid out in order in the third column. The fourth, fifth and sixth column contain the values of the elements and of the intervals. Such values are expressed in the fourth column by means of the major and minor semitones; in the fifth column they are expressed as the number of the parts into which the octave is divided, while in the last column I present the difference by which the logarithms of the tempered ratios are larger or smaller than the logarithms of the perfect ones. Their excess is indicated with the sign + and their defect is indicated with the sign -. If we compare these differences with the logarithm of the comma 53950, one discovers easily to which and how many portions of said comma they correspond, for instance, whether they correspond to one third, to one four or to three fourteenth of it etcetera.

[Riccati, The laws of counterpoint, 725; text: Sistema generale temperato, che divide l'Ottava in parti 12. o sia in dodici Semituoni medj, in cui si suppone (S) = (s) = 1. B #, C, D b, C #, D b, C 2#, D, E 2b, D #, E b, D 2#, E, F b, E #, F, G 2b, F #, G b, F 2#, G A 2b, G #, A b, G 2#, A B 2b, A 2#, B, C b, Semituono minore, maggiore, Tuono, Terza, diminuita, superflua, Quarta, Quinta, Sesta, Settima, Ottava, 1, 25/24, 21/20, 16/15, 15/14, 10/9, 9/8, 8/7, 7/6, 6/5, 5/4, 32/25, 4/3, 7/5, 10/7, 3/2, 25/16, 8/5, 5/3, 12/7, 7/4, 16/9, 9/5, 28/15, 15/8, 2/1, - (S) + (s), (S) - (s), 2(s), (S) + 9s, 2(s), (S) + 2(s), 2(S) + (s), (S) + 3(s), 2(S) + 2(s), 3(S) + (s), 2(S) + 3(s), 3(S) + 2(s), 4(S) + (s), 3(S) + 3(s), 4(S) + 2(S), 3(S) + 4(s), 4(S) + 3(s), 5(S) + 2(s), 4(S) + 4(s), 5(S) + 3(s), 4(S) + 5(s), 5(S) + 4(s), 6(S) + 3(s), 5(S) + 5(s), 6(S) + 4(s), 5(S) + 6(s), 6(S) + 5(s), 7(S) + 4(s), 6(S) + 6(s), 7(S) + 5(s), 8(S) + 4(s), 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 00000, 250858, 501717, 752575, 1003433, 1254292, 1505150, 1756008, 2006867, 2257725, 2508583, 2759442, 30103000, + 73570, + 35965, - 29429, - 48774, + 44142, - 09808, - 78201, + 83107, - 39237, + 34333, - 68666, + 04905, + 43870, - 43870, - 04905, + 68666, - 34333, + 39237, - 83107, + 78201, + 09808, - 44142, + 487774, + 29429]

[Riccati, The laws of counterpoint, 726; text: Sistema generale temperato, che divide l'Ottava in parti 55, ponendo (S) = 5, (s) = 4. C, D 2b, A 4# E 4 b, B 3 #, C #, D b, E 3b, B 3#, C 2 #, D, E 2b, B 4#, F 3 b, C 3#, D #, E b, F 3b, C 4 #, G 4b, D 2#, E, F b, G 3b, D 3#, E #, F, G 2b, D 4#, A 4b, E 2#, F #, G b, A 3b, E 3#, F 3#, G, A 2b, E 4#, B 4b, F 3#, G #, A b, B 3b, F 4#, C 4b, G 3#, A, B 2b, C 3b, G 3#, A #, B b, C 3b, G 4#, D 4b, A 2#, B, C b, D 3b, A 3#, B #, C, Semituono minore, maggiore, Tuono, Terza, diminuita, superflua, Quarta, Quinta, Sesta, Settima, Ottava, 1, 25/24, 21/20, 16/15, 15/14, 10/9, 9/8, 8/7, 7/6, 6/5, 5/4, 32/25, 4/3, 7/5, 10/7, 3/2, 25/16, 8/5, 5/3, 12/7, 7/4, 16/9, 9/5, 28/15, 15/8, 2/1, (S) - (s), - 2(S) + 3(s), 2(S) - 2(S), - (S) + 2(s), (s), (S), 2(S) - (s), - (S) + 3(s), 2(s), (S) + (s), 2(s), - (S) + 4((s), 3(S) - (s), 3(s), (S) + 2(s), 3(S), 4(s), 4(S) - (s), (S) + 3(s), 2(S) + 2(s), 3(S) + (s), 4(S), (S) + 4(s), 2(S) + 3(s), 3(S) + 2(s), 4(S) + (s), (S) + 5(s), 5(S), 2(S) + 4(s), 3(S) + 3(s), 4(S) + 2(s), 5(S) + (s), 2(S) + 5(s), 3(S) + 4(s), 4S + 3s, 5S + 2s, 2S + 6s, 6S + s, 3S + #, 4S + 4s, 5S + 3s, 6S + 2s, 3S + 6s, 7S + s, 4S + 5s, 5S + 4s, 6S + 3s, 7S + 2s, 4S + 6s, 5S + 5s, 6S + 4s, 7S + 3s, 4S + 7s, 8S + 2s, 5S + 6s, 6S + 5s, 7S + 4s, 8S + 3s, 5S + 7s, 6S + 6s, 7S + 6s, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 00000, 54733, 109465, 164196, 216931, 273664, 328396, 383129, 437862, 492595, 602060, 546793, 711525, 766258, 820991, 875724, 930456, 985189, 1039922, 1094655, 1149387, 1204120, 1258853, 1313585, 1368318, 1423051, 1477784, 1532516, 1587249, 1641982, 1696715, 1751447, 1806180, 1860913, 1915645, 1970387, 2025111, 2079844, 2134576, 2189309, 2244042, 2298775, 2353507, 2408240, 2462973, 2517705, 2572438, 2627171, 2681904, 2736636, 2791369, 2846162, 2900835, 2955567, 3010300, + 41643, + 07038, - 06623, - 25969, + 35020, - 18930, - 30591, + 42057, - 25554, + 16089, - 32177, + 09466, + 16504, - 09466, + 32177, - 16089, + 25554, - 42057, + 32591, + 18930, - 35020, + 25969, + 06623]

[-729-] [Riccati, The laws of counterpoint, 729; text: Sistema generale temperato di Monsieur Sauveur, in cui l'Ottava si divide in parti 43., e ponesi (S) = 4, (s) = 3. C, D 2b, B 2#, C #, D b, B 3#, E 3b, C 2#, D, E 2b, C 3#, F 3b, D #, E b, F 2b, D 2#, E, F b, D 3#, G 3b, E #, F, G 2b, E 2#, F #, G b, E 3#, A 3b, F 2#, G, A 2b, E 4#, F 3#, G #, A b, B 3b, A, B 2b, G 3 #, C 3b, A #, B b, C 2b A 2#, B, C b, A 3#, D 3b, B #, C, Semituono minore, maggiore, Tuono, Terza, diminuita, superflua, Quarta, Quinta, Sesta, Settima, Ottava, 1, 25/24, 21/20, 16/15, 15/14, 10/9, 9/8, 8/7, 7/6, 6/5, 5/4, 32/25, 4/3, 7/5, 10/7, 3/2, 25/16, 8/5, 5/3, 12/7, 7/4, 16/9, 9/5, 28/15, 15/8, 2/1, (S) - (s), - (S) + 2(s), (s), (S), - (S) + 3(s), 2(S) - (s), 2(s), (S) + (s), 2(S), 3(s), 3(S) - (s), (S) + 2(s), 2(S) + (s), 3(S), (S) + 3(s), 2(S) + s(s), 3(S) + (s), (S) + 4(s), 4(S), 2(S) + 3(s), 3(S) + 2(s), 4(S) + (s), 2(S) + 4(s), 3(S) + 3(s), 4(S) + 2(s), 2(S) + 5(s), 5(S) + (s), 3(S) + 4(s), 4(S) + 3(s), 5(S) + 2(s), 3(S) + 5(s), 4(S) + 4(s), 5(S) + 3(s), 3(S) + 6(s), 6(S) + 2(s), 4(S) + 5(s), 5(S) + 4(s), 6(S) + 3(s), 4(S) + 6(s), 7(S) + 2(s), 5(S) + 5(s), 6(S) + 4(s), 7(S) + 3(s), 5(S) + 6(s), 6(S) - 5(s), 7(S) + 4(s), 5(S) + 7(s), 8(S) + (3)s, 6(S) + 6(s), 7(S) + 5(s), 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22,

23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 00000, 70007, 140014, 210021, 280028, 350035, 420042, 490049, 560056, 630063, 700070, 770077, 840084, 910091, 980098, 1050105, 1120112, 1190119, 1260126, 1330133, 1400140, 1470147, 1540154, 1610161, 1680168, 1750175, 1830183, 1890189, 1960196, 2030203, 2100210, 210217, 2240224, 2310231, 2380238, 2450245, 2520252, 2590259, 2660266, 2730273, 2800280, 2870287, 2940294, 3010301, +32733, - 01872, -00259, -19604, + 32474, -21476, - 19862, +30602, - 21735, +10998, - 22099, + 20739, +68867, -08867, -16739, + 22099, - 10998, +21735, -30602, +19862, +21476, - 32474, + 19604, + 05872]

[Riccati, The laws of counterpoint, 731; text: Sistema generale, che accettando l'ottimo temperamento, divide l'Ottava in parti 74., e pone (S) = 7, (s) = 5. C, A 4#, D 2b, B 2#, E 4 b, C #, A 5#, F 5b, D b, B 3#, E 3b, C 2 #, F 4b, D, B 4#, E 2b, C 3#, F 3b, D #, B 5#, G 5b, E b, C 4 #, F 2b, D 2#, G 4b, E, C 5#, A 6b, F b, G 3b, E #, C 6#, A 5b, F, D 4#, G 2b, E 2#, A 4b, F #, D 5#, B 6b, G b, A 3b, F 2#, D 6#, B 5b, G, A 2b, F 3#, B 4b, G #, E 5#, C 5b, A b, F 4#, B 3b, G 2#, C 4b, A, F 5#, D 6b, B 2b, G 3#, C 3b, A #, F 6#, D 5b, B b, G 4#, C 2b, A 2#, D 4b, B, G 5#, E 6b, C b, A 3#, D 3b, B #, G 6#, E 5b, C, Semituono minore, maggiore, Tuono, Terza, diminuita, superflua, Quarta, Quinta, Sesta, Settima, Ottava, 1, 25/24, 21/20, 16/15, 15/14, 10/9, 9/8, 8/7, 7/6, 6/5, 5/4, 32/25, 4/3, 7/5, 10/7, 3/2, 25/16, 8/5, 5/3, 12/7, 7/4, 16/9, 9/5, 28/15, 15/8, 2/1, -2(S) + 3(s), (S) - (s), - (S) + 2(s), 2(S) - 2(s), (s), -2(S) + 4(s), 3(S) - 3(s), (S), -(S) + 3(s), 2(S) - (s), 2(s), 3(S) - 2(s), (S) + (s), - (S) + 4(s), 2(S), 3(s), 3(S) - (S), (S) + 2(s), - (S) + 5(s), 4(S) - 2(s), 2(S) + (s), 4(s), 3(S), (S) + 3(s), 4(S) - (s), 2(S) + 2(s), 9(s), 5(S) - 2(s), 3(S) + (s), (S) + 4(s), 4(S), 2(S) + 3(s), 6(s), 5(S) - (s), 3(S) + 2(s), (S) + 5(s), 4(S) + (s), 2(S) + 4(s), 5(S), 3(S) + 3(s), (S) + 6 (s), 6(S) - (s), 4(S) + 2(s), 2(S) + 5(s), 5(S) + (s), 3(S) + 4(s), (S) + 7(s), 6(S), 4(S) + 3(s), 2(S) + 6(s), 5(S) + 2(s), 3(S) + 5(s), 6(S) + (s), 4(S) + 4(s), 2(S) + 7(s), 7(S), 5(S) + 3(s), 3(S) + 6(s), 6(S) + 2(s), 4(S) + 5(s), 7(S) + (s), 5(S) + 4(s), 3(S) + 7(s), 8(S), 6(S) + 3(s), 4(S) + 6(s), 7(S) + 2(s), 5(S) + 5(s), 3(S) + 8(s), 8(S) + (s), 6(S) + 4(s), 4(S) + 7(s), 7(S) + 3(s), 5(S) + 6(s), 8(S) + 2(s), 6(S) + 5(s), 6(S) + 5(s), 4(S) + 8(s), 9(S) + (s), 7(S) + 4(s), 5(S) + 7(s), 8(S) + 3(s), 6(S) + 6(s), 4(S) + 9(s), 9(S) + 2(s), 7(S) + 5(s), 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 00000, 40680, 81359, 122039, 162719, 203399, 244078, 284758, 325438, 366198, 406797, 447477, 488157, 528836, 569516, 610196, 650876, 691555, 732235, 772915, 813595, 854274, 894954, 934634, 976314, 1016993, 1057673, 1098353, 1139032, 1179712, 1220392, 1261072, 1301751, 1342431, 1383111, 1423791, 1464479, 1505150, 1545830, 1586509, 1627189, 1667869, 1798549, 1749338, 1789908, 1830388, 1871268, 1911947, 1952627, 1993307, 2033986, 2074666, 2115346, 2156026, 2196705, 2237385, 2278065, 2318745, 2359424, 2400104, 2440784, 2481464, 2522143, 2562823, 2603503, 2644182, 2684862, 2725542, 2766222, 2806901, 2847581, 2888261, 2928941, 2928241, 2969620, 3010300, + 26111, - 08494, +04471, -14878, +30582, - 23368, - 10403, + 22087, - 18897, + 07214, - 14526, + 11685, + 03190, - 03190, - 11685, + 14526, - 07214, + 18897, -22087, + 10403, + 23368, - 30582, + 14878, -04471]

[Riccati, The laws of counterpoint, 735; text: Sistema generale temperato di Cristiano Ughenio, in cui l'Ottava si divide in parti 31., e ponesi (S) = 3, (s) = 2. C, B 2#, D 2b, C #, D b, C 2#, D, E 3b, D, C 3#, F 3b, D #, E b, D 2#, F 2b, E, D 3#, F b, E #, G 3b, F, E 2#, G 2b, F #, G b, F 2#, A 3b, G, A 2b, G #, A b, G 2#, B 3b, A, G 3#, B 2b, A 2#, C 2b, B A 3#, C b, B #, D 3b, c, Semituono minore, maggiore, Tuono, Terza, diminuita, superflua, Quarta, Quinta, Sesta, Settima, Ottava, 1, 25/24, 21/20, 16/15, 15/14, 10/9, 9/8, 8/7, 7/6, 6/5, 5/4, 32/25, 4/3, 7/5, 10/7, 3/2, 25/16, 8/5, 5/3, 12/7, 7/4, 16/9, 9/5, 28/15, 15/8, 2/1, - (S) + 2(s), (S) - (s), (s), (S), 2(s), 2(S) - (s), (S) + (s), 3(s), 2(S), (S) + 2(s), 2(S), + 2, (S) + 4(s), 3(S) + (s), 2(S) + 3(s), 4(S), 3(S) + 2(s), 2(S) + 3(s), 4(S), 3(S) + 2(s), 2(S) + 4(s), 4(S) + (s), 3(S) + 3(s), 4(S) + 2(s), 3(S) + 4(s), 5(S) + (s), 4(S) + 3(s), 3(S) + 5(s), 5(S) + 2(s), 4(S) + 4(s), 5(S) + 3(s), 4(S) + 5(s), 6(S) + 2(s), 5(S) + 4(s), 4(S) + 6(s), 6(S) + 3(s), 5(S) + 5(s), 6(S) + 5(s), 6(S) + 4(s), 5(S) + 6(s), 7(S) + 3(s), 6(S) + 5(s), 5(S) + 7(s), 7(S) +

4(s), 6(S) + 6(s), 8(S) + 3(s), 7(S) + 5(s), 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 97106, 194213, 291319, 388426, 485532, 582639, 679345, 776852, 873958, 971064, 1068171, 1165277, 1262382, 1359490, 1456597, 1553703, 1650810, 1747916, 1845023, 1942129, 2039236, 2136342, 2233448, 2330555, 2427661, 2524768, 2621874, 2718981, 2816087, 2913194, 3010300, + 16925, - 17680, + 11032, -08313, +27953, - 25993, + 02721, +10277, - 14960, + 01964, -03928, + 12997, - 04683, +04683, - 12997, +03928, - 01964, + 14960, - 10277, - 02721, + 25993, - 27957, + 08313, - 11032]

[-737-] [Riccati, The laws of counterpoint, 737; text: Sistema generale temperato del Signor Henfling, che divide l'Ottava in parti 50., e pone (S) = 5, (s) =3. C, B 2#, D 2b, C #, B 3#, E 4b, D b, C 2#, B 4#, E 3b, C 3#, F 4b, E 2b, D #, C 4#, F 3b, E b, D 2#, F 2b, E, D 3#, G 4b, F b, E #, D 4#, G 3b, F, E 2#, G 2b, F #, E 3#, A 4b, G b, F 2#, E 4#, A 3b, G, F 3#, A 2b, G #, F 4#, B 4b, A b, G 2#, B 3b, A, G 3#, C 4b, B 2b, A #, G 4#, c 3b, B b, A 2#, B, A 3#, D 4b, c b, B #, A 4#, D 3b, c, Semituono minore, maggiore, Tuono, Terza, diminuita, superflua, Quarta, Quinta, Sesta, Settima, Ottava, 1, 25/24, 21/20, 16/15, 15/14, 10/9, 9/8, 8/7, 7/6, 6/5, 5/4, 32/25, 4/3, 7/5, 10/7, 3/2, 25/16, 8/5, 5/3, 12/7, 7/4, 16/9, 9/5, 28/15, 15/8, 2/1, - (S) + 2(s), ((S) - (s), (s), - (S) + 3(s), 2(S) - 2(s), (S), 2(s), - (S) + 4(s), 2 (S) - 2(s), (S) + (s), 3(s), 3(S) - 2(s), 2(S), (S) + 2(s), 4(s), 3(S) - (s), 2(S) + (s), (S) + 99s), (S) + 3(s), 3(S), 2(S) +2(s), (S) + 4(s), 4(S) - (s), 3(S) + (s), 2(S) + 3(S), ((S) + 5(s), (S) + 5(s), 4(S), 3(S) + 2((s), 2(S) + 4(s), 4(S) + (s), 3(S) + 3(s), 2(S) + 5(s), 5(S), 4(S) + 2(s), 3(S) + 4(s), 2(S) + 6(s), 5(S) + (s), 4((S) + 3(s), 3(S) + 3(s), 5(S) + 2(s), 4(S) + 2(s), 4(S) + 4(s), 3(S) + 6(s), 6(S) + (s), 5(S) + 3(s), 4(S) + 3(s), 6(S) + 2(s), 5(S) + 4(s), 4(S) + ^s), 7(S) + (s), 6(S) + 3(S), 5(S) + 5(s), 4(S) + 7(s), 7(S) + 2(s), 6(S) + 4(s), 5(S) + 6(s), 7(S) + 3(s), 6(S) + 5(s), 5(S) + 7(s), 8(S) + 2(s), 7(S) + 4(s), 6((S) + 4(s), 6(S) + 6((s), 5(S) + 8(s), *(S) + 3(s), 7(S) + 5(s), 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 60206, 120412, 180618, 240824, 301030, 361236, 421442, 481648, 541854, 602060, 662266, 722472, 782678, 842884, 903090, 963296, 1023502, 1083708, 1143914, 1204120, 1264326, 1324532, 11384738, 1444944, 1505150, 156356, 1625562, 1689768, 1745974, 1806180, 1866386, 1926592, 1986798, 2047004, 2107210, 216416, 22227622, 2287828, 2348034, 1408340, 2468446, 2528652, 2588858, 2649064, 2709270, 2769476, 28296882, 2889888, 2950094, 301300, + 03330, - 31275, + 20743, +01398, +24073, - 39877, + 22142, -07202, - 09134, - 05804, +11609, +14939, - 16336, + 16336, - 14939, - 11609, + 05804, + 09132, + 07202, - 22142, + 39877, - 290073, - 01398, -20743]

[-140-] [Riccati, The laws of counterpoint, 740; text: Sistema generale temperato, che divide l'Ottava in parti 19., e pone (S) = 2, (s) =1. C, D 2b, C #, D b, C 2#, D, E 2b, D #, E b, D 2#, E, F b, E #, F, G 2b, F #, G b, F 2#, G, A 2b, G #, A b, G 2#, A B 2b, A #, B b, A 2#, B, C b, B #, C, Semituono minore, maggiore, Tuono, Terza, diminuita, superflua, Quarta, Quinta, Sesta, Settima, Ottava, 1, 25/24, 21/20, 16/15, 15/14, 10/9, 9/8, 8/7, 7/6, 6/5, 5/4, 32/25, 4/3, 7/5, 10/7, 3/2, 25/16, 8/5, 5/3, 12/7, 7/4, 16/9, 9/5, 28/15, 15/8, 2/1, (S) + (s), (s), (S), 2(s), (S) + (s), 2(S), (S) + 2(s), 2(S) + (s), (S) + 2(s), 2(S) + 2(s), 3(S) + (s), 2(S) + 3(s), 3(S) +2(s), 4(S) + (s), 3(S) + 3(s), 4(S) + 2(s), 3(S) + 4(s), 4(S) + 3(s), 5(S) + 2(s), 4(S) + 4(s), 5(S) + 3(s), 4(S) + 5(s), 5(S) + 4(s), 6(S) + 3(s), 5(S) + 5(s), 6(S) + 4(s), 5(S) + 6(s), 6(S) + 5(s), 7(S) + 4(s), 6(S) + 6(s), 7(S) + 5(s), 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,, 14, 15, 16, 17, 18, 19, 158437, 316874, 475311, 633747, 792184, 1109058, 1267495, 142932, 1584368, 1742805, 1901243, 2059679, 2218116, 2376553, 2534989, 2693436, 281863, 3010300, - 18851, - 53456, +36587, +17242, +17736, -36214, +53829, - 35721, + 00372, - 184479, +17614, + 18108, -35348, + 35348, - 18108, - 17614, + 18479, -00372, +35721, -53829, +30214, - 17736, -17242, - 36587]

Third chapter

Exposition of a new method to establish an enharmonic systems that presents itself as adequately general.

[signum] 1. A system is called adequately general when its elements of a different species

that are distant no more than one comma one from the other can be considered equal advisedly. If the participations consider the comma as negligible, this will have to be applicable also to the quantities that are equal to it or smaller than it is. A system that distributes quantities that are greater than the comma goes against the idea of the temperaments that were introduced only to distribute the comma and for no other reason. Once laid out the series of ratios $2/1$, $3/2$, $4/3$, $5/4$, $6/5$, $7/6$, $8/7$, $9/8$ and $10/9$, which were called by the ancients sesquiprima, sesquialtera, sesquiterza, sesquiquarta etcetera, we have noted several times that the participations have the aim to join the major tone $9/8$ and the minor one $10/9$ in a medium tone and to abolish the comma [-742-] $81/90$, which is the difference between them. Meanwhile, the extreme systems of twelve and nineteen notes attribute to themselves irregularly a greater freedom. In the first one, the two ratios $6/5$, of the minor third, and $7/6$, of the augmented second, and also the three $8/7$, of the diminished third, $9/8$ and $10/9$ of the major seconds are fused together, while it sets out to divide the quantity $26/25$, which is the difference between the ratios $6/5$, $7/6$ and $8/7$, $10/9$, which quantity is noticeably greater than the two commas. The second system of nineteen parts distinguishes the minor third $6/5$ from the augmented second $7/6$ and the diminished third $8/7$ from the major seconds $9/8$ and $10/9$, but then it unites in a median ratio the proportion $7/6$ of the augmented second and $8/7$ of the diminished third abolishing their difference $49/48$ that is close to $5/3$ c. The two tone $9/8$ and $10/9$ are the most distant ratios that must be joined together by the good participations, which, consequently, must distinguish the ratios $6/5$, $7/6$, $8/7$ and $9/8$ one from the other. The principal union of the tones $9/8$ and $10/9$ produces the secondary unions of the two major semitones $15/14$ and $16/15$ of the two minor semitones $21/20$ and $25/24$, the first two of which, namely, $15/14$ and $21/20$, constitute the major tone $9/8$, while the the other two, namely, $16/15$ and $25/24$ constitute the minor tone $10/9$. The two major semitones differ by the fraction $225/224$, which exceeds slightly one third of the comma, while the difference between the two minor ones is $126/125$, which corresponds to a little less than two thirds of the comma. Said two fractions multiplied with each other constitute the comma $81/80$.

[signum] 2. What we have explained so far teaches us to distinguish two species of harmonies in our music. Some have a principal ratio, which can be called perfect ratio, which must be preserved as untouched as possible by the participations. This first species of harmonies contains all the ones whose ratios do not contain even numbers that are larger than the four most simple ones, namely, 1., 3., 5. and 7., namely, the aequisonances, [-743-] the perfect consonances, the imperfect consonances and the semiconsonances. If we are presented sometimes with some different ratio that expresses such intervals, it must be considered to be altered. The altered ratio of the minor third is $25/21$, which derives from reducing the major third by a minor semitone $21/20$. The ratio $25/18$, which represents the major fourth and originates from widening the fourth by the minor semitone $25/24$, must also be placed among the other altered ones.

The harmonies species of harmony recognise more than one ratio. The good participations must establish one that lays in the middle among them and can perform the role of anyone of them. The the union of the tones $9/8$ and $10/9$, which requires, consequently, that the two major semitone $15/14$ and $16/15$ and the two minor semitones $21/20$ and $25/24$ should be united in an median ratio, has opened the way to such ratios, which I call extreme, because they contain the well tempered ratios. If we unite the semitones $16/15$ and $21/20$, one major and the other one minor, we obtain the medial note $28/25$, which deserves to be considered as the difference between an imperfect consonance and a semi-consonance, namely, between the major third $5/4$ and the major fourth $7/5$ and between the minor sixth $8/5$ and the minor fifth $10/7$. The shared union of the various tones, of the various major semitones and of the minor ones produces the shared union of the intervals that added to the former create the octave, namely, of the minor sevenths, of the major sevenths and of the diminished octaves. Similarly, one must consider among the extreme intervals the diminished fourth and the interval that added to it constitutes the octave, namely, the augmented fifth. The simplest ratio of the diminished fourth is $9/7$, which is an extreme ratio because it contains the off number 9., which is larger than the four simplest ones, namely, 1. 3. 5. and 7. The ratio $9/7$ is produced by adding the major tone $9/8$ to the diminished third $8/7$. If one adds to the diminished

third $8/7$ the minor tone $10/9$, the opposite extreme ratio is created, which is $80/63$. The ratio $32/25$ of the diminished fourth, which derives from adding the median tone $28/25$ to the diminished third $8/7$, is halfway between the two mentioned earlier. [-744-] The augmented fifth $25/16$, which equals the sum of two perfect major thirds, corresponds to the diminished fifth $32/35$ as the interval that added to it forms the octave. I order the harmonies of the two species considered just now in the two tables placed herewith.

[Riccati, The laws of counterpoint, 744; text: Intervalli dotati d'un solo esponente giusto. Unisono $1/1$, Quinta $3/2$, Terza maggiore $5/4$, Sesta $5/3$, superflua $7/4$, Seconda $7/6$, Quarta [[superflua]] [maggiore add. supra lin.] $7/5$, Compimento all'Ottava, $2/1$, Quarta $4/3$, minore $8/5$, $6/5$, $8/7$, Settima $12/7$, $10/7$, Intervalli, ed elementi di limite dotati di più esponenti. Tuono $9/8$, medio $28/25$, $10/9$, Semituoni maggiori $15/14$, $16/15$, minori $21/20$, $25/24$, Quarte diminuite $9/7$, $32/35$, $80/63$, Settime $16/9$, $25/14$, $9/5$, maggiori $28/15$, $15/8$, Ottave diminuite $40/21$, $48/25$, Quinte superflue $14/9$, $25/16$, $63/40$]

[signum] 3. Someone may object that it appears strange that one should ascribe an equal rank to certain ratios, as for instance to the three ratios of the diminished fourths, namely, $9/7$, $32/25$ and $80/63$. Since we observed earlier (book 1., chapter 1., [signum] 13.) that the degree of simplicity of two ratio is inversely proportional to the largest odd numbers that occur in the ratios themselves, it follows that the simplicity of the ratio $9/5$ correspond to the one of the ratio $80/63$ in the proportion $63:9$ or $7:1$. Now, as it appears evident that [-745-] the simpler the ratio, the more principal it has to be called, one cannot understand the reason why an equal or similar degree of principality is ascribed to the two ratios $9/7$ and $80/63$. The same argument has to be applied to the ratios $9/7$, $32/25$ and $80/63$, when they are compared to each other. The objection would appear unassailable, were I not to explain in which way I deem the ratios $9/7$, $32/25$ and $80/63$ to be of the same degree of principality.

The main way in which an interval is employed is either through inequality of harmony or through inequality of fundamental harmony. If the diminished fourth is employed in those roles, the ear will always refer it to its simplest ratio $9/7$, which, therefore, must be considered perfect, while the other two, $32/25$ and $80/64$, are considered as altered.

A given interval appears in a secondary role in counterpoint when one employs it as derivative melody. I did not fail to note in the appropriate place (book 1., chapter 3., [signum] 16.) that, while the derivative passages of the melody are considered little or nothing in themselves, one pays attention only to the place from which they originate and to the place to which they lead. Under this less principal aspect, the most important place belongs to the ratio $32/25$, because it is the difference between two imperfect consonances, namely, the minor sixth $8/5$ and the major third $5/4$. The second place is allotted to the ratio $80/63$, because it is the difference between a semi-consonance and one of the simplest dissonances, in whose ratios there is no odd number larger than nine, namely, as it is the difference between the minor fifth $10/7$ and the major second $9/8$ or between the augmented fourth $7/5$ and the minor seventh $16/9$.

The Reader shall be able to deduce easily from what has been said that I do not consider the ratios $9/7$, $32/25$ and $80/63$ as belonging to the same degree in any of these meanings. However, it is because of the temperaments that all three of these deserve [-746-] consideration. I have already demonstrated that they derive from adding to the diminished third $8/7$ the three tones, the major one $9/8$, the medium one $28/25$ and the minor one $10/9$. The third, when diminished by the good participations, must be preserved as almost perfect, as it consists of the two major semitones $15/14$ and $16/15$, the sum of which is very close to the sum of two medium semitones. Therefore, one should draw the conclusion that, the three ratios $9/7$, $32/25$ and $80/63$, which belong to the diminished fourth, require the consideration that one assigns to the three tones $9/8$, $29/25$ and $10/9$ on account of the temperaments.

[signum] 4. Now, I am going to approach the promised explanation of the method. I take the key of C and the unit as the base and I lay out in proportion with it all the ratios that do

not contain an odd number greater than seven and thus need to be maintained within their perfect measure. I employ only those that have a fixed ratio and I exclude the extreme ones, whether intervals or elements, because they do not have a fixed ration. Therefore, I obtain the first series of letters that I present to the Reader in the following table, where the aforementioned intervals are laid out one after the other according to their value. Moreover, as the system has to be general, which term means, as everyone knows, that the same sequence of ratios is allotted to each particular letter, I take as base one by one each of the twelve letters of the first series and I apply to them in the same order all the ratios that do not contain an odd number larger than seven, so that we obtain the two scales that one sees recorded under the one of C in the table. For instance, in the second series I place first $G = 3/2 \cdot 8/7 = 12/7$, then its augmented second $7/6$, so that I obtain $A \# = 3/2 \cdot 7/6 = 7/4$, and so forth.

[signum] 5. If we consider every scale separately, we note the property according to which all the intervals that we encounter ascending from the base towards [-747-] the octave present themselves to us also when we descend from the octave towards the base, which conforms precisely to the idea of the general systems. The reason depends from the fact that twelve intervals were applied to each octave, half of which are intervals that, added to the six belonging to the other half, form the octave. If two intervals added together form the octave, a ratio taken upwards, for instance, from C to c, and the other one taken downwards from c to C determine the same sound. Since the octave consists of the fifth and of the fourth, the same sound G is produced both by the fifth C G taken upwards and by the fourth c G taken downwards. Thus, equally, the same sound F derives from the ascending fourth C F and from the descending fifth c F. So, introduced two ascending ratios in relation to the base C that added together complete the octave, I also find the same ratios when I descend, namely, the fifth c F and the fourth c G. The example that I presented, which refers to the first scale and to the couple of intervals of the fifth and of the fourth, has to be applied to the other couples of intervals. Thus, one shall discover that in any scale, both ascending and descending, one finds the same relationships between intervals that complement each other.

[signum] 6. Moreover, I note that, if the thirteen scales are joined into one, the property that I have already explained shall remain true, and the same ratios shall be allotted to the key of C both ascending and descending. The total scale is created by adding to the first particular scale of C the twelve following ones. I divide them into six couples, so that, taken the ratios on which they are founded two by two, one is always the one that added to the other forms the octave. They have been laid out in the table in this order. For instance, $5/2$ is the base of the second scale, while $4/3$, which added to the former produces the octave, is the base of the third one. Now, with the help of the scale that belongs to G and put on one side the common intervals, the seven following ratios are added to the first one which belongs to C, namely, $B b = 9/5$, $B = 15/8$, $C \# = 21/20$, $D b = 15/14$, $D = 9/8$, $F b = 9/7$ and $E \# = 21/16$. The intervals that added to them form the octave [-748-] join the total scale through the third built on F. They are the following: $D = 10/9$, $D b = 16/15$, $C b = 10/21$, $B = 28/15$, $B b = 16/9$, $G \# = 14/9$ and $A 2b = 32/21$. Everyone can see that, on the basis of what was said in the previous paragraph, the first scale is enriched by an equal number of ascending and descending ratios. The ratios that are introduced in ascending by the second scale, are introduced in descending by the same scale, and, equally, the same ratios are provided by the third scale in ascending and by the second in descending.

Our fourteen ratios, when considered as descending, could be expressed in the following way, by subtracting from the octave $2/1$ the ratio that one wants to take downwards. If a fraction, for instance $10/9$, indicates an interval that is added or taken upwards, the inverse fraction, namely, $9/10$, indicates the same interval as detracted or taken downwards. $B b = 9/10 \cdot 2/1$, $B = 15/16 \cdot 2/1$, $C \# = 21/40 \cdot 2/1$, $D = 9/16 \cdot 2/1$, $F b = 9/14 \cdot 2/1$, $E \# = 21/32 \cdot 2/1$, $D = 5/9 \cdot 2/1$, $D b = 8/15 \cdot 2/1$, $c b = 20/21 \cdot 2/1$, $B = 14/15 \cdot 2/1$, $B b = 8/9 \cdot 2/1$, $G \# = 7/9 \cdot 2/1$, $A 2b = 16/21 \cdot 2/1$. Once we made the appropriate comparisons, we shall observe that the same sound B b introduced by the second scale is employed in the minor seventh C B b $9/5$ upwards and in the major second c B b $9/10$ taken downwards. Conversely, the same sound D introduced by the third scale produces the major second C D $10/9$ in ascending and the minor seventh c D $5/9$ in descending. I will not tire the Reader with

other examples. What I said of the second and third scale must apply to the other pairs of scales, namely, the fourth and fifth, the sixth and seventh ad so on, all of which shall add to the total scale an equal number of ascending and descending ratios.

[-748, a, b-] [Riccati, The laws of counterpoint, 748a, 748b; text: C 1. E 2b 8/7, D # 7/6, # b 6/5, E 5/4, F 4/3, F # 7/5, G b 10/7, G 3/2, A b 8/5, A 5/3, B 2b 12/7, A # 7/4, G 3/2, F 4/3, A 5/3, E b 6/5, E 5/4, A b 8/5, F # 7/5, G b 10/7, D # 7/6, B 2b 12/2, A # 7/4, E 2b 8/7, B 2b 3/2 . 8/7 = 12/7, A # 3/2 . 7/6 = 7/4, B b 3/2 . 6/5 = 9/5, B 3/2 . 5/4 = 15/8, C 3/2 . 4/3 . 1/2 = 1, 3/2 . 7/5 . 1/2 = 21/20, D b 3/2 . 10/7 . 1/2 = 15/14, D 3/2 . 3/2 . 1/2 = 9/8, E b 3/2 . 8/5 . 1/2 = 6/5, E 3/2 . 5/3 . 1/2 = 5/4, 3/2 . 12/7 . 1/2 = 9/7, 3/2 . 7/4 . 1/2 = 21/26, A 2b 4/3 . 8/7 = 32/21, G # 4/3 . 7/6 = 14/9, A b 4/3 . 6/5 = 8/5, A 4/3 . 5/4 = 5/3, B b 4/3 . 4/3 = 16/9, B 4/3 . 7/5 = 28/15, C b 4/3 . 10/7 = 40/21, C 4/3 . 7/2 . 1/2 = 1, D b 4/3 . 8/5 . 1/2 = 16/15, D 4/3 . 5/3 . 1/2 = 10/9, E 2b 4/3 . 12/7 . 1/2 = 8/7, D # 4/3 . 7/4 . 1/2 = 7/6, c b 5/3 . 8/7 = 40/21, B # 5/3 . 7/6 = 35/18, C 5/3 . 6/5 . 1/2 = 1, C #, 5/3 . 5/4 . 1/2 = 25/24, D 5/3 . 4/3 . 1/2 = 10/9, D # 5/3 . 7/5 . 1/2 = 7/6, E b 5/3 . 10/7 . 1/2 = 25/21, E 5/3 . 3/2 . 1/2 = 5/4, F 5/3 . 8/5 . 1/2 = 4/3, F # 5/3 . 5/3 . 1/2 = 25/18, G b 5/3 . 12/7 . 1/2 = 10/7, F 2# 5/3 . 7/4 . 1/2 = 35/24, G 2b, 6/5 . 8/7 = 48/35, F # 6/5 . 7/6 = 7/5, G b 6/5 . 6/5 = 36/25, G 6/5 . 5/4 = 3/2, A b 6/5 . 4/3 = 8/5, A 6/5 . 7/5 = 42/25, B 2b 6/5 . 10/7 = 12/7, B b 6/5 . 3/2 = 9/5, c b 6/5 . 8/5 = 48/25, C 6/5 . 5/3 . 1/2 = 1, D 2b 6/4 . 12/7 . 1/2 = 36/35, C # 6/5 . 12/7 . 1/2 = 21/20, G b 5/4 . 8/7 = 10/7, F 2# 5/4 . 7/7 = 35/24, G 5/4 . 6/5 = 3/2, G # 5/4 . 5/4 = 25/16, A 5/4 . 4/3 = 5/3, A # 5/4 . 7/5 = 7/4, B b 5/4 . 10/7 = 25/14, B 5/4 . 3/2 = 15/8, C 5/4 . 8/5 . 1/2 = 1 C # 5/4 . 5/3 . 1/2 = 24/25, D b 5/4 . 12/7 . 1/2 = 15/14, C 2# 5/4 . 7/4 . 1/2 = 35/32, C 2b 8/5 . 8/7 = 64/35, B 8/5 . 7/6 = 28/15, c b 8/5 . 6/5 = 48/25, C 8/5 . 5/4 . 1/2 = 1, D b 8/5 . 4/3 . 1/2 = 16/15 D 8/5 . 7/5 . 1/2 = 28/25, E 2b 8/5 . 10/7 . 1/2 = 8/7, E b 8/5 . 3/2 . 1/2 = 6/5, F b 8/5 . 8/5 . 1/2 = 32/25, F 8/5 . 5/3 . 1/2 = 4/3, G 2b 8/5 . 12/7 . 1/2 = 48/35, F # 8/5 . 7/4 . 1/2 = 7/5, A b 7/5 . 8/7 = 8/5, G 2# 7/5 . 7/6 = 49/30, A 7/5 . 6/5 = 42/25, A # 7/5 . 5/4 = 7/4, B 7/5 . 4/3 = 28/15, B # 7/5 . 7/5 = 49/25, C 7/5 . 10/7 . 1/2 = 1, C # 7/5 . 3/2 . 1/2 = 21/20, D 7/5 . 8/5 . 1/2 = 28/25, D # 7/5 . 5/3 . 1/2 = 7/6, E b 7/5 . 12/7 . 1/2 = 6/5, D 2# 7/5 . 7/4 . 1/2 = 49/40, B 3b 10/7 . 8/7 = 80/49, A 10/7 . 7/6 = 5/3, B 2b 10/7 . 6/5 = 12/7, B b 10/7 . 5/4 = 25/12, c b 10/7 . 4/3 = 40/21, C 10/7 . 7/5 . 1/2 = 1, D 2bb 10/7 . 10/7 . 1/2 = 50/49, D b 10/7 . 3/2 . 1/2 = 15/14, E 2b 10/7 . 8/5 . 1/2 = 8/7, E b 10/7 . 5/3 . 1/2 = 25/21, F 2b 10/7 . 12/7 . 1/2 = 60/49, E 10/7 . 7/4 . 1/2 = 5/4, F 7/6 . 8/7 = 4/3, E 2# 7/6 . 7/6 = 49/36, F # 7/6 . 6/5 = 7/5, F 2# 7/6 . 5/4 = 35/24, G # 7/6 . 4/3 = 14/9, G 2# 7/6 . 7/5 = 49/30, A 7/6 . 10/7 = 5/3, A # 7/6 . 3/2 = 7/4, B 7/6 . 8/5 = 28/15, B# 7/6 . 5/3 = 35/18, C 7/6 . 12/7 . 1/2 = 1, B 2# 7/6 . 7/4 . 1/2 = 49/48, D 3b 12/7 . 8/7 = 96/49, C 12/7 . 7/6 . 1/2 = 1, D 2b 12/7 . 6/5 . 1/2 = 36/35, D b 12/7 . 5/4 . 1/2 = 15/14, E 2b 12/7 . 4/3 . 1/2 = 8/7, E b 12/7 . 7/5 . 1/2 = 6/5, F 2b 12/7 . 10/7 . 1/2 = 60/49, F b 12/7 . 3/2 . 1/2 = 9/7, G 2b 12/7 . 8/5 . 1/2 = 48/35, G b 12/7 . 5/3 . 1/2 = 10/7, 12/7 . 12/7 . 1/2 = 72/49, G 12/7 . 7/4 . 1/2 = 3/2, C 7/4 . 8/7 . 1/2 = 1, B 2# 7/4 . 7/6 . 1/2 = 21/20, C 2# 7/4 . 5/4 . 1/2 = 35/32, D # 7/4 . 4/3 . 1/2 = 7/6, D 2# 7/4 . 7/5 . 1/2 = 49/40, E 7/4 . 10/7 . 1/2 = 5/4, E # 7/4 . 3/2 . 1/2 = 21/16, F # 7/4 . 8/5 . 1/2 = 7/5, F 2# 7/4 . 5/4 . 1/2 = 35/24, G 7/4 . 12/7 . 1/2 = 3/2 F 3 # 7/4 . 7/4 . 1/2 = 49/32, G 3 b 8/7 . 8/7 = 64/49, F 8/7 . 7/6 = 4/3, G 3b 8/7 . 6/5 = 48/35, G b 8/7 . 5/4 = 10/7, A 2b 8/7 . 4/3 = 32/21, A b 8/7 . 7/5 = 8/5, B 3b 8/7 . 10/7 = 80/49, B 2b 8/7 . 3/2 = 12/7, c 2b 8/7 . 8/5 = 64/35, c b 8/7 . 5/4 = 40/21, D 3b 8/7 . 12/7 = 96/49, C 8/7 . 7/4 . 1/2 = 1]

[-749-] [Riccati, The laws of counterpoint, 749; text: Scala nascente dalla unione delle soprascritte. C, B 2#, D 2b, C #, D b, C 2#, D, E 2b, D #, E b, D 2#, F 2b, E, F b, E #, G 3 b, F, E 2#, G 2b, F #, D 3b, B 2, c b, B, c 2b, B b, A #, B 2b, A, B 3b, G 2#, A b, G #, A 2b, F 3#, G, A 3b, F 2 #, G b, 1, 49/48, 50/49, 36/35, 25/24, 21/20, 16/15, 15/14, 35/32, 10/9, 28/25, 9/8, 8/7, 7/6, 25/21, 6/5, 49/40, 60/49, 5/4, 32/25, 9/7, 21/16, 64/49, 4/3, 49/36, 48/35, 25/18, 7/5, 96/49, 49/25, 35/18, 48/25, 40/21, 15/8, 28/15, 64/35, 9/5, 25/14, 16/9, 7/4 , 12/7, 42/25, 5/3, 80/49, 49/30, 8/5, 25/16, 14/9, 32/21, 49/32, 3/2, 72/49, 35/24, 36/25, 10/7, 89549, - 1809, 87739, + 32796, 122344, 177288, 211893, 280287, 299632, 369181, 457575, 492180, 511525, 579919, 669468, 757207, 791812, 881361, 879552, 969100, 1072199, 1091444, 1180993, - 21154, 1159839, 1249387, 1338936, 1371732, 14266675, 1461280, 3010300, 2920751, 2922561, 2887956, 2833012, 3798409, 2730013, 2710668, 2621119, 2552725, 2518120, 2498775, 2430381, 2340832, 2253093, 2218488,

2128939, 2130746, 2041200, 1938200, 1918856, 1829307, - 21154, 1850461, 1760916 [[1760964 ante corr.], 1671364, 1638568, 1583625, 1549020]

[-750-] Although my method has as its only aim to apply to the letter C and then to all the other included in the first scale the intervals that do not contain an odd number larger than seven, it occurred as necessary consequence that the ratios of the extremes employed in our music and some altered ratios found their way into the total scale, as well as many other ratios that, although not employed, concur to complete the system and to render it general. The diminished fourth, expressed by both fractions $32/25$ and $9/7$, the two diatonic tones $10/9$ and $9/8$, which comprehend as their half the simplest of the chromatic ones $28/25$, the two major semitones $16/15$ and $15/14$ and, finally, the two minor semitones $15/14$ and $21/20$ belong notably to the first category, left aside the intervals that added to them complete the octave. The second category contains the augmented third, which is last chromatic interval and has no application as well as the enharmonic intervals. The simplest enharmonic elements that originate from our scale are the following: the diminished seconds $64/63$, $50/49$ and $36/35$ and the twice diminished second $49/48$. So that the Reader may learn how these constitute the two tones $9/8$ and $10/9$, the two major semitones $15/14$ and $16/15$ and the two minor semitones $21/20$ and $25/24$, I take from the scale the distribution of the tones F G $9/8$ and G A $10/9$. I have chosen these because they are the difference between the fourth C F and the fifth C G, and between the fifth C G and the major sixth C A, and because they are divided into their semitones $15/14$ and $25/20$ and $16/15$ and $25/24$ by the letters G b $10/7$ and A b $8/5$ which are represented by fixed ratios.

[Riccati, The laws of counterpoint, 750; text: F, G 2b, F #, G b, G, A 2b, G #, A b, G 2#, A, $4/3$, $48/35$, $7/5$, $10/7$, $35/24$, $3/2$, $32/21$, $14/9$, $8/5$, $49/30$, $5/3$, $36/35$, $49/48$, $50/49$, $64/63$, $15/14$, $21/20$, $16/15$, $25/24$, $9/8$, $10/9$, $28/25$]

At the same time the Reader has the opportunity to learn the division of the median tone $28/5$, which, as we noted earlier ([signum] 2.) [-751-] consists of the two semitones $21/20$ and $16/15$. It occurs between the letters G b $10/7$ and A b $8/5$, while the letter G $3/2$ divides it into the aforementioned semitones $21/20$ and $16/15$.

[signum] 8. Had I been required to proceed further to arrive to the general principles after I have laid out the thirteen scales with the rules explained above, I would have not know how to comport myself. In fact, as long as I base each of my scales on a relationship that is expressed by a fixed ratio that must not be altered in an audible way, the matter proceeds perfectly. However, if I takes as my base an extreme ratio, for instance the tone represented by several ratios, as I lack a stable foundation, the matter would have become much more convoluted and difficult. Meanwhile, however, we have accomplished the aim for our scale to be general and circular. The fact that certain letters altered with the sharps and others that accept the flats are strictly connected and fused together provides us with some evidence of the circulation. I took care to separate these letters in the scale with dotted lines. Moreover, since the confluence between occurs always between the letters that differ one from the other by the difference occurring between the enharmonic elements of difference species, namely, the diminished second and the twice diminished second, if such difference can be overlooked, said elements are presumed to be equal, which leads us directly to the system by Ughenio, which divides the octave in thirty-one parts. When the eight pairs of letters separated by a dotted line in the octave C c contained by the scale are supposed to be joined into a single value, one finds that the octave is divided into thirty-one elements. The difference between the diminished second and the twice diminished one can be called a four times diminished third. C D 2b is a diminished second, C B 2# a twice diminished second and B 2# D 2b, which is their difference, is a four times diminished third. The other seven pairs of letters, namely, D 2# F 2b, E # G 3b, E 2# G 2b, F 2# A 3b, F 3 # A 2b, G 2# B 3b, B # D 3b, that join together appropriately in the same value, are related to each other in the same way. I have taken care to mark along side the scale the values of the four times diminished third expressed in logarithms. I find three species of them:

the largest, [-752-] or + 32796, which is less than two thirds of the comma 81/80, which equals 53950, is the difference between the diminished second 36/35 and the twice diminished second 49/48. I place the plus sign in front of it because the diminished second 36/35 is larger than the twice diminished 49/48. The median one, or – 21154, corresponds to the difference between them diminished second 64/63 and the twice diminished second 49/48. Finally, the smallest – 1809, expresses the difference between the diminished second 50/49 and the twice diminished one 49/48. I placed the minus sign in front of the last two differences because the diminished seconds 64/63 and 50/49 are smaller than the twice diminished 49/48. These differences, one positive and the other two negative, that derive from the property that the twice diminished second 49/48 is smaller than the diminished second 35/36 and larger than the other two 64/63 and 50/49, teach us manifestly that the enharmonic elements of different species must be presumed to be equal. In truth, if the temperaments have the task to establish the equivalence between the diminished seconds 36/35 and 64/63, which differ by the comma 81/80, how much more securely shall we have to presume the twice diminished second 49/48, which is included between them, to be equal to both? Moreover, a physical assimilation occurs between the twice diminished seconds 49/48 and the diminished second 50/49, since their difference equals 1809., which is one thirtieth of a comma, which is a completely inaudible quantity. However, the diminished second 50/49 needs to be equal to the two ratios 36/35 and 64/63. Therefore, a similar correspondence of equality will have to be presumed between the twice diminished second 49/48 and the diminished ones, 36/35 and 64/63. Moreover, that the presumption of equality between the diminished seconds 36/35, 50/49 and 50/49, 64/63 is in accordance with the idea of participation is proved by the fact that the difference between the first two ratios is the same as the difference between the two minor semitones 21/20 and 25/24, while the difference between the second two ratios is equivalent to the difference between the two major semitones 15/14 and 16/15. To assert the equality between the semitones of the same species is an immediate consequence of the fundamental suppositions regarding the temperaments, which is the coming together of the [-753-] tones 9/8 and 10/9.

[signum] 9. If simplicity deserves the first prerogative of a musical system, I am flattered that my system exceeds by a long way all the enharmonic systems that have been invented by different theorists in this respect as well. The odd largest odd number that occurs in the ratios that constitute the scale is forty-nine. If one examines the enharmonic system by Signor Henfling, which he himself published in the *Miscellanea Berolinensia* of the year 1710, one find very complex odd numbers that extend from the number one to the number 10125. This number can be seen in one of the ratios, 10125/8192, ascribed to the twice augmented second, which is C D 2# if one takes the key of C as base of the system. Said interval is expressed in my scale by the ratio 49/40. The ancient Greeks and Romans, who did not allow in music the number 5, used to mark the major 5/3 and the minor third with the ratio 6/5 with the ratios 27/16 and 32/27, and the major third 5/4 and the minor sixth 8/5 with the ratios 81/62 and 128/81. Those modern authors who exclude from music the number seven incur a similar error. If I limit myself to observe the mere semi-consonances and semi-dissonances employed so widely in counterpoint, Signor Henfling indicates the major fourth 7/5 with the ratios 45/32 and 25/18, the minor fifth 10/7 with the ratios 45/32 and 25/18, the augmented second 7/6 with the ratios 75/64 and 125/108, the diminished seventh 17/ with the ratios 128/75 and 216/125, and finally the augmented sixth 7/4 with the ratios 125/72 and 225/128 and the diminished third 8/7 with the ratios 144/125 and 256/225. If one considers that the semi-consonant or semi-dissonant ratios, as we want to call them, are always half-way between the two by Signor Henfling, this would suffice to convince the lauded theorist that the number seven must be allowed in counterpoint. It is indeed very strange that, while the major fourth, the augmented second and the augmented sixth are employed all the time in music with the intervals that added to them produce the octave as such an important musical condiment, and while the ear draws pleasure from the simpler proportions that [-754-] contain the number seven, music theorist struggle so much to realise that said number does not deserve to be banished from music, which is the punishment to which it was sentenced by them.

[signum] 10. I had been convinced for a long time that Cristiano Ughenio's system was the

best of the general one and that it contained the best participation. I was encouraged strongly to think so by the fact my enharmonic constitution, which divides the octave into thirty-one elements, pointed manifestly to it. Now, since said constitution originates only from the consonant and semi-consonant ratios, it appeared undeniable that the general system that conforms to the character of the mentioned constitution and that presumes equal the enharmonic elements of different species, namely, the diminished second and the twice diminished one, would be involved in the best temperament of those consonant and semi-consonant ratios. My observing that, if I added the differences ascribed to the consonant and semi-consonant ratios by a general system, this sum corresponded to the smallest one in the Ughenio's system, added weight to my conjecture. In the table below, I compare the the most reasonable systems of forty-seven, seventy-four, thirty-one and fifth parts and I highlight the differences by which they alter the ratios that do not contain an odd number larger than seven. I shall record these ratio while omitting only half of the ratios that complement them to form the octave, while, when I add together the differences I shall not pay attention to plus and minus signs, since the ear dislikes the positive and by the negative differences, which is the same as to say by the wider or narrower intervals.

[-755-] [Riccati, The laws of counterpoint, 755; text: Differenze, e somme d'esse in riguardo agli intervalli consonanti, e semiconsonanti nei quattro Sistemi di parti 43. 74. 31. 50. Ragioni, Sistema, Terza diminuita, 8/7, -19862, -10403, +02721, +22142, Seconda superflua, 7/6, + 30602, +22087, + 10277, -07202, Terza minore, 6/5, -21735, - 18897, - 14960, - 09134, maggiore, 5/4, + 10998, + 07214, +01964, - 05804, Quarta, 4/3, + 10739, + 11685, + 12997, +14939. [[superflua]] [maggiore add. supra lin.], 7/5, +08867, +03190, - 04683, - 16336, Somma delle differenze, 102803, 73476, 47602, 75557]

[signum] 11 Were the ear to deduce the goodness of a participation from the sum of the differences that alter the ratios that do not contain an odd number larger than seven, there is no doubt that Ughenio's system of thirty-one notes would prove the best one, since the sum of the differences in it is very small and much smaller than in the other system noted above. A similar, but inferior, degree of goodness would be allotted to the systems of seventy-four and fifty notes, as the sum of the differences is almost equal. The last degree of goodness would be allotted to the system of forty-three merides, in which the sum of the differences exceeds the sum of the differences contained in the other systems represented in our table. However, the ear has no way to know the sum of the differences that alter the consonant and semi-consonant ratios, therefore it cannot ascertain the degree of goodness of particular participation on the basis of this sum. It is the correct distribution of the differences, which does not tolerate that any ratio is altered more than its nature requires, that satisfies the ear. In architecture we note that the eye, which is not aware of an error if it is well distributed, becomes then aware of a smaller error if it is not [-756-] distributed adequately. Who shall ever compare the participation of the system of fifty parts with the participation of the system of seventy-four parts, although the sum of the differences is almost the same in both systems? In the system of fifty parts the alteration of the perfect consonances equals 14939, while the imperfect consonances are altered at the most by 9134., and the semi-consonances 22142. One discovers at first sight the imperfection noted elsewhere (chapter 1., [signum] 16.) namely, that one takes into account the perfect consonances less than the imperfect ones. Moreover, the largest difference of the semi-consonances, namely, 22142, is too small as to the quantity by which the perfect consonances are out of tune, which is 14939. If one takes the value of the alteration 14939 as the largest possible, the semi-consonances tolerate to be altered by the value 24858, according to the law established earlier (chapter 1., [signum] 2.). It is necessary, therefore, to resort to a system in which, while the perfect consonances are as perfect as possible, the amount by which the imperfect consonances and the semi-consonances depart from perfection are appropriately proportional. Such effect is obtain only partially by the system of thirty-one parts. Even if the difference of the perfect consonances from perfection is reduced to 12997, it is still too wide, while the alteration of the imperfect consonances 14960 is still smaller than it should be, while the greater alteration of the

semiconsonances 10277 is absolutely too small. The system of seventy-four parts lends us a hand. If one compares the alteration of the perfect consonances 11685 with the greater difference of the imperfect consonances, one finds that they are closely represented by the ratio of the odd numbers 3. and 5. according to the law just quoted. In this way, the variation from the respective perfect intervals are proportional in such a way that the impression that they produce on the ear is equal between them and it is the smallest of all the possible ones. If we leave the mentioned system to embrace another one, we are presented with an approximation of the perfect consonances or of the imperfect ones that displeases the ear even more. Therefore, albeit in the system of seventy-four parts the greater alteration of the semi-consonances 22087 does not prove proportional to the aforementioned ones of the perfect consonances [-757-] and of the imperfect ones, this is no reason to assume that the aforementioned system cannot contain the perfect participation. A system half-way between the one of seventy-four and the one of forty-three parts divides the differences of the perfect consonances by altering the semi-consonances to a greater degree, while the system of forty-three merides renders proportional the greater alterations of the imperfect consonances and of the semi-consonances. Now, there is a drawback in both of these scenarios. In fact, if one leaves aside the semi-consonances the differences of the minor third and of the major sixth are harsher on the ear than any alteration ascribed to the consonant and semi-consonant intervals by the system of seventy-four parts.

[signum] 12. The impression made on the ear by the difference between an imperfectly tuned interval and its perfect tuning is proportional to the size of such difference and to the simplicity of the altered ratio, but the simplicity of the ratios are in the same reciprocal proportion as the odd largest numbers that occur in those ratios. Therefore, the impressions made on the ear maintain a double proportionality, direct as to the size of the differences themselves, and inverse as to the largest odd numbers that are involved in the mentioned ratios. If the alterations are proportional to the odd larger number, the ear shall be also affected according to the principle established earlier (chapter 1. [signum] 2.). Given that the impression caused by the difference between an altered perfect consonance and its truly perfect state is expressed by said difference, which is arbitrary, it follows that, in order to obtain the measure of the impressions on the ear caused by the imperfect consonances and by the semi-consonances, one will have to multiply the differences of the first ones by $\frac{3}{5}$ and the differences of the second ones by $\frac{3}{7}$. Thanks to this operation and compared the measures of the impressions produced by the tempering of the perfect and imperfect consonances and of the semi-consonances, one shall find that, as we established earlier, they correspond to each other according to a double proportionality: according to the direct ratio of the differences and according to the inverse ratio of the odd numbers 3., 5. and 7. The following table contains the measures of the impressions produced by the differences that modify the perfect consonances, the imperfect ones and the semi-consonances in the general systems of forty-three, seventy-four, thirty-one and fifty parts. [-358-] I mark the various impressions in the way explained above, namely, those of the perfect consonances according to their differences, the ones of the imperfect consonances according to their differences multiplied with the fraction $\frac{3}{5}$ and the ones of the semi-consonances by the differences multiplied with the fraction $\frac{3}{7}$.

[Riccati, The laws of counterpoint, 758; text: Impressioni prodotte nell'orecchio dalle differenze, colle quali i Sistemi generali 43. 74. 31. 50. modificano le consonanze perfette, imperfette, e semiconsonanze. Ragioni consonanti, semiconsonanti, Sistema di parti, Terza diminuita, $\frac{8}{7}$, 08521, 04458, 10066, 09489, Seconda superflua, $\frac{7}{6}$, 13115, 09466., 04404, 03087, Terza minore, $\frac{6}{5}$, 13041, 11336, 08976, 05480, maggiore, $\frac{5}{4}$, 0659, 04328, 01178, 03482, Quarta, $\frac{4}{3}$, 10739, 11685, 12997, 14939, [[superflua]] [maggiore add. supra lin.], $\frac{7}{5}$, 03801, 01367, 02007, 07001, Somma delle impressioni, 55807, 42642, 30728, 43478]

The greatest impressions that occur in the system of forty-three parts are the two adequately similar ones of the augmented second 13115. and of the minor third 13041. The greatest two that occur in the system of seventy-four parts are the two that are closest to equality, namely, the one of the

fourth 11685. and the one of the minor third 11338. The greatest in the system of thirty-one parts is the one of the fourth 12997., and, similarly, the greatest in the system of fifty parts is the one of the just mentioned fourth 14939. The smallest, 11685, of these greatest impressions belongs to the system that divides the octave in seventy-four parts, which, therefore, contains the best temperament.

[signum 13.] In the choice of the best system among [-759-] the general ones, just as one must not consider the smallest sum of the differences, thus one must not consider the smallest sum of the impressions. The smallest sum of the impression is found in the system of thirty-one parts, followed in order by the systems of seventy-four, fifty and forty-three parts. Were the sum of the impressions to provide the rule for the aforesaid choice, it would follow that the system of fifty parts would proceed almost in the same way as the system of forty-three parts. As I demonstrated at [signum] 11. that the opposite is true, one has to apply to the sum of the impressions everything that was said with regard to the sum of the differences in the paragraph quoted above.

[signum] 14. Someone will ask me what practical application the general system may have. I reply that they would have little or no application at all. If we want to apply a general system to a particular instrument, it is necessary that we equip it with the same number of notes or sounds within the octave as the parts into which the octave itself is divided by the general system. For instance, one cannot take from the general system just some sounds rather than all of the ones of the chromatic scale, because in this way the aim of the general systems, namely that all the musical ratios participated in the same way correspond to each sound taken as the base, becomes compromised. Therefore, if we leave aside many useless subtleties, we obtain the same effect by applying to the chromatic constitution the temperament that is more closely embraced by the general system from which one wanted to borrow a number of sounds. Who shall ever imagine to build an organ or a harpsichord with seventy-four, fifty or forty-three keys for each octave? Signor Conte Lorenzo Magalotti mentions an instrument with thirty-one keys in each octave built by Signor Nigetti from Florence (a) [(a) Lettera scientifica VII, page 80 of the Venetian edition of the year 1734. add. infra lineas]. However, these are museum instruments, whose function is to allow someone who has studied tuning in depth to let us hear its fine tuning and [-760-] good harmony. How will anybody be able to perform our modern extremely fast compositions that can barely be played on our ordinary instruments of twelve keys for every octave on a harpsichord that has four black keys where our common ones have only one, or on an instrument that has a black key where our common ones have none? Instruments with nineteen notes for every octave were known up to Zarlino's time. Such instruments had two black keys between the diatonic white keys that lay at the distance of one tone, and a single black key between the diatonic white keys that lay at the distance of a major semitone. These instruments have been sidelined because of the excessive difficulty in playing them. Therefore, the only instruments played commonly are the normal organ and harpsichords with twelve keys for each octave, which can be tuned equally according to the general system of twelve semitones. However, since the aforementioned system proves too harsh, practical musicians choose an unequal tuning that allows all the tones, whether with the major third or with the minor third, produce a different impression on the ear. I leave it to the following chapter to explain this matter in detail, but I conclude for now that the general tempered system shall never move commonly from theory to practice, as they remain purely as ingenious speculation that are not to be overlooked in an accomplished and complete treatise on counterpoint.

Fourth Chapter

On the nature and properties of the keyboard instruments commonly used

[signum] 1. The accurate examination of the character and properties of our keyboard instrument is extremely important in the practice of counterpoint, in order that one may be aware of what sort of harmony and melody one has under one's fingers, so that one can employ them appropriately, as the inspiration of different emotions requires. If we start from [-761-] the origin of the mentioned instruments, it is well known that the ancients introduced into the diatonic system the sound B b by means of the conjoined tetrachord called by them synemmenon in order to soften the tritone F B.

[Riccati, The laws of counterpoint, 761; text: Scala Diatonica antica. A, B b, B, C, D, E, F, G, a, Semituono, Tuono]

Before the addition of the sound B b, the diatonic scale based on the note called Proslambanomenos, or adopted, to which was assigned later the first letter of the alphabet A, divided the octave A a into two semitones and five tones all undivided. When the tone A B was divided into two semitones by the note B b, this division offered the opportunity to divide into two the other four tones as well, and this is the origin of the keyboard instruments commonly employed, which contain twelve keys in each octave, seven of whom are white and five are black.

[signum] 2. After considering that the octave consists in our instruments of twelve semitones, five major and five minor, some less skilled musicians had the idea to divide the octave itself into twelve equal semitones, in such a way that a black key may be employed equally as a sharp of the white key immediately beneath it and as a flat of the white key immediately above it. The eye find this agreeable by virtue of its simplicity, but the ear rejects it, because it considers the temperament embraced by the general system of twelve medium semitones as too imperfect. In the previous chapters I mentioned the aforesaid system, so I refer the Reader to them. For now, it is sufficient to consult the scale placed at the end of the second chapter, which illustrates clearly how badly it works, especially with regard to the imperfect consonances. As to its practical use, I have never encountered an organ or a harpsichord tuned according to the system of twelve semitones. To tell the truth, all the tones of the same species, either with the major third or with the minor third, should [-762-] sound exactly the same in an instrument tuned in this way. Now, in which instrument does one not feel that the the tones C and B with the major third or A and F with the minor third, for instance? One certainly does not require a fine ear to distinguish such a patent difference. Therefore, if the instruments commonly used do not admit a general and equal temperament, by virtue of which every key can be employed with equal reason as a sharp or as a flat of the adjacent keys respectively above and below, it follows that, properly speaking, each key shall be called in one way rather than in the other one. I said 'properly speaking' because I know very well that the organs and the harpsichords are played as if they were tuned according to the system of twelve semitones, which requires necessarily that the same key has to be played under different roles. Of these roles, which in practice are restricted to two, one is legitimate and the the other one alien, but it can be assigned to the key by virtue of a sufficient approximation. For instance, the white key, to which the letter F truly belongs, is also taken as E #. In order to inform the Reader of the true names of the seven white keys as determined by the diatonic system, it is necessary to investigate the ones of the black keys. By common agreement of all the the tuners of keyboard instruments, one observes that we ascend by eleven fifths all well tempered starting from the black key located between D and E. On this basis, one determines the names suited to each key through the following series of fifths.

E b B b F C G D A E B F # C # G #

The black key between D and E, which lays at the distance of two fifths has to be called E b. The black key between A and B has to be called B b, because it lays at the distance of a fifth from F. Ancients musicians called this key as well with the same name, as we have noted. These are followed by the seven natural letters corresponding to as many diatonic white keys, then the [-763-] sequence ends with three letters altered by the sharp sign. The first of them, F #, which lays a fifth above B, derives from the true name that belongs to the black key between F and G. The name of the second, C #, applies to the black key between C and D, which lays two fifths above B and a fifth above F #. Finally, the third one is G #, which is the name of the black key between G and A, which forms a fifth with C #, a double fifth with F # and a triple fifth with B. The consequence of the established names is that, when one compares the two keys G # and E b, they correspond each other at a diminished sixth, which exceeds the fifth G # D # by the diminished second D # E b. In fact, when our commonly used instruments are tuned well, when we employ the interval G # E b as a fifth, the ear lets us know that said fifth is considerably wider. I present to the Reader the scale of the common harpsichords, which is the simple chromatic scale belonging to the tones of F with the

major third and D with the minor third, as one can deduce easily from what was explained elsewhere (chapter 5., [signum] 16.).

C C # D E b F F # G G # A B b B c

[signum] 3. At this point one may wonder why three sharps and two flats were chosen and placed in those established positions rather than elsewhere. In order to solve this difficulty, I say first of all that both the flats and the sharps were to be understood regularly according to the series of the fifths and not by leap. If one wanted to open the way to a single flat, this had to be necessarily B b. If one wanted to add first the second, then the third one and so on, one had to proceed in order by introducing first E b and then A b, D b and G b. The same reasoning applies to the sharps. In fact, if one proceeds by fifths, the first one that we encounter is the F #, the second one is the C #, the third one G #, the fourth one D # and the fifth one is A #. This is the sequence according to which the three that are found in the harpsichords have been introduced. Had anyone introduced the sharps or the flats by leap ignoring such a rule, for instance by placing D b instead of C #, so that one encountered a double leap, namely, [-764-] from F # to G # and omitting the intervening C # and from E b to D b omitting the intervening A b, one would incur the drawback that I shall explain. While the change from C # to D b does not adduce any advantage, as it does not eliminate the diminished sixth G # E b, nevertheless it is damaging, because, instead of the two well-tempered fifths F # C # and C # G #, two intervals of no use, F # D b, diminished sixth, and D b G #, [[twice]] augmented fourth are admitted. In fact, if one wants to employ them as fifths, one shall find that the first one is wider by a diminished second and the latter narrower by the same amount. Therefore, the absurdity consists in that instead of employing two fifths that are nearly perfect, one substitutes to them another two that are out of tune. Moreover, while in our instruments eleven legitimate fifths would correspond to eleven notes, in the hypothesis that I reject only nine legitimate fifths would correspond to eleven notes. Apart from this, since one proceeds to tune according to the series of the fifths, as it is known, when such series is interrupted, the usual rule to tune the sharps or the flats by leap is compromised as well.

[signum] 4. Secondly, I say that, as far as harmony and melody is concerned, it does not matter that the black keys are employed as sharps or flats, as long as one observes the rule that I prescribed. They could have been taken all as sharps or flats and one would have been able to choose any mixed combination. The only consequence that the different supposition would imply would consist in the fact that the diminished would change position. For instance, if one employs five sharps, one had eleven notes would have their fifth nearly perfect, but the diminished sixth would fall between A# and F. Similarly, if one employs five flats, the aforementioned diminished sixth would occur between B and G b.

In fact, although the choice is indifferent as to harmony and melody, the same cannot be said in relation to the comfort of the hand and the ease of playing. It is certain that the most perfect tones with the major third or minor third are employed more frequently than the others. [-765-] Such are the ones in whose scale all the keys retain their [true add. supra lin.] and legitimate name. Moreover, it is certain that the tones are also easier to play the smaller the number of black keys contained in their scale, as they cause the hand to make a detour and to step out of its trajectory, so to speak. Therefore, the combination in which the most perfect and most commonly employed tones were also the easiest to handle had to be preferred. For instance, if one employed five sharps, it would occur that the best tone would be the one that required five sharps, or black keys, and two white ones, or the other one that requires four black keys and three white ones. Conversely, there would be certain less perfect tones that require a flat, or a black key, and six white ones or two flats, or two black keys, and five white ones. In order to avoid such a drawback, it shall be necessary to divide the number five into two parts as much as possible equal, namely, into the numbers two and three, because in this way the the tone among the best ones that is the most difficult to handle requires four white keys and three black ones. It seems indifferent to choose three sharps and two flats, as it was done, or, conversely, three flats and two sharps, but, in practice, as the sharps are more used than the flats, it is clear that the chosen division is the best of all. The double sharp is often employed in musical compositions. Many indicate it in this way. The double flat is never used

and so far no particular sign has been invented to indicate it.

[signum] 5. After determining the origin and the scale of the common keyboard instrument, I set out to illustrate their tuning. At first it occurs to us to think that they should be tuned according to the best participation, which orders the narrowing of the fifth and of the minor third proportionally, reducing the former by three fourteenth parts of the comma and the latter by five fourteenth parts of the comma. Should we want to temper the tuning of an instrument in this way, we should reduce the fifths to such an extent that, [-766-] after we have tuned three consecutive ones aequisonant to a major sixth, the reductions of said imperfect consonance and of each fifth would make the same impression on the ear. If we add a new fifth narrowed as much as the others, the major third aequisonant to four fifths must be found to be audibly perfect, since one struggles to hear the widening of one seventh of the comma ascribed to it by the best temperament. Following the two mention leads of the proportional alterations of the fifth and the major sixths and of the adjustment of the major thirds, which, however, must be widened slightly, one would be able to perfect the tuning of the eleven fifths that constitute the scale of the common instruments, eight of which must be taken above C sol fa ut, and three beneath it. Up to the point in which one employed the keys in their true role without changing their names, one would hear a very pleasant harmony and melody. However, if the names of the keys are changed according to the different needs, some imperfect consonances would prove too harsh on the ear and the fifth G # D # or A b E b would be found to be unbearable when it is widened by the logarithm 68314., namely, more than one and a quarter comma. We shall discover how much wider is this fifth if we realise a given temperament in this very simple way. Twelve fifths exceed seven octaves by 58856., and, if the sum of the eleven fifths that determine the scale of our instruments were narrower by that amount, the twelfth fifth G # D # or A b E b would be perfect. Therefore, the sum of the reductions of the eleventh mentioned fifths exceeds the quantity 58856. as much as the twelfth fifth G # D # and A b E b is wider than the perfect one. Because of the excessive widening of our fifths, not only the temperament of Monsieur Saveur, which widens the fifth by 59268., but also my third participation, which narrows each of the eleven fifths by $9521 = \frac{3}{17} c$ (and their sub by 104732) and widens [-767-] the twelfth fifth by 45875., which is greater than five sixth of the comma or 54950, need to be excluded from the tuning of the organs and of the harpsichords.

[signum] 6. Therefore, it is necessary to establish which difference can be tolerated in the twelfth fifth G # D # or A b E b. After studying the practice of the most expert tuners and builders of organs and harpsichord, I understood finally that they establish the aforementioned fifth as wider by about half a comma. If our fifth on a particular instrument is wider by 23644, or a little less than half a comma, on the basis of what was said in the previous paragraph the sum of the eleven fifths that determine the scale of our common instruments shall be found to be narrower by 82500. Now shall we discover that the sum of these reductions is distributed perhaps equally among all of the eleven fifths, so that each of them is narrower by 7500., a quantity that is a little smaller than one seventh of the comma? Were this the case, all the tones either with the major third or with the minor third in which one does not change the name of any key would produce on the ear the same impression in relation to the harmony and to the melody, which is something that certainly does not occur. If we compare the tones with the major third C and D, who does not find that the seconds one shines more than the first one? It is clear that an expert ear, when it hears an organ or a harpsichord being played, can distinguish very well the tones of the same nature in which no names of key are changed, so that, for instance, one hears being some state with confidence: "The organist is playing now in the tone F with the major third." Such occurrences shall be explained if we take to consider the way in which the most expert practitioners tune their keyboard instruments.

Starting from C sol fa ut, they tune eight fifths upwards and three downwards, according to what the scale already established of our common instruments requires. As to the six fifths corresponding to the white keys, which to be succinct I shall call diatonic, they reduce them in such a way that their narrowing and the widening of the major thirds produce a similar impression on the ear. Up to this point, they follow my third participation in relation to the diatonic sounds, in line with the system of 55 parts of the comma, [-768-] which participation balances the alterations of the

fifth and of the major third, as well as those of the fourth and of the minor sixth, and distributes perfectly the comma between the aforesaid pairs of consonances. The aforementioned proportional alterations, which produce on the ear the same impressions, serve as a rule for the tuners of such instruments in order to adapt to the diatonic system a temperament that deserves to be considered the best one, if not absolutely, at least in relation to the consonances of the fifth and major third and of the fourth and minor fifth. However, because, if we carried on with the five remaining fifths that contain one or two black keys and that I shall call chromatic, the twelfth fifth G # D # and A b E b and many imperfect consonances in which a key leaves its proper name and takes on another one that does not belong to it, would be too harsh on the ear, they make the five remaining fifths more perfect and less narrow, while they tolerate a more audible difference in the major third. In this way an unequal temperament, which is responsible for the varied character according to which the tones of the same nature differ one from the other while the all the keys retain their original name, is applied to the instruments.

[signum] 7. In order to determine with precision the reduction of each of each of the five chromatic fifths, I employ the same principle that has led me earlier (chapter 1., [signum] 17.) to the choice of the best temperament. The widening of the twelfth fifth G # D # and A b E b must be proportional to the imperfect consonance that is the most out of tune among the true imperfect consonances in relation to whom the keys maintain their proper name. Such is the minor third G # B, whose alteration is equivalent to the comma, detracted the reductions of the three chromatic fifths B F #, F # C # and C # G #. All of the other truly minor thirds are less narrow, since we must subtract from the comma the reductions of three fifths, among whom there is at least [-769-] a diatonic fifth, which, because it is narrower than the chromatic ones, renders the alterations of said thirds smaller than the alteration of the third G # B. The true major E G #, which is wider than the others, appears to be less altered than the minor third G # B by the quantity 9521., by which the diatonic fifth E B is narrowed according to my third participation. If the alterations of the fifth G # D # and of the minor third G # B are proportional, namely, they are represented by the proportion 3:5, they produce a similar impression on the ear. For instance, if one adjusts a little the fifth G # D #, it shall occur that, since the chromatic fifths B F #, F # C #, C # G #, E b B b and B b F must be closer to the perfect fifth in order to achieve this, the imperfection of the minor third G # B shall become greater and thus it shall affect the ear more than the proportional alterations of the fifth G # D # and of the minor third G #B. One encounters a similar drawback if one reduces the alteration of the minor third G # B by reducing further the aforementioned chromatic fifths. In this way one increases the widening of the twelfth fifth G # D #, which, consequently, offends the ear more than the proportional alterations of the fifth G # D # and of the minor third G # B. The alterations of the aforementioned fifth and minor third shall be established as proportional to the number three and five, after one has realised this calculation. A Reader a little familiar with mathematical analysis shall be able to discover its demonstration by oneself. (a) [(a) I present here the mentioned demonstration, should one require it. If we call x the quantity by which each of the five fifths containing one or two black keys must be reduced, I observe that, if we presume that each of the six fifths consisting of two white keys is reduced by 9521., the widening of the twelfth fifth G # D # shall equal the reduction of the eleven true fifths, namely, $6 \cdot 9521 + 5x$ minus the quantity 58856 by which the fifth G # D # would be narrower if the remaining eleven were perfect. The widening of the major sixth B G # is equivalent to the comma 53950 minus $3x$, which is the sum of the reductions of the three fifths B F #, F # C # and C # G #. However, the two alterations of the fifth G # D # and of the major sixth B G # must be represented by the ratio 3:5. Therefore we shall have the equation $6 \cdot 9521 + 5x - 58856 : 53059 = 3x : 3 : 5$, and, consequently, the equation $53950 \cdot 3 - 9x = 6 \cdot 9521 \cdot 5 + 25x - 58854 \cdot 5$, from which we deduce $(53950 \cdot 3 + 58856 \cdot 5 - 6 \cdot 9521 \cdot 5)/34 = x$, in conformity with the law that I established earlier. add. infra lineas]. If I add to the trebled comma, which equals $3 \cdot 53950 = 3 \cdot 53950 = 161850$ the quantity 58856 multiplied by five, which is the amount by which the twelfth fifth G # D # would be reduced and the remaining fifths would be perfect, namely, $5 \cdot 58856 = 294280$, the result would be the sum 456130. I subtract from this the sum of the sixth diatonic fifths multiplied also by five. Since each diatonic fifth must be reduced

according to my third participation by 9521., said sum multiplied by five equals 285630 in our case. Performed the indicated subtraction, let us divide the remainder 170500 by 34 and the quotient 5014 12/17, or nearly 5015. shall be equal to the quantity by which each of the five chromatic fifths must become narrower, so that the twelfth fifth G # D # and the minor third G # B may prove modified appropriately according to the ratio 3:5. In fact, according to the method explained earlier ([signum] 5.), once the logarithm 58856., by which twelve perfect fifths exceed seven octaves, is subtracted from the sum 82207 of the reductions of the eleven consecutive fifths, the remainder 23345. is equivalent to the widening of the twelfth fifth G # D #. Moreover, if one subtracts the sum 15045 of the reductions of the three chromatic fifths B F #, F # C #, C # G # from the comma 53950, one finds the alteration 38905 of the minor third G # B. Now, the discovered alterations of the mentioned fifth and minor third, 23345 and 38905, are [-771-] represented adequately by the ratio 3:5, as one can discover through calculation. The smallest alteration derives from having established the chromatic fifths as narrow by the approximate quantity 5015. that by the exact quantity 5014 12/17. This has been done in order to avoid the fractions.

[signum] 8. I place under the eyes of the Reader the described unequal participation of our instruments. I know perfectly well that one shall find two instruments that are tuned rigorously in the same manner. The alteration of the twelfth fifth G # D #, A b E b shall sound sometimes larger and sometimes smaller, and the unequal temperament shall be more or less regular. Nevertheless, my participation is such that the ones employed by experienced musicians in our common instruments bear at least a very close resemblance to it, if they are not entirely identical to it. An incontrovertible proof will be provided by the observation that, if one accepts my unequal temperament, one will be able to account for the different character of the tones which benefits music to such a high degree. The reduction of each of the sixth diatonic fifths by 9521 conforms exactly to my third participation and resemble closely the system of fifty-five commas that reduces the fifths by 9466. The five remaining chromatic fifths, which together with the six diatonic ones determine the scale of our common instruments, can be seen reduced in my unequal temperament each by the quantity 5015., according a system that is inaudibly better than the one of twelve semitones, which reduces the fifths by 4905. The twelfth fifteenth G # D #, A b E b, contrary to all others, is wider by 23345., a quantity roughly equivalent to three sevenths of the comma. It is useful to note with regard to the double form G # D # and A b E b of the aforementioned fifth that the true name of the lower black key, namely, G #, is preserved in the first one, while the second one assumes the less exact name D #. In fact, since, as everyone knows, the sharp is equal to a minor semitone, according to the rules, in our case the difference between D and D # equals a major semitone. In the second form, while the upper black key preserves its proper name of E b, the lower black key [-772-] changes its name and it is called A b instead of G #. In this case also the flat written next to A is wider than the true flats, because A b is at the distance of a major semitone from A.

[Riccati, The laws of counterpoint, 772; text: Temperamento ineguale de' comuni Stromenti da tasto. Serie di Quinte. A b E b, 3/2, +23345, E b B b, - 05015, B b F, F C, - 09521, C G, G D, D A, A E, E B, B F #, F # C #, C # G #, G # D #]

[signum] 9. I have promised earlier (chapter 1., [signum] 18.) that I would explain how it is possible to understand that an instrument is tuned according to the system of forty-three merides invented by Monsieur Sauveur. The lauded author states in the Records of the Royal Academy of Sciences of Paris for the year 1711. that, after dividing the octave on a monochord according to the systems of thirty-one, forty-three, fifty and fifty-five parts, if I tune the string of said monochord so that it is in unison with the C sol fa ut of a harpsichord tuned very accurately, by placing the bridge under the string and by tuning it in unison with each key of the harpsichord, he managed to discover that [-773-] the bridge coincided precisely or appropriately with the divisions of the system of forty-three merides, while it departed from the divisions of the other systems when they were very different from those of the system of forty-three parts. As to the systems of thirty-one and fifty

parts, the statement of this French author must not be challenged. As for the rest, I do not agree with him entirely. In order to avoid any misunderstanding, it is necessary to distinguish the white keys from the black ones. In relation to the white keys, I concede also that in a well-tempered instrument one shall find very often a number of unisons that coincide exactly or approximately with the corresponding divisions of the monochord according to forty-three merides. Two different tunings differ mostly in the sound of the key B, which is the most distant from the key C in the series of the diatonic fifths, as it lays at the distance of five fifths from it. If we multiply by five the difference of the reductions through which the different tuning modify constantly, as I presume, each diatonic fifth, the product shall express the difference between the sounds of the key B. My unequal temperament reduces the diatonic fifths less than the system of forty-three merides by the quantity 12118. If I multiply this quantity by 5, I obtain the product 6090 equivalent roughly to one ninth of the comma 53950., which is the quantity by which the sound of the key B tuned according to my system exceeds the quantity of the same key tuned according to the Monsieur Saveur's system. If the tuning of an instrument is half-way between my unequal participation and the system of forty-three merides, in that case the sound B shall be similar to the corresponding ones of one and of the other temperament, while differing from both by one eighteenth of the comma, which is a very small quantity, and, I am about to say, inaudible. If the sounds B of the instrument and of Monsieur Saveur's system are equivalent, it is clear, on the basis of what was said, that one shall find greater correspondence between the other diatonic sounds. In a sufficiently well-tempered instrument the white keys can accept with precision the participation of this French theorist as well. In fact, if each diatonic fifth is reduced by 10739. merides according to the system of [-774-] forty-three merides, and, since the twelfth fifth G # D #, A b E b tolerates to be widened by 30103., which is a little more than half a comma, the five chromatic fifths are found to be tempered according to the system of twelve semitones, as each of them is reduced by 4905.

If we move from the white to the black keys, I shall never consider an instrument to be well-tempered when not only the sixth diatonic fifths, but also the five chromatic ones are narrower according to Monsieur Sauveur's system. The reason adduced by me earlier ([signature] 1.) consists in the fact that those consonances in which the name of a key is changed, are too far removed from being perfect, and the fifth G # D # and A b E b in particular is too wide by 59268. which is a larger quantity than the comma. Moreover, practice teaches us that all the tones, either with the major third or with the minor third in which a key retains its proper name, produce a difference impression on the ear in relation to harmony and melody. This occurs, as I noted in the sixth paragraph, because all of the eleven fifths of which the scale of our common instruments consists are not tuned equally. Therefore, one shall be able to say in truth that in the best tuned organs and harpsichords the seven diatonic keys correspond in many cases to the system of forty-three merides at least approximately. I have used these words because such correspondence will be found with greater frequency with the system of fifty-five commas, to which my unequal participation corresponds in relation to the white keys. There, the five black keys exceed the system of fifty-five parts by a small amount, as they lean, one more or less than the other, towards the system of twelve semitones. F # is the black key that approaches more closely the system of fifty-five parts, since it is higher than the system requires by 4174., which is an amount so small that it is almost completely inaudible. The black key furthest removed from the mentioned system is G #, since it is widening reaches 13076., which is a little less than one fourth of the comma. Our key sounds higher than the system of forty-three merides would allow it to sound by 23258. [-775-] or by two wider parts of the comma, which shall notify us without danger of error if said key in a given system is tuned according to my unequal participation or according to the mentioned system.

I conclude the present paragraph with an important reflection. If we establish the principle that the twelfth fifth G # D # and A b E b must not be considerably higher by more than half a comma, it follows that the sum of the eleven remaining fifths is narrower by 85000. When we distribute the sum of these reductions, we must ensure on one hand that the five chromatic fifths are not tuned in a way that is worse than the one required by the system of twelve semitones as a consequence of tuning the sixth diatonic fifths too accurately; on the other hand, we must ensure

that the diatonic fifths are not narrow enough because the chromatic fifths are tuned too well, so that the tones employed most often sound too harsh. If I am not mistaken, I believe that my unequal participation avoids both of these drawbacks.

[signum] 10. Some of the calculations that I illustrated in the previous paragraph shall be performed with the greatest ease, once one has deduced the tempered scale of our instruments from my unequal participation. Once this has been done, one shall be able to compare the sounds contained in that scale itself with the correspondent ones of the general systems, whose scale have been laid out by me orderly at the end of the second chapter. Therefore, I move on to discover the sound of the mentioned scale within the space of an octave and I shall express them as logarithms, as we are used to doing. First of all, the quantity 0., logarithm of one, is assigned to low C, while the logarithm 3010300., logarithm of two, or of the octave, is assigned to high c. The sound G equals a fifth reduced by 9521, or 1751392. In order to ascertain the value of the sound D, which lays at the distance of two fifths, each narrower by 99521, minus the octave from C, I double the tempered fifth place above and then I subtract the octave 3010300., so that I obtain $D = 492484$. Added [-776-] to D the fifth D A, which is also reduced by 9521, we encounter $A = 2245876$. With a similar method, namely, by adding fifths reduced by the amount required by my participation and by subtracting the octave, we find, one after the other, the sounds E, B, F #, C # and G #, which are recorded in the scale placed below.

It remains for us to investigate the sounds F, B b and E b generated by the three fifths F C, B b F and E b B b placed below C. Since these fifths are equivalent to the fourths C F, F B b, B b E b taken upwards, which are wider by the same quantity by which the aforementioned fifths and added to them form the octave, one can understand at first sight that F is equivalent to a fourth wider by 9521, or 1258908. In order to obtain the sound B b, I add the quantity of the sound F B b, which is wider by a mere 5015, to the value of $F = 1258908.$, and the result is $B b = 2513310$. Finally, if I add the fourth B b E b, itself wider by 5015., to $B b = 2513310$. and I subtract the octave 3010300, we obtain $E b = 787412$.

The table containing the tempered scale of the keyboard instruments commonly used contains four columns. The first one contains the natural letters and those altered by the sharp or flat sign. In the second one can see that I placed the names of the elements and of the intervals in relation with the base C of the scale. The third one contains the values of the sounds that constitute the scale expressed in logarithms. Finally, the fourth and last column illustrates the differences by which the logarithms of the tempered ratios are larger or smaller than the logarithms of the perfect ratios or of the extreme ones that are recorded diligently and in order in that column itself. I note the numbers expressing enlargement with the sign + and the one expressing reduction with the sign – according to the method adopted by me in the table of the general systems (chapter 2., [signum 24].).

[-777-] [Riccati, The laws of counterpoint, 777; text: Scala temperata de' comuni Stromenti da Tasto. C, C #, D, E b, E, F, F #, G, G #, A, B b, B, c, Semituono minore, Seconda maggiore, Terza, Quarta, [[superflua]] [maggiore add. supra lin.] Quinta, superflua, Sesta, Settima, Ottava, 00000, 227556, 492484, 757412, 984968, 1258908, 1481958, 1751392, 1983454, 2243876, 2513310, 2736360, 3010300, $25/24 + 50268$, $21/20 + 15663$, $10/9 + 34909$, $28/25 + 00304$, $9/8 - 19041$, $6/5 - 34400$, $5/4 + 15868$, $4/3 + 09521$, $7/5 + 20678$, $3/2 - 09521$, $14/9 + 64598$, $15/16 + 45254$, $63/40 + 10648$, $5/3 + 25388$, $16/9 + 14535$, $25/14 - 04810$, $9/5 - 39415$, $28/15 + 25692$, $15/8 + 06347$, 2/1]

[signum] 11. Certainly, I must not omit the division of a monochord according to my unequal temperament, thanks to which one can prove in practice to what extent a well-tempered instrument conforms [-778-] to such temperament. It is well known that different parts of the same string that are capable to produce different sounds correspond to each other in a ratios that are the inverse ones of the one expressing the sounds themselves. Take two sounds, for instance C and G, that relate to each other according to the proportion 1:3/2, the lengths of string that produce them, measured on the same string, shall be expressed by the ratio 1:2/3 or 2:4/3. Observe that the lengths

2 and $\frac{4}{2}$ are correspond to the lengths of the sound that added to the sounds 1 and $\frac{3}{2}$ form the octave. Therefore, given a string measuring 2. in length and expressed the sound C as the number one, as per usual, we shall discover the portions of the string that generate the sounds contained in the octave C c if we consider the sounds that added to the former ones produce the octave. Let us consider each unit of the string 2. divided into 3000. parts, so that the entire string is divided into 6000. parts, which is the produce of 2. and 3000. It is clear that we shall a similar distribution of the string 2. if we multiply all the portions of the string 2. by 3000. We must deduce then that the parts of the string 6000. that we required are equal to the product between 3000 and the sounds that added to the former ones produce the octave, expressed in my usual fashion.

Now, since the sounds of my unequal temperament are expressed as logarithms, let us establish the following series of operations. Let us subtract the logarithm of a sound from the one of the octave 3010300. so that we obtain the logarithm of the sound that added to the former constitutes the octave. Let us add the logarithm of the number 3000. to the latter one. The deriving sum shall be the logarithm of the portion of the string 6000. that generates the given sound. Let us consult the tables of the logarithms. The number corresponding at the logarithm found shall be equal to the required portion of the string. Moreover, since it will happen very rarely that the mention logarithm coincides exactly with any logarithm contained in the tables, we shall find the two that precede and follow it immediately and we shall select the one to which it is closer. Thus, the corresponding number found in the tables will represent the part of the string that must produce the desired sound. The greatest error that one may incur consists in $\frac{1}{2}$. I select the worst case scenario [-779-] and I presume that such error is committed in determining the length of the string B, which is the shortest of the ones contained in the octave C c. The diatonic system assigns to it the length 3200. Now, the proportion $3200:3200 + \frac{1}{2}$, or $6400:6400 + 1$ is equivalent to one seventy-ninth of the comma, an easy calculation to perform with logarithms. Such small quantities are not audible. In fact, should one even want to consider them, one should take the difference between the given logarithm and the closest to it in the tables, and then the difference between the two logarithms in the table that are immediately before and after the given one, and, established a fraction in which the first difference is the numerator and the second one the denominator, it shall equal the quantity that has to be added or subtracted, according to whether the given logarithm is larger or smaller than the closest to it contained in the tables.

I did not select the division of the monochord into 6000 parts at random. The instrument that I am employing in this experiment is divided into sixth half-ounces of the Venetian foot. The base of the mobile bridge that is placed under the string is equal to one of said half-ounces and it is divided into ten portions each of them equal to $\frac{1}{5}$ of a line. Thus, one has an easy division of the monochord into 600 parts. Moreover, for reasons of greater precision, every tenth part of the base of the bridge is presumed divided into ten parts, so that the base of the bridge is divided into ten parts and the entire monochord into 6000.

[signum] 12. In the table that I present to the Readers, the first column contains the letter that constitute the scale of the common instrument. The second column contains the logarithms of the portions of the monochord capable of producing the sounds of said tempered scale according to my unequal participation. The third and last column shows the aforesaid portions expressed with four numbers. The first two indicate the parts or half-ounces into which the monochord is divided in reality. The third indicates the number of the parts into which the base of the [-780-] bridge equivalent to half of one ounce. Finally, the fourth number indicates the tenths of one part of the base of the bridge corresponding to $\frac{3}{5}$ of a line. As to the irrelevant fractions that derive from this, I have discussed them sufficiently a little earlier ([signum] 11.). As to the fourth number indicating the subdivisions of a part of the base of the bridge, practice is able to distinguish them up to a point. The eye can distinguish with sufficient accuracy one fourth, one half and three fourths of one of said parts that it is sufficient for us to consider. In this way, the greatest error that one can commit amounts to one eighth of one part of the base of the bridge. Let us fail by this amount to determine the length of the shortest string B, which corresponds to 320. of the mentioned portions according to the diatonic system. The proportion $320:320 + \frac{1}{8}$ or $2560:2560 + 1$ is equivalent to $\frac{1}{32}$ of the

comma, which is a very small difference, so much so that the sharpest ear cannot distinguish it. I note that in the fourth number of the number of the third column the value $2\frac{1}{2}$ indicates one fourth of a portion of the base of the bridge, 5. indicates half of it and $7\frac{1}{2}$ three fourths of it. According to whether the fourth number is closer to one or the other of said three numbers, I consider one fourth, one half or three fourths. If it comes closer to ten than to $7\frac{1}{2}$, it should be considered equal to one parts of the base of the bridge.

The table just now described shall be followed by three small tables containing the values of the notes B, E b and G # according to the various temperaments that can be more or less relevant when one examines the tuning of an organ or of a harpsichord. I pointed out earlier ([signum] 9.) that the note B varies more than any other diatonic sound in the different tunings. One must deduce that the greatest alteration in length, except only for the diatonic notes, occurs in the note B, which, as a consequence of this, shall be the most suited to determine which participation has been applied to the white keys of an instrument. [-781-] The chromatic notes E b and G # complete the series of the eleven fifths that determine the scale of the instruments commonly used, eight of them being taken above C, and three beneath it. Therefore, if several tunings are compared, the greatest difference in length shall be observed in the note E b in relation to the ascending fifths. The difference in size of this last note shall exceed all the other ones, as it is the note that is most remote from C in the series of the fifths. The portions of the monochord tuned perfectly to the C sol fa ut of an instrument that are in unison with the notes E b and G # of said instrument indicate to me how much narrower the sum of the three descending fifth and the sum of the eight ascending ones is, as well how much wider the twelfth fifth G # D #, A b E b is. In fact, if determine the portion of the monochord that is in unison with B, I shall learn through it how much narrower is the sum of the five diatonic fifths and the sum of the three chromatic one, all of whom are ascending, which provides me with a sufficiently complete knowledge of the tuning of the instrument. I considered redundant to record the different lengths of the portion of the monochord that produces the sound F in the various participations. In fact, since F lays at the distance of only one fifth from C, the differences are very small and hard to observe. Our three short tables have been introduced to avoid mathematical calculations for the most part or at least for a good part.

[Riccati, The laws of counterpoint, 782,1; text: Divisione d'un monocordo giusta il mio ineguale Temperamento. Logaritmi delle proporzioni del Monocordo, C, 37781512, 60, 00, C # 37555956, 56, 94, - 219/763, D, 37289028, 53, 57, - 188/810, E b, 37924100, 50, 40, - 205/861, E 36796544,, 47, 82, + 448/908, F, 36522604, 44, 90, + 141/967, F #, 36299554, 42, 65, + 364/1018, G, 36030120, 40, 09, - 241/1084, G #, 35798058, 38, 00, + 222/1143, A, 35537636, 35, 79, + 19/1213, B b, 35045152, 31, 95, + 443/1359, c, 34771212, 30, 00]

[Riccati, The laws of counterpoint, 782,2; text: Porzioni del Monocordo, che nei notati Temperamenti sono unisone alla corda B. Logaritmi delle porzioni del Monocodo. B, Sistema 43., 35051240, 32, 00, - 260/1358, Mio ineguale Temperamento. 35045152, 31, 95, + 443/1359, 55., 35044876, 31, 95, + 167/1359, 12. 35022070, 31, 78, + 532/1366]

[Riccati, The laws of counterpoint, 783; text: Porzioni del Monocordo, che nei notati Temperamenti sono unisone alla corda E b. Logaritmi delle porzioni del Monocodo. Sistema 43. 37011436, 50, 25, + 75/864, 55. 37015254, 50, 29, + 4388/864, Mio ineguale temperamento. 37024100, 50, 40, - 205/861, 12. 37028937, 50, 45, + 325/860, G #, 35821317, 38, 21, - 453/1136, 35811134, 38, 21, - 395/1140, 35798058, 38, 00, + 222/1143, 35774645, 37,80, - 273/1149]

[signum] 13. I took the opportunity to widen my understanding of these matters from the fact that Signor Don Pietro Nachini, an extremely skilled organ builder, had built an organ for the church of the Ospitale di Santa Maria and was tuning it diligently. When I asked him what rule he employed to tune the instrument well, he answered that he tuned all the eleven fifths in sequence reducing the by the same amount, so that the ear would receive the same impression from the

narrowing of the fifths and from the widening of the eight truly major thirds. The Reader can realise at first sight that [-784-] Signor Nachini applies my third participation in tuning the organ, in which the alterations of the fifth and of the major third are in proportion. He added that the twelfth fifth $G \# D \#$ or $A b E b$ and the four major thirds $B D \#$ and $C b E b$, $F \# A \#$ and $G b B b$, $C \# E \#$ and $D b F$, and $G \# B \#$ and $A b C$ cannot be reduced to good harmony, and that he employed is the only one capable to tune organs and harpsichords correctly, because, if one does it differently, what the improvement granted to a consonance is removed from another one with much displeasure of the ear. There is no doubt that, if one does not consider the poor quality of the consonances in which one key changes name and if one compares, as Signor Nachini does, the alterations of the fifth and of the major third, the tuning adopted by him deserves to be considered the best one. I asked him how much wider was the fifth $G \# D \#$ or $A b E b$ was in the instruments tuned by him and he replied that it was wider by one comma. In fact, if my third participation is applied to the instruments commonly used, the mentioned fifth is wider by 45875., a quantity that, since it exceeds five sixths of the comma, it not very far removed from a whole comma. In order to be completely sure of the stated widening of said fifth and since I did not want to trust my own ears, I asked the many times lauded Padre Maestro Francescantonio Vallotti to ascertain experimentally if said fifth was also wider by one comma in the organs of the basilica of Saint Anthony of Padua, which also had been built by Signor Nachini. Padre Vallotti examined said fifth accurately and he found it to be wider by one comma, as Signor Nachini had told me.

I note here in passing that Signor Nachini's practice confirms incontrovertibly my theory, by which I established that the difference affect the ear in a similar way when they are proportional to the largest odd number that occur in the musical ratios. If one establishes the alterations of the fifth $3/2$ and of the major third $5/4$ as proportional to the numbers 3. and 5. we obtain the fifth $G \# D \#$ which is widened by nearly one comma. However, since Signor Nachini had the sole scope that the alterations of the fifths [-785-] and of the major third should produce an equal impression on the ear, widens said fifth $G \# D \#$ by one comma. Therefore, the imperfections of the fifth and of the major third affect the ear in a similar way when they are represented by the ratio 3:5.

In the letter (a) [(a) 25. may 1751. add. infra lineas], in which Padre Vallotti informs me of the observed widening of the fifth $G \# D \#$, he cannot approve of Signor Nachini's method for tuning the organ in relation to the black keys, and, expressing himself in a rather modest way, he says that he wants to believe that the mention widening of the fifth is accidental rather than produced deliberately on the basis of an established system. He says that he had discussed the matter on other occasions with organ makers of the same opinion of Signor Nachini, as to the practice, and that he had succeeded easily to convince them to raise the intonation of the note $G \#$, so that the note $G \# D \#$ may become tolerable, while at the same time the third $F A b$ does not result so languid that the ear struggles to recognise it as a minor third, which occurs if one narrows the note $G \#$ in the aforementioned way. He states that a fifth that is widened by a whole comma is intolerable. He says that he experimented this alteration many years before, when he himself considered the tuning of our instruments. Finally, he adds that he met the famous Signor Giuseppe Tartini, that he discussed the matter with him and that he found him to be of the same opinion, as they both agreed that said fifth must be widened by half of one comma.

Padre Maestro Vallotti informs me in a later letter (b) [(b) 4. June 1751. add. infra lineas] that Signor Tartini repeated the experiment on the violin and that he found that a fifth that is wider by one comma is unbearable for a fine ear. He says that it was certain that that particular fifth was wider by one comma, since Signor Tartini took care to make this clear by means of the third sound produced in the air by two sounds of the violin, a very beautiful experiment [-786-] mentioned by me elsewhere (book 1., chapter 2., [signum] 7. and book 3., chapter 1., [signum] 8.). The letter concludes with the assurance that he shall never embrace Signor Nachini's ideas in tuning his own harpsichord. The shared opinion of two great music masters, such as Padre Vallotti and Signor Tartini are, has removed the doubts engendered in me by the words of Signor Don Pietro Nachini, who is great expert in his profession, and confirmed me in my opinion that my unequal temperament is the best one that can be employed in a keyboard instrument.

[signum] 14. Keyboard instruments can be tuned according to my unequal participation by relying only on the ear or by employing the division of the monochord express in the table placed at the end of the eleventh paragraph. If we want to rely only on the ear, as we normally do, we have to tune the four fifths C G, G D, D A and A E, a little narrow, and, if the major third C E determined by them is found to be a little wide, so that the ear is equally aware of the reduction of the fifths and of the widening of the major third, the temperament is realised to perfection. However, if the major third C E proves narrow or perfect rather than too wide, it must be altered so that it is a little wider, while the difference produced in the fifth A E must be distributed among the other three, namely, C G, G D, D A. After this, it shall be appropriate to consider the major sixth E and G E and the minor third A C and E G, which are the intervals that added to the sixths produce the octave. The sixth will have to sound wider but not harsh, while the third will have to sound narrow, but not languid. Then, one has to tune the fifths E B and F C, which are equal to the four mentioned above. This will be accomplished by comparing them to the major thirds G B and F A, which must prove as widened as the third C E. One must not fail to check the major sixths D B and F D, and of the minor thirds B D and D F, in which one must find the same conditions already noted with regard to the major sixths C A and G E and to the minor thirds A C and E G. After we have executed the participation of the white diatonic keys, I suggest the following method to obtain the participation of the black [-787-] chromatic keys. Tune the major third E G # a little wider than the third C E, ensuring that it sounds piquant, but not harsh. Proceed then to temper the minor third C E b, which must be a little narrower than the diatonic minor thirds without becoming languid in character. The tuner must ensure to play the major third E b G, which must sound wider, but agreeable and half-way between the two thirds C E and E G #. If the interval G # E b is equivalent to a tolerably widened fifth and as piquant on the ear as the major sixth B G #, we have achieved the desired aim. If we find that the fifth G # D # and A b E b are perfect or nearly perfect, we have evidence that one or the other or both of the thirds E G #, C E b are altered too much. It shall be easy to remedy this inconvenience if one has observed calibrated those thirds, so that the fifth G # D # or A b E b and the major sixth B G # produce a similar impression on the ear. When we lower the note G # or A b and we raise the note E b or D #, we must ensure that on one side the minor third F A b does not prove too languid, while we must avoid that the major sixth B D # sounds too prominent. However, should the fifth G # D # or A b E b prove too wide, it shall be appropriate to mitigate it by raising the major third E G # or by reducing the minor third C E b by altering one or both of the thirds themselves by ensuring that they adduce the least possible discomfort to the ear. After the intervals are tempered in such a way, the instrument can be said to be in tune, as it proves extremely easy to temper the the five chromatic fifths (three between B and G #, namely, B F #, F # C #, C # G #, and two between Eb and F, namely, E b B b, B b F) as almost perfect by distributing appropriately their small reductions so that they may sound almost equal.

I would never advise anyone to employ only my division of the monochord to tune their instrument. When two sounds of the instrument must produce a consonance, especially a perfect one, if I commit two small and opposite errors in tuning them in unison with the corresponding portions of the monochord, by making one slightly too high and the other one slightly too low, the consonance [-788-] proves always badly tempered, and in many cases either too narrow or too wide and the ear results considerably unhappy with it. If one tunes an organ or a harpsichord in this way, one must revise the tuning and eliminate the errors made with the judgement of the ear by re-distributing the difference as appropriate. The main application of my division of the monochord is not as a method to tune the instrument, but as an instrument to examine its tuning. I shall suggest a way to make partial use the monochord that proved very useful in practice and leads securely to my unequal participation. Tune the fifth C G a little narrow, as usual, and then tune the monochord to a scrupulous unison with the C sol fa ut of the instrument. Correct the tuning by altering the fifth, which, according to my unequal temperament must equal approximately the portion of the monochord $\frac{40}{1}$ or forty parts of the ones into which the monochord is divided with the addition of a tenth part of the base of the mobile bridge. Once we have determined such portion by means of said bridge, if it is found to be perfectly in tune with the G sol re ut of the instrument, we are

completely satisfied. If it is not so, we must again compare the monochord with the sound C sol fa ut of the instrument in order to remove the little error that might have crept in. Repeated experiments have taught me that such correction is very useful and, I would say, almost necessary in order to avoid the mistakes in tuning the monochord with the C sol fa ut of the instrument. If the mentioned tuning is not absolutely correct, it becomes useless to employ the divisions of the monochord to temper the instrument. After this, measure with the bridge the portion 38, 0 and tune with it perfectly the G # of the instrument, as my unequal participation requires. This is use that I think one should make of the monochord. Then, divide the interval C G # into two major thirds C E and E G # by relying solely on the judgement of the ear and make it no more [-789-] than a little piquant. The tuner should then move on to tune the fifth G # D # or A b E b a little wider. When one decides how wide it should be, one should take care that the third C E b should prove narrow and the third E b should prove wide, but both must sound pleasant. After establishing the sounds E, G # and E b, on which the tuning of the instrument depends, the rest of the participation can be provided with ease. After tempering the four fifths between C and E as equally as possible, the two remaining diatonic fifths, E B and F C, must be tuned similarly narrow. This shall be obtained by checking the major thirds G B and F A which must be as wide as the third C E. Finally, one must adjust the five chromatic fifths, three between B and G # and two between E b and F, by distributing the smallest differences as accurately as possible, and thus the entire tuning of the instrument according to my unequal participation shall be complete. I inform the Reader that my monochord, whose length 60,0 (I omit the last number indicating the hundredth parts of the base of the bridge) is tuned at the unison with the C sol fa ut sounded by a four foot pipe on an eight foot organ and expressed by the note written in the second space in the bass clef or between the second and the third line. Now, in order to judge the imperfect consonances correctly, it shall be useful that their sound shall be contained within the boundaries of the higher octave 30, 0. and 15, 0 than within those of the lower one 60, 0. and 30, 0., or at least that the lower sound belongs to the lower octave and the higher one to the higher one. For instance, if one has to divide the interval C G # into two major thirds C E and E G # according to the method proposed, I tune exactly the octaves of C and G #, which must be in unison with the portions of monochord 30, 0 and 19, 0, and I place the middle sound E between these sounds following the rules described a little earlier. The consonances that are too low, especially the imperfect ones, produce a certain beating that makes them less pleasant and upset considerably the judgement of the ear. The explanation of such an effect has been adduced on another occasion (book 3., chapter 5., [signum] 11.).

[signum] 15. So that the Reader may form a distinct idea of the commonly used keyboard instruments, I lay out all the species of the elements and of the intervals employed in counterpoint in as many separate series as they appear in those instruments. I shall order these series in the same way in which I ordered the series of the fifths in the seventh paragraph. After I have laid out the series of a given species of intervals, I shall follow it with the series of the intervals that added to the previous ones constitute the octave, as long as they are employed in music. Then I shall explain both of these series through my considerations.

[-791-] [Riccati, The laws of counterpoint, 791; text: Progressioni di tutti gl'intervalli minori del'Ottava usitati nel Contrappunto come si ritrovano nei comuni stromenti accordati giusta il mio ineguale Temperamento. Serie di Semituoni minori. F b F, C b C, G b G, D b D, A b A, E b E, B b B, F F #, C C #, G G #, D D #, A A #, E E #, B B #, F # F 2 #, C # C 2 #, 21/20, 16/15, 25/24, + 62047, - 06347, + 57541, - 10853, + 53035, - 15359, + 48529, - 19865, + 50268, + 15663, + 45762, + 11157, +50268, + 15663, + 54774, + 20169, + 53035, - 15359, + 57541, - 10853, + 62047, - 06347, + 62047, - 06347, + 57541, - 10853, + 53035, - 15359]

[-792-] 16. The minor semitones that take place in our instruments are twelve, namely, as many as the number of the keys, since a minor semitone corresponds to a particular key. The same occurs in the case of all the other elements and intervals, which I say once and of all. In the proposed series the minor semitones appear to be more than twelve because the same keys called

with different names are involved in forming two minor semitones. The same keys plays the following pairs of semitones, according to what the composition requires: F b F, E E #, C b C, B B#; G b G, F # F 2#; D b D, C # C 2#. The number of the intervals called minor semitones in a strict sense, namely, those in which both keys preserve their proper name, are five in number. They are: E b E, B b B, F F #, C C # and G G #. These are the ones that are employed more frequently in counterpoint. In order that our common instruments may be employed as general and circular, said semitones prove rather larger than they should, since they all exceed the ratio 21/20 at least by 11157. and at most by 20169. The best participation requires that the minor semitone must equal 21/20 – 07629. Were the chromatic fifths that I reduced by 05015. were to be reduced by 09521, or as much as the diatonic ones, the minor semitones would not be wider than the ratio 21/20, but by 06651. Therefore, in relation to the temperament assigned to the diatonic system, whose greater perfection is noticed by the ear, the minor semitones B b B, F F # prove too wide by 04506, the minor semitones E b E, C C # by 09012 and the minor semitone G G# by 13518. The widening of the semitones F F#, C C # and G G # depend on the fact that the chromatic notes altered by the sharps F #, C # and G # prove higher than the diatonic participation requires by the noted differences 04506., 09012. and 13518. Conversely, the widening of the semitones B b B and E b E derive from the fact that the chromatic notes altered by the flat, namely, B b and E b, are lower than the diatonic participation requires by the differences 04506 and 09012. I invite the Reader to observe that the best tuned chromatic notes are the ones that occur in the natural scales of the tones closest to the diatonic tones C with the major third and A with the minor third. For instance, the notes that are raised and lowered less respectively, F # and B b, belong to the scales of the natural tones G and F with the major third that are subordinate to the similar tone C as well as to the tones E and E with the minor third that are subordinate to the similar tone A. Now, a sensitive and expert ear realises the widening and from the reduction more or less considerable of the sharps and of the flats respectively and employs them as an element to distinguish one from the other those tones with the major third or with the minor third in which every key maintains its proper name.

[signum] 17. After discussing sufficiently the five minor semitones that we call in this way in a strict sense, I move on to make a few observation on the other ones that, albeit they are in fact major semitones, nevertheless they are employed as minor semitone improperly and by approximation. These are found between seven pairs of keys, and five of them are expressed in two different ways, according to whether one changes the name of one of the keys or of the other one. The remaining two are notated in a single way with the letters D D # and A A #, in which the second letters is obtained by altering the first one. Were we to alter the names of the lower notes, we would be presented with the forms E 3b B 3b and B 2b B b containing the double flats that are not employed in musical compositions. All of our seven semitones are half-way between the ratios 21/20 belonging to the minor semitone and 16/15 belonging to the major semitone in such a way, nevertheless, that they are closer to the latter than to the former. The smallest semitone of the seven that we are discussing is A b A = G # G 2 #, which is smaller by 19865. than the semitone 16/15 and is larger than the semitone 21/20 by 48529 or a little less than the comma. The largest semitones are F b F = E E # and C b C = B B #. They coincide with the ratio 16/15, as it exceeds them only by 06347., while they are removed from the ratio 21/20 by 64047, a quantity that is very close to the smallest element or enharmonic diesis $64/63 = 68393.$, which is the difference between the two semitones 16/15 and 21/20. [-794-] I pointed out several times that the major and minor semitones differ from each other by a diminished second, which is equivalent to 79876. according to the best participation. My unequal temperament reduces said difference as much as it is allowed by the good tuning of the instruments commonly used, so that a semitone can be employed instead of another one. If we compare the smallest of our semitones, namely A b A = G # G 2# 21/20 + 48529. with the largest among the five semitones that are really minor, which is G G # 21/20 + 20269., their difference 28360 only reaches about half of the comma. In fact, if one compares one of the largest of the seven semitones, for instance C b C = B # 21/20 + 62047. with one of the smallest of the five ones, for instance B b B 21/20 + 11157., the comma exceeds slightly their difference 50890. It is worth mentioning that, if the series of the semitones is reduced to only twelve terms, the first of

which is C b C and the last one E E #, even the seven semitones that are employed as minor semitones are less wide and better tuned according to whether the notes altered by the sharp or by the flat signs contained in them belong to the natural scales of tones that are closer to the two diatonic ones, namely, C with the major third and A with the minor third, or otherwise. It is enough to cast an eye over the mentioned series to become convinced that this is true. If one exceeds the mentioned terms, namely, C b C and E E #, we do not find any newer semitones, but new expressions of the ones that are contained within the series. If we exceed the series on the side of the term E E #, the semitone B B # is the same as C b C, which is removed from it by twelve semitones, F # F 2# is the same as G b G and so on. This is why the semitones B B#, F # F 2#, C # C 2# etcetera do not continue to grow, as B B # is equal to E E #, while the others F # F 2#, C C 2# etcetera become gradually smaller, as the former corresponds to A A # and the latter to D D #. As the minor semitone is also called augmented unison, our semitones, which exceed the truly minor ones by a sort of [-795-] enharmonic diesis deserve the name of more than augmented unisons. These more than augmented intervals, which the ear notices without paying too much attention, derive from raising the higher note, as it occurs in the semitones B B #, E E # etcetera or from lowering the lower note, as it occurs in the semitones F b F, C b C etcetera. At this point one must consider that although the same keys are involved in the semitones B B # and C b C, nevertheless they make a very different impression on the ear. When I employ the two keys as B and B #, what I said earlier convince me that the higher note B # is raised too much. The complete opposite occurs when I employ the two keys as C b C, since the ear then believes that the note C b has been lowered too much. The noted enharmonic enlargements and reductions of intervals introduce considerable alterations in the tones in which one or more keys change name, and because of these alterations they assume a particular and distinguishing character. For instance, in the tone F with the minor third, besides the two flats B b and B b that, as we have seen, are a bit lower than what the temperament of the diatonic sounds would require, we have the third note A b and the sixth note D b reduced by an enharmonic element. The effect of the enlargements and reductions that we are discussing shall be understood even better, if one considers that, as the major and minor intervals are turned into semi-augmented and semi-diminished, they cause them to become closer to an alien ratio that does not correspond to the one of the major and minor intervals. We have seen that the more than augmented unisons are closer to the ratio 16/15 pertaining to the major or minor semitones than to the ratio 21/20, which is one of the ones belonging to the augmented unisons. We will observe soon ([signum] 23., [signum] 25.) that the semi-diminished and semi-augmented thirds come closer to the ratios 7/6 and 9/7 belonging to the augmented second and to the diminished fourth than to the ratios 6/5 and 5/4 pertaining, as we know, to the major third and to the minor third. It is easy to understand that the substitution of a ratio [-796-] placed instead of another one must produce considerable variety both in the harmony and in the melody.

[Riccati, The laws of counterpoint, 796; text: Serie di Semituoni maggiori, o Seconde minori. Settime, E b F b, B b C b, F G b, C D b, G A b, D E b, A B b, E F, B C, F # G, C # D, G # A, D # E, A # B, E # F #, B # C #, F 2# G #, 21/20, 16/15, 15/14, , + 15663, - 52731, + 11157, - 57237, + 20169, - 48225, - 15359, - 34704, - 10853, - 30198, - 06347, - 25692, - 10853, - 19865, - 39210, + 15663, - 48225, F b E b, C b B b, G b F, D b C, A b G, E b D, B b A, F E, C B, G F #, D C #, A G #, E D #, B A # F # E #, C # B #, G # F 2#, 40/21, 15/8, 28/15, - 15663, + 52731, - 11157, + 57237, - 11157, + 57237, - 20169, + 48225, + 15359, + 34704, + 10853, + 30198, + 15359, + 19865, + 39210]

[-797-] [signum] 18. The same twelve pairs of keys adjacent one to the other are involved in the minor semitones and in the major semitones, which are normally called minor seconds. The truly major semitones [are add. supra lin.] seven and are found among the keys that preserve their proper name. The semitones that are employed as major despite being minor are five, and they are called semi-diminished seconds. Each of them receives two names, according to whether the form of the lower key or the one of the higher key is altered. Among the seven seconds truly minor, the

best tuned are the two diatonic ones E F and B C, which are represented by the ratio 16/15, as they are smaller than it is only by 06347., which is less than one eighth of a comma. In order to employ our common instruments as general, we tolerate that our seconds are a little smaller than required by the best participation, which widens them by 03853 above the ratio 16/15. the remaining five chromatic minor seconds become gradually smaller the further they depart from the two diatonic ones. F # G and A B b are smaller than the two diatonic ones by 04506, C # D and D E b by 09012., and G # A by 13518. Our reductions depend on the fact that the sharps and the flats rise and fall respectively more than they should in relation to the temperament of the diatonic keys, as I strived to clarify a little earlier ([signum] 16.).

The five semi-diminished seconds, each of whom can be expressed in two form, are contained all somehow within the two ratios 21/20, belonging to the minor semitone, and 16/15 belonging to the major semitone, although they are closer to the former than to the latter. If we consider the sequence of twelve seconds all commonly called minor, which begins from F G b and ends on A # B, the five semi-diminished seconds G A b, C D b, F G b, D # E and A # B are wider and better tuned the closer they are to the two minor diatonic seconds E F and B C. If [-798-] we move beyond the established boundaries F G b and A # B, we go back to the seconds contained in the series but expressed in a different form, which, therefore, start widening again instead of becoming progressively smaller. If the diminished seconds, which become such either by increasing enharmonically the lower sound or by reducing enharmonically the higher one, are employed as melodic passages leading from an interval to another one, they introduced an enharmonic alteration in the interval in which a key or a note maintains its own proper name, while the other one changes it. For instance, we can move from the fifth C G to the semi-diminished sixth C A b by means of the semi-diminished second G A b.

I will not say much about the sevenths that are called all commonly major. Since they form the octave when they are added to their corresponding minor seconds, the narrower a minor second, for instance B C, is than the one expressed by the ratio 16/15, the wider the major seventh C B than the one expressed by the ratio 11/8, which added to the ratio 16/15 forms the octave. Therefore, it easy to apply to the major sevenths what I said of the unequal temperament of the minor seconds. The truly major sevenths added to as many truly minor seconds form the octave. The five semi-diminished seconds require, in order to form the octave, five semi-augmented sevenths, which also are expressed in two ways, depending on whether the name of the higher key or the one of the lower one is changed. As to the truly major sevenths, I do not fail to inform the Reader that my participation presents them as perfectly tuned when they are considered as dissonances added to the consonant accompaniment. The major seventh is expressed by the ratio 15/8 as a fundamental dissonance. The most altered among the truly major sevenths contained in the series placed above is A G # 15/8 + 19865. however, the difference 19865. is very small in relation to a ratio containing [-799-] the number 15., as it would not produce a greater impression on the ear than the one that would be produced by the difference 06611. added to an imperfect consonance or by the difference 03973. added to a perfect consonance. The semi-augmented sevenths are represented by the ratio 15/8 belonging specifically to the major seventh in the same way the minor seventh, the eleventh and the thirteenth, as they are found in the best tones of our instruments, represent the ratios attached to the dissonances themselves represented by the dissonances 7/4, 11/4 and 13/4.

[-800-] [Riccati, The laws of counterpoint, 800, Serie di Seconde maggiori, minori. C b D b, G b A b, D b E b, A b D b, E b F, B b C, F G, C D, G A, D E, A B, E F #, B C #, F # G #, C # D #, G # A #, D # E #, A # B #, E # F #, 10/9, 9/8, 8/7, + 43921, - 10029, + 18331, - 50063, + 39415, - 14535, + 34909, - 19041, D b C b, A b G b, E b D b, B b A b, F E b, C B b, G F, D C, A G, E D, B A, F # E, C # B, G # F #, D # C #, A # G #, E # D #, B #, A #, F 2#, E #, 9/5, 16/9, 7/4, - 43921, + 10029, - 18331, + 50063, - 39415, + 14535, - 34909, + 19041]

[-801-] [signum] 19. Only ten of the twelve seconds all commonly called major deserve that name in a rigorous sense. In fact, the other two require the name of semi-augmented seconds, since they

are truly two diminished thirds and since they are wider than the major seconds by an enharmonic element. The simplest expressions of the twelve seconds, ten major and two semi-augmented, are contained in the series beginning from A b B b and ending on C # D #. Any second placed outside said series is merely a different expression of the second in the series that lays at the distance of twelve fifths from it. It is possible to note in relation to the truly major seconds that both keys or notes retain their proper name or they both change it. When both keys change their name, one and the other letter, which correspond to each other at the interval of a major second, is altered either by the unison more-than-augmented, which is the same as saying by a sharp that is larger than the ordinary ones, or by the more-than-diminished unison, which is the same as saying by a flat that is larger than the ordinary ones. For instance, if D and E lay at the distance of a major second, the same distance shall intervene between D # and E #, since both letters are modified by the more-than-augmented unison.

The diatonic system and the chromatic one provide us with five major seconds each. All of them are contained between the minor tone $10/9$ and the major tone $9/8$, although they are clearly closer to the latter than to the former. In the best temperament the major second is equal to $10/9 + 39828$ and to $9/8 - 23133$. In my unequal participation the major diatonic seconds are closer to the major tone, as it establishes them equal to $9/8 - 19041$. The aim of this is to enable our instruments to be employed as based on the general system. The chromatic major seconds are even closer to the ratio $9/8$, namely, E F # and B b C by 04506, and B C #, F # G # and E b F by 09012. One must note that two are the best tuned chromatic major seconds, namely, E F # and B b C, which are adjacent to the diatonic ones A B, F G. [-802-] We have already ascertained (book 3., chapter 1., [signum] 7.), that the true expression of the dissonance of the ninth added to the consonant accompaniment is $9/4$, which is aequisonant to $9/8$, as it lays at the distance of an octave from it. Now, as the intervals that differ by the simple or compound octave are reduced or widened by the same quantity, the greatest alteration applied to the tenth major ninths is 19041. at most, which is a very small quantity in relation to a ratio containing the odd number nine. The aforementioned dissonances corresponds to the major tenth, one of the sounds of the consonant accompaniment with the major third according to the second $10/9$. The greatest alteration allotted to the ten major seconds considered in relation to the ratio $10/9$ is 43921, which produces on the ear the same impression that would produce an imperfect consonance altered by 14640., a quantity that exceeds one fourth of the comma by a small amount. The diatonic major seconds are more suitable to be employed as represented by one or by the other ratio $9/8$ and $10/9$, because the differences $9/8 - 10941$. and $10/9 + 34909$. are distributed in the best way and are the closest to equality among all the others.

The two semi-augmented seconds A b B b = G # A # and C # D # = D b E b are the only ones that remain to be considered. Each can be expressed in two ways, according to whether the name of one or of the other key or note is changed. They are contained between the ratios $9/8$ and $8/7$, as they are wider than the first one by 18331. and narrower than the first one by 50063. Our seconds lay at the distance of 41453. from the median tone of the best participation and at the distance of the median diatonic tone of my unequal temperament by 37372. Consequently, they are considerably less close to the ratio $8/7$ than to the two aforementioned median tones. It is very noticeable the difference between the seconds that we are discussing and the minor tone $10/9$, which they exceed by 72281. if the semi-augmented seconds are [-803-] employed as melodic passages moving from an interval to the other, they produce an enharmonic alteration in the interval in which a key preserves its proper name and the other key changes it. When they are employed as represented by the ratio $9/8$ by which one moves from the fourth to the fifth, they produce only the reduction, in the former, and the widening, in the latter, that was mentioned earlier ([signum] 7.) which corresponds to 23345. This occurs only in the fifth G # D# = A b E b, and in the fourth D # G # = E b A b that added to it forms the octave. Much greater enharmonic alterations are produced when one wants to represent our semi-augmented seconds with the ratio $10/9$, which is the difference between the fifth and the major sixth and between the minor third and the fourth. We shall observe later ([signum] 24.) that three minor thirds are narrower than the ratio $6/5$ by 67266.,

while three major thirds, which added to the former produce the octave, are wider than the ratio $5/3$ by the same amount. Even in harmony the two semi-augmented seconds are employed more willingly as differences between the octave and the ninth expressed by the ratio $9/8$ than as differences between the ninth and the major tenth expressed by the ratio $10/9$. The most appropriate form that the aforementioned seconds can take is the one of inverted minor sevenths or of differences between the octave and the ninth expressed by the ratio $9/8$, rather than as differences between the ninth and the major tenth expressed by the ratio $10/9$. The best form that the mentioned seconds can assume is of minor inverted sevenths or of differences between the minor seventh and the octave. I noted earlier (book 3., chapter 1., [signum] 7.) that the minor sevenths enjoy the privileges that they enjoy in counterpoint because they deviate from the simple ratio $7/4$ only by an enharmonic element. Now, since the inversion of $7/4$, or the interval that complements it to form the octave, is $8/7$, it must be represented by the major seconds employed as the inversions of the minor sevenths. The two semi-augmented seconds of my unequal temperament do not simply represent the ratio $8/7$, but they embrace it with adequate approximation, as they are narrower than it is by 50063. Such difference produces the same impression on the ear that would be produced by the difference 35759. in relation to an imperfect consonance. We did point out earlier ([signum] 7.) that the truly minor third [-804-] G # B is narrower by 38905 in my unequal participation. The name of semi-augmented second suits the ratio $8/7$ well, since it is contained within the two ratios $9/8$ of the major second and $7/6$ of the augmented second. When I said that our two seconds take the form of inverted minor sevenths or of differences between the minor seventh and the octave, I adopted a common way to express myself. Had I spoken more precisely, I should have said that they are employed very gainfully as inverted semi-diminished sevenths, or as differences between the semi-diminished seventh and the octave. The semi-diminished sevenths and the semi-augmented seconds, which added together form the octave, are the ones that embrace the ratios $7/4$ and $8/7$ in the instruments commonly used.

[signum] 20. Everything that I said on the different temperament of the major seconds can be applied entirely to the minor sevenths, which added to the former form the octave, with this sole difference, namely, that a major second is narrower or wider than a given ratio, while, vice versa, the corresponding minor seventh is wider or smaller compared to the ratio that added to the former given ratio forms the octave. Since the ten truly minor sevenths are contained between the two ratios $9/5$ and $16/9$, they are employed very appropriately as derived from the fundamental accompaniment of the major third, fifth and ninth. $5:9$ is the ratio between the major third and the ninth, while $9:16$ is the ratio between the ninth and the double octave, which can be called inverted ninth with an appropriate name, as I explained amply elsewhere (book 3., chapter 1., [signum] 8.). Since our ten sevenths differ from the ratio $7/4$ belonging to the fundamental dissonance that is called commonly minor seventh by a value contained between 87435. and 78423., they are employed in this form thanks to the enharmonic representation that they maintain of the ratio $7/4$. [-805-] Now, I move on to the two semi-diminished seventh, which represent the proportion $9/5$ enharmonic, which exceeds them by the quantity 7228. These semi-diminished sevenths are contained between the ratios $16/9$ and $7/4$, as they are narrower than the first one by 18331. and they exceed the second one by 5063. Therefore, they are employed as either of them with suitable accuracy. If we listen intently to the semi-diminished sevenths considered by themselves and unrelated to the tone to which they belong, E b D b = D # C # and B b A b = A # G #, we shall hear a softer and coherent harmony than the one produced by the minor sevenths. When the twelve sevenths that are all called minor collectively are employed in counterpoint as fundamental dissonances, they awaken the idea of the ratio $7/4$, and, the closer they are to this ratio, the the milder the impression that they make on the ear. However, it is true that the greatest perfection of the two aforesaid sevenths produces some enharmonic alterations in some consonances. Therefore, in the best keys, because of the precision of the consonances, the minor sevenths are widened by an enharmonic element above the ratio $7/4$ and the ear hears them as perfect, because this is required by the scale of the mode, which is generated by three consonant accompaniments. I invite the Reader to check what I wrote earlier on this matter (book 3., chapter

1., [signum] 7.). Moreover, the semi-diminished sevenths introduce in the tones to which they belong a certain particular character that is very effective in expressing in music certain emotions, as I shall strive to explain in the appropriate place (chapter 6. [signum] 31.). Meanwhile, I must not overlook a notable property that one notices in the temperament of the tone E b with the major third. In said temperament we take into account proportionately the fourth E b $\frac{4}{3}$ A b and the semi-diminished seventh B b $\frac{7}{4}$ A b, which is considered added to the consonant accompaniment of the fifth note B b. The letter A b could represent [-806-] two sounds, a lower one, if one considers it as seventh of B b, and another one higher, if one considers it as fourth of E b. The difference consists in the enharmonic diesis $\frac{64}{63}$. My unequal temperament places a middle sound between these two, so that it can perform the role of either of them, obtaining through this device that the alterations of the fourth E b A b = $\frac{4}{3} - 23345$ and of the semi-diminished seventh B b A b = $\frac{5}{4} + 50063$. are closely proportional to the simplicity of those ratios, or, in other words to the odd numbers three and seven, and thus make an equal impression on the ear.

[-807-] [Riccati, The laws of counterpoint, 807, Serie di Terze diminuite. Superflue. D F b, A C b, B D b, F # A b, C # E b, G # B b, D # F, A # C, E # G, B # D, F # A, C 2# E, G 2# B, D 2# F #, A 2# C #, $\frac{9}{8}$, $\frac{8}{7}$, - 19041, - 87435, - 14535, - 82929, - 10029, - 78423, - 50063, F b D, C b A, G b E, D b B, A b F #, E b C #, B b G #, F D #, C A #, G E #, D B #, A F 2#, E C 2#, B G 2#, F # D 2#, C # A 2#, $\frac{16}{9}$, $\frac{7}{4}$, + 19041, + 87435, + 14534, + 82929, + 10029, + 78423, + 50063]

[signum] 21. The diminished thirds and the augmented sixth are intervals that belong artificially to the tones with the minor third. The former are found between the fourth artificial note and the seventh natural one [-808-] as well as between the artificial seventh and the ninth note of said tones. If we consider the diminished thirds as harmonies or as fundamental melodic passages, they accept, as we have seen earlier (chapter 3. [signum] 2.) the ratio $\frac{8}{7}$, while the ratio $\frac{7}{4}$, which added to the former produces the octave, is assigned to the augmented sixths. Rigorously speaking, there are only two diminished thirds, C # E b and G # B b, and two augmented sixths, E b C # and B b G #, in the keyboard instruments commonly used. This is indicated clearly by the fact in these both of the keys preserve their own name and by the fact that the remaining ten in each sort of interval change the name of one of the keys. Our two couples of diminished thirds and augmented sixth embrace the convenient ratios $\frac{8}{7}$ and $\frac{7}{4}$ with the difference $- + 50063$., which, as I noted a little earlier ([signum] 19.) makes a smaller impression on the ear than the one caused by the difference 38905. that affects the minor third G # B. The difference 50063. must be tolerated, because the aforementioned two diminished third as major seconds, namely, D b E b = C # D # and A b B b = G # A #, and also so that, if we add the former above the fourth A b D b = G # C # and the latter below the fourth B b E b = A # D # we shall obtain in both cases the fifth A b E b = G # D # that is augmented by no more than 23345. The remaining ten thirds, commonly called diminished are narrower than the ratio $\frac{8}{7}$ by one harmonic element which is contained between the quantities 78423 and 87435 inclusive. Therefore, if we wanted to speak with precision, they would need to be called more-than-diminished. For the opposite reason, the intervals that added to them form the octave, which are called commonly augmented sixths, would need to be called sixths more than augmented, because, in fact, they are wider than the ratio $\frac{7}{4}$ by an enharmonic element. The more-than-diminished thirds accept very accurately the ratio $\frac{9}{8}$ belonging to the [-809-] major seconds, while the more-than-augmented sixths accept the ratio $\frac{1}{9}$, which belongs to the minor sevenths. For this reason the more-than-diminished thirds and the major seconds, the more-than-augmented sixths and the minor sevenths share two by two the value accepted by the diminished thirds and by the more-than-augmented seconds, and by the augmented sixths and by the semi-diminished sevenths, as one deduces clearly from what I said in the present paragraph and in the preceding two.

The two couples of diminished thirds C # E b and G # B b and of augmented sixths E b C # and B b G # belong harmonically half to the tone G and half to the tone D with the minor third, which are more perfect than the other tones of a similar nature in this respect. If we ascend by a

fifth, we encounter firstly the tone A, which contains the more-than-diminished third D # F and the more-than-augmented sixth F D #. One can see that the last two mentioned third and sixths are modified by the enharmonic element $- + 78423$. The third more-than-diminished A # C and the sixth more-than-augmented C A # are assigned to the tone E, and they are altered by the enharmonic element $- + 82929$, which is larger than the preceding one by the difference 904506. The next table shall suggest to the Reader the remaining values without the need for me to go any further and it shall show with ease the tones to which the thirds and the sixth belong together with their alterations. These tones are found by continuing to ascend in fifths or by descending in fifths or, similarly, by ascending in fourths starting from the letter G. I limit myself to inform the Reader that the enharmonic alterations of our thirds and of our sixths derive from the widening or reducing a sound according to whether those intervals suit the tones that one encounters ascending or descending in fifths, which, as we shall see, produces a different effect on the ear. For instance, the two thirds A # C and F # A b are more than diminished, the former by the upper alteration of the sound A and the latter by the lower alteration of the sound A b. [-810-] In the thirds and in the sixths that we are discussing the harshness is increased for two reasons, namely because they appear altered when they are employed as representing the ratios $8/7$ and $7/4$ and because they accept the ratios $9/8$ and $16/9$, which are more complex than their own, namely, $8/7$ and $7/4$.

[Riccati, The laws of counterpoint, 810; text: F b G, C b D, G b A, D b E, A b B, E b F #, B b C #, F G #, C D #, G A #, D E #, A B #, E F 2#, B C 2#, F # G 2#, C # D 2 #, $7/6$, $6/5$, + 96956, - 25388, + 92450, - 29894, + 87944, + 34400, + 83438, - 38906, + 55078, + 92450, - 29894, G F b, D C b, E D b, B A b, F# E b, C # B b, G # F, D # C, A # G, E # D, B # A, F 2# E, C 2# B, G 2# F#, D 2# C #, $12/7$, $5/3$, - 96956, + 25388, - 92450, + 29894, - 87944, - 34400, - 83438, + 38906, - 55078, - 92450, + 29894]

[-811-] [signum] 22. The augmented seconds and the diminished sevenths, which are ratios belonging to the chromatic genus, are employed in the tones with the minor third. The augmented seconds occur between the third natural note and the fourth artificial one and between the sixth natural note and the seventh artificial one, while, consequently, the diminished sevenths, which added to the former complete the octave, are contained between the fourth artificial note and the tenth natural note and between the artificial seventh note and the natural thirteenth note. On the organ and the harpsichord we count only three genuine augmented seconds, namely, E b F #, B b C # and F G #, and an equal number of diminished sevenths, namely, F # E b, C # B b and G # F. They embrace their suitable ratios $7/6$ and $12/7$ with the differences ± 55078 , which are closely proportional to the ones of the fifth A b E b = G # D # = $3/2 + 23345$. and of the minor third G # B = $6/5 - 38905$. The mentioned differences deserve to be applied, so that, when the augmented second are employed as minor thirds and the diminished sevenths are employed as major sixths, the former do not appear too narrow and the latter too wide, causing them to be unable to represent their corresponding ratios $6/5$ and $5/3$.

The remaining intervals, nine for every species, which are employed in our instruments as augmented seconds and diminished sevenths, are truly minor thirds and major sixths, which take up an alien character by altering the name of one key and by abandoning their own role. Since the aforesaid intervals alter their proportions $7/6$ and $12/7$, the first by becoming wider by an enharmonic element and the second one by becoming smaller by the same quantity, which consists of a value contained between 87944. and 96956. inclusive, they need to be called more-than-augmented seconds and more-than-diminished sevenths, strictly speaking. If we start from the augmented second F G # and from the diminished seventh G # F and we ascend in fifths, we encounter seconds and sevenths, such as C D #, G A # and D # C, A # G, that become more-than-augmented and more-than-diminished thanks to the enharmonic alteration [-812-] of one sound, which is the higher one in the former and the lower one in the latter. On the contrary, if we descend by fifths starting from the augmented second E b F #, and from the diminished seventh F # E b, we encounter seconds and sevenths, for instance A B B, D b E and B A b, E D b, that are made more-

than-augmented and more-than-diminished respectively by reducing enharmonically one sound, which is the lower one in the former and the higher one in the latter. In order to understand fully the different effect produced on the ear by the noted semi-augmented and semi-diminished intervals and in which opposite ways they are achieved, I propose to the Reader the two accompaniments D # F # A C and B D F A b commonly called of the minor third, [minor add. supra lin.] fifth and diminished seventh, which are very often are applied to D # and B, artificial seventh notes of the tones E and C with the minor third. The first accompaniment originates from the accompaniment D F # A C of the major third, fifth and minor seventh in which the natural note D is substituted with the artificial note D #, which exceeds too much above the natural one by the more-than-augmented unison equal to a major semitone. Such excess alters the chord D # F # A, which employs it as representing a consonance. Since this chord must be of the minor third and minor fifth, it becomes of the semi-diminished third and of the semi-diminished fifth. As the ear recognises the enharmonic alteration of the base D #, consequently, it considers altered the seventh D # C as well, which alteration is also softened by the fact that that seventh accepts the elegant proportion 5/3. The accompaniment B D F A b derives from substituting the artificial note B to the natural B b in the accompaniment B b D F A b of the major third, fifth and diminished seventh, which, as I noted earlier ([signum] 20.) is more pleasant to the ear per se than the accompaniment of the major third, fifth and minor seventh. Moreover, the accompaniment B D F, which is employed [-813-] as a consonant accompaniment consists, as counterpoint requires, of the minor third and minor fifth, while the more-than-diminished seventh embraces the simple ratio 5/3. the comparison of the two accompaniments D # F # A C and B D F A b shall suffice to allow us to understand manifestly how the seventh, which becomes more-than-diminished because of the enharmonic lowering of the higher sound, must sound milder to the ear than the other one that becomes such because of the enharmonic heightening of the lower sound.

The two couples of augmented seconds and diminished sevenths E b F #, B b C # and F # E b, C # B b belong to the tone G with the minor third, while the two other couples, B b C #, F G # and C # B b, G # F, belong to the tone D, which is a fifth higher than the tone G. If we continue to ascend in fifths, the tone A acquires the more-than-augmented second C D # and the more-than-diminished seventh D # C besides the augmented second F G # and the diminished seventh G # F. If we continue with the same order, the more-than-augmented seconds and the more-than-diminished sevenths C D #, G A # and D # C, A # G belong to the tone E; G A #, D E # and A # G, E # D belong to the tone B, while D E #, A B # and E # D, B # A belong to the tone F # and so on. Moreover, since the augmented seconds E b F # and B b C # and the the diminished sevenths F # E b and C # B b belong to the tone G, if we descend by a fifth, the tone C shall recognise as its own the augmented second E b F #, the diminished seventh F # E b, the more-than-augmented second A b B and the more-than-diminished seventh B A b. If we continue to descend in fifths we shall learn that the more-than-augmented seconds and the more-than-diminished sevenths, namely, A b B, D b E and B A b, E D b, belong to the tone F; D b E, G b A and E D b, A G b belong to the tone B b, and G b A, C b D and A G b, D C b belong to the tone E b and so on.

[-814-] [Riccati, The laws of counterpoint, 814; text: Serie di Terze minori. maggiori, D b F b, A b C b, E b G b, B b D b, F A b, C E b, G B b, D F, A C, E G, B D, F # A, C # E, G # B, D # F #, A # C #, E # G #, B # D #, F # A #, 6/5, 7/6, - 34400, - 38906, + 55078, - 67266, +55078, - 29894, - 25388, -38906, F b D b, C b A b, G b E b, D b B b, A b F, E b C, B b G, F D, C A, G E, D B, A F #, E C #, B G #, F # D #, C # A #, G # E #, D # B #, A # F #, 5/4, 12/7, + 34400, + 38906, - 55078, + 67266, - 55078, + 29894, + 38906]

[-815-] [signum] 23. The truly minor third on the keyboard instruments are nine, and the truly major sixths are also nine. Four of each species of interval are diatonic and five are chromatic. The diatonic minor thirds are narrow by the quantity 25388., and, consequently the major sixths, which added to the former constitute the octave, are wide by the same quantity according to my third participation. The alteration of the chromatic minor thirds and major sixths becomes greater as

they become more distant from the diatonic minor thirds and major sixths. The chromatic minor thirds and major sixths are altered more than the diatonic ones according to the following quantities: $G \flat B \flat$, $F \sharp A$ and $B \flat G$, $A F \sharp$ by 04506; $C \flat E \flat$, $C \sharp E$ and $E \flat C$, $E C \sharp$ by 09012, and $G \sharp B$ and $B G \sharp$ by 13518. When a flat occurs in the chromatic minor thirds and major sixths, the former are narrower and the latter wider than the diatonic ones because the flats, as I observed when I dealt with the minor semitones ([signum] 16.), become gradually narrower of what would occur even if the black keys shared the temperament of the white keys. On the contrary, if the chromatic minor thirds and major sixths contain a sharp, the reductions of the former and the enlargements of the latter are larger than those of their respective diatonic counterparts, because the sharps are tuned rather sharper than the temperament of the white keys would require, were it extended to the black keys as well. I shall discuss in the sixth chapter ([signum] 4.) the different characters of the minor third and of the major sixth. We shall see, for instance, that the minor third is tender in the fundamental accompaniment of the minor third and fifth and that the major sixth proves tranquil in the accompaniment of the fourth and sixth derived from the fundamental accompaniment of the major third and fifth. In such occasions the flats render the minor thirds more emotional and the major sixths more serious than their corresponding diatonic ones. Conversely, the sharps reduce the emotional charge of the minor thirds and increase the cheerfulness of the major sixths, which in the case of the major sixth $B G \sharp$ begins to be regarded as cheekiness. If I now turn to the tones with the minor third to provide an example of the perfect [-816-] correspondence between theory and experience, which expert ear does not feel that the tone G is considerably more emotional than the tone D , which is itself more emotional than the tone A ? Moreover, who does not discern that the tones E , B , $F \sharp$ and $C \sharp$ become gradually less emotional and more irate? We shall observe in the quoted chapter their very attractive function in imitating the different characters and feelings expressed by the words the poems to be set to music.

[signum] 24. The three thirds $E \flat G \flat = D \sharp F \sharp$, $B \flat D \flat = A \sharp C \sharp$ and $F A \flat = E \sharp G \sharp$, which are called commonly minor thirds, deserve the name of semi-diminished, while, conversely, the three sixths that added to them form the octave, $G \flat E \flat = F \sharp D \sharp$, $D \flat B \flat = C \sharp A \sharp$ and $A \flat F = G \sharp E \sharp$, normally called major, deserve the name of semi-augmented. In fact, the former are narrower than the ratio $6/5$, while the latter exceed ratio $5/3$ by the difference 67266 , which is very close to the smallest enharmonic diesis $64/63 = 68393$. The aforementioned thirds and sixths come close to the ratios $7/6$ and $12/7$ through the differences ± 55078 ., which are closely proportional to the ones of the fifth $A \flat E \flat = G \sharp D \sharp = 3/2 + 23345$ and of the minor third $G \sharp B = 6/5 - 38905$. the semi-diminished thirds and the semi-augmented sixths [[one]] considered in relation to the ratios $7/6$ and $12/7$ are, strictly speaking, merely the difference between the fifth and the diminished seventh and between the diminished seventh and the twelfth. Moreover, since the minor seventh enjoys the privileges that we know, since it is employed in representation of the ratio $7/4$, typical of the semi-diminished seventh, it follows that the minor third, which is the difference between the fifth and the minor seventh, and the major sixth, which is the difference between the minor seventh and the twelfth, are employed as representing the ratios $7/6$ and $12/7$. It was very important that the Reader should understand well the character of the fractions $7/6$, $12/7$ considered as third and sixth. The most refined sort of harmony requires that our third and sixths should be rendered semi-diminished and semi-augmented by the enharmonic reduction of one of their sounds, the higher one in the former and the lower one in the latter. This occurs in the forms [-817-] of the two intervals that contain the flats. Compare the two chords normally called minor fifth and minor seven, namely, $E \flat B \flat D \flat$ and $G \sharp D \sharp F \sharp$. One shall observe that, while the third $B \flat D \flat$ has become semi-diminished through the narrowing of the sound $D \flat$, the fifth $E \flat B \flat$ remains untouched, while the seventh $E \flat D \flat$ become closer to its privileged ratio $7/4$. Conversely, while the semi-diminished condition of the third $D \sharp F \sharp$ originates from the enharmonic widening of the sound $D \sharp$, the fifth $G \sharp D \sharp$ is made less in tune without adducing any benefit to the seventh $G \sharp F \sharp$.

In fact, if the aforementioned thirds and sixths are employed in musical compositions as consonances, then they shall be employed as representation of the ratios $6/5$ and $5/3$, which the ear

shall discern as altered, although the alteration is rendered milder by the fact that our intervals approach the semi-consonant ratios $7/6$ and $12/7$. If the tone requires that the thirds and sixths that we are discussing are written with flats, the ear shall ascribe their semi-diminished and semi-augmented character to the enharmonic reduction of one of their sounds. Such reduction shall ensure for instance that the fundamental semi-diminished third sound plaintive, while the semi-augmented sixths in the semi-augmented accompaniment of the fourth and sixth sound less tranquil and more serious than the major sixths. However, if the key requires that aforesaid thirds and sixths are expressed with the sharps, the ear shall convince itself that that semi-diminished and semi-augmented quality depend on the enharmonic raising of one sound, which shall ensure that the semi-diminished thirds in the accompaniment of the semi-diminished third and fifth are heard as less tender and that the semi-augmented sixths of the accompaniment of the semi-augmented fourth and sixth take on a cheeky character.

It would be too long-winded to specify to which tones our third and sixth can belong naturally or artificially. Therefore, I ask the Reader to be satisfied that I concentrate a while on the three fundamental accompaniments [-818-] of the semi-diminished third and fifth [[E b B b D b]] E b G b B b, B b D b F and F A b C, which, as they are full of emotion, are admirably useful on counterpoint. The base notes E b, A b and F of the accompaniment written above constitute the system of the melody of the tone B b commonly called with the minor third, but which needs to be called more accurately with the semi-diminished third, since it originates from assigning to the three notes E b, B b and F of its melodic system an equal number of accompaniments of the semi-diminished third and fifth. The two tones F and C, which are believed commonly by musicians to be with the minor third, are mixed in reality. This means that they are partly with the minor third and partly with the semi-diminished third. In the tone F the first note F and the fourth note B are allotted the accompaniments of the semi-diminished third and fifth, while the fifth note C is assigned the accompaniment of the minor third and fifth. The tone C assigns the accompaniment of the semi-diminished third and fifth only to the fourth note F, which is the less perfect of the ones that constitute the system of the melody, since two accompaniments of the minor third and of the fifth are assigned to the other two, namely, the first one C and the fifth one G. Now a master of the chapel can express any degree of emotion by means of these three tones. The tone B b appears extremely sad and tearful. A lesser degree of sadness is heard in the tone F, and such sadness is reduced and tempered again in the tone C. The emotion is contained in the tone mentioned last to such a degree of perfection that the ear enjoys it very much. The enjoyment is heightened by the tone E b with the major third, which is ascribed the same scale of the aforementioned tone C and supplies it with more accompaniments. The narrowed note A b mitigates the cheerfulness typical of the tones with the major third and it introduces an sentimental element to the aforementioned tone.

[-819-] [Riccati, The laws of counterpoint, 819; text: Serie di Terze maggiori. Minori, F b A b, C b E b, G b B b, D b F, A b C, E b G, B b D, F A, C E, G B, D F #, A C #, E G #, B D #, F # A #, C # E #, G # B #, D # F 2#, A # C 2#, $5/4$, $9/7$, + 29386, + 62252, - 60092, + 57746, - 64598, + 25078, + 20374, + 15868, + 20374, A b F b, E b C b, B b G b, F D b, C A b, G E b, D B b, A F, E C, B G, F # D, C # A, G # E, D # B, A # F #, E # C #, B # G #, F 2# D #, C 2# A #, $8/5$, $14/9$, -29386, -62252, + 60092, - 57756, + 64598, - 25078, -15868, - 20374]

[-820-] [signum] 25. On the organ and on the harpsichord we count eight thirds that are truly major and as many sixths that are truly minor. Three of each species of interval belong to the diatonic genus and five belong to the chromatic one. The diatonic major thirds are wider, while, consequently, the minor sixths that added to the former constitute the octave are narrowed by the quantity 15868., as my third temperament requires. The alterations of the chromatic major thirds and of the minor sixths become considerably wider as they become more distant from diatonic major thirds and minor sixths. We can see that the chromatic major thirds and minor sixths are altered more than the diatonic are by the following differences, namely, B b D, D F # and D B b, F # D by 04506. ; E b G, A C # and G E b, C # A by 09012. and E G # and G # E by 13518. If the chromatic

major thirds and the chromatic minor sixths contain one flat, the former are wider while the latter are narrower than their correspondent diatonic ones because the flats are a little lower than what would occur if one applied to the black keys as well the temperament of the white keys. On the contrary, if the chromatic major thirds and the minor sixths contain a sharp, the widening of the former and the narrowing of the latter exceed the ones of the corresponding diatonic major thirds and minor sixths because the sharps are made to be considerably wider than the participation of the white keys would require if it was extended to the black keys as well. When I discuss (chapter 6. [signum] 4.) the different forms that the major third and the minor sixth can assume in relation to the arousal of stronger or weaker passions, I shall draw attention to the fact that, for instance, the major third sounds happy in the perfect accompaniment of the major third and fifth and that the minor third sounds sentimental in the accompaniment of the fourth and minor sixth derived from the fundamental one of the minor third and fifth. In such circumstances the flats make the major thirds more serious and the minor sixths more emotional, while the sharps under those conditions increase the cheerfulness of the major thirds and reduce the emotional charge of the minor sixths.

[-821-] To avoid being prolix, I shall restrict my observations to the fundamental accompaniments of the truly major third and fifth, and I shall demonstrate how they influence the character of the various tones. An equal number of accompaniment of the major third and fifth is assigned to the eight major thirds. They are the following: E b G B b, B b D F, F A C, C E G, G A D, D F # A, A C # E and E G # B. As these accompaniments partake of the nature of the major thirds that contribute to their formation. The three diatonic ones sound cheerful and suave. Of the three chromatic accompaniments containing sharps, D F # A and A C # E,, sound cheerful, while E G # A sounds a little daring and piquant. The two remaining chromatic accompaniments with the flat sign contain in themselves greater seriousness than the diatonic ones. From the properties of our accompaniments it is easy to deduce the properties of the tones with the major third that consist of them. Since the tone of C consists of three diatonic accompaniments, it sounds quieter and calmer than any other. If we continue on the side of the sharps, the tone G is rather more spirited than the tone C, since the chromatic chord D F # A contributes to its formation. The tone D, which admits in its formation the two accompaniments D F # A and A C # E is defined by a cheerfulness of character that becomes a little cheeky in the tone of A consisting of the three chromatic accompaniments D F # A, A C # E and E G # B, all of which include sharps. If we descend towards the flats, the tones F and B b contain more gravity than the tone C, because they contain chromatic accompaniments with flats, which are B b D F in the first one and B b D F, E b G b in the second one. If the Readers to compare my theory with experience, they shall judge whether it is useful to explain the variety of the truly major tones in which all the keys preserve their own name.

As to the tones with the minor third, I do not want to fail to inform the Reader that, since their fifth note is allotted the accompaniment of major third and fifth very often, such accompaniment proves so much less sparkling and cheerful, the more [-822-] emotional the tones with the minor third are. Note for now as a prove of this that the accompaniments E G # A, A C # E, D F # A and G A D are allotted to the fifth note of the tones A, D, G and C respectively, which, as we have seen earlier (paragraphs 23. and 24.) become gradually more sentimental and depressed. This element as well contributes to differentiate the tones with the minor third.

[signum] 26. The four thirds B D # = C b E b, F # A # = G b B b, C # E # = D b F and G # B # = A b C, which are called commonly major thirds, need to be called semi-augmented; conversely, the four sixths that added to the former constitute the octave, namely, D # B = E b C b, A # F # = B b C b, E # C # = F D b and B # G # = C A b, which are called commonly minor sixths, require the name of semi-diminished sixth, because the former exceed the ratio 5/4 and the latter are narrower than the ratio 8/5 by an enharmonic element that is equal to a quantity contained between 62252. and 57746. inclusive. The aforementioned thirds are narrower than the ratio 9/7 and the corresponding sixth are wider than the ratio 14/9 at least by the quantity 60092. and at most by the quantity 64598. Therefore, if one considers their absolute differences, those thirds and sixths are contained almost exactly between the proportions 5/4, 9/7 and 8/5, 14/9. However, if we consider the differences in relation to the simplicity of the ratios written above, we discover easily that the

semi-augmented thirds and the semi-diminished sixths are closer to the ratio expressing a dissonance than to the one expressing a consonance, and that, moreover, they embrace the former with sufficient approximation, since the differences 60092. and 64598. produce the same impression as the one that would be caused by the alterations by the values 20031. and 21533. assigned to a perfect consonance.

If we consider the semi-augmented third and the semi-diminished sixth in relation to the ratios $9/7$, $14/9$, they are simply the difference between the diminished seventh $7/4$ and the ninth $9/4$, and between the ninth $9/4$ and the semi-diminished fourteenth $7/2$. I note here, as at did at paragraph 24., that, since the minor seventh has very many privileges, because it is employed as representation of the ratio $7/4$ belonging [-823-] of the semi-diminished seventh, we deduce by legitimate conclusion that the major third, difference between the minor seventh and the ninth, and the minor sixth, difference between the ninth and the minor fourteenth, are employed as representing the ratios $9/7$ and $14/9$. If we compare two consonant fundamental accompaniments with the addition of the semi-diminished seventh and of the ninth in one case, and of the minor seventh and ninth in the other one, as, for instance, in this case: B b D F A b C, A C # E G B, we shall find that the first one is calmer and more harmonious than the second one, because in the first case the third A b C is expressed with sufficient accuracy by the ratio $9/7$, while in the latter the third G B is barely able to represent it. After having illustrated in detail the character of the fractions $9/7$ and $14/9$ considered as a third and a sixth, I do not fail to point out that the most exact harmony requires that our thirds and sixths become semi-augmented and semi-diminished because of the harmonic reductions of one sound, which is the low one in the former, and the high one in the latter. This is achieved when the seventh, which is the base of those thirds, or the fourteenth, which is the highest sound of those sixths, are semi-diminished instead of minor, as it occurs in the chords presented above, namely, B b D F A b C. Were the thirds and sixths that we are discussing to become semi-augmented and semi-diminished because of the enharmonic widening of the ninth, which is the high sound in the former and the low one in the latter, we would worsen the tuning of the ninth without any advantage of the seventh. What I just mentioned occurs in the accompaniment C # E G # B D #.

[signum] 27. However, the semi-augmented thirds and the semi-diminished sixths are employed very often as consonances. In that case the ear relates them to the ratios $5/4$ and $8/5$, that it recognises as altered, while it remains bitten and gains scant relief from the fact that said thirds and sixths embrace the ratios $9/7$ and $14/9$, which are very distinct dissonances. From this we can deduce the reason why, when the semi-diminished thirds [-823-] and the semi-augmented sixths are considered as consonances they offend the ear less than the semi-augmented thirds and the semi-diminished sixths, although the former deviate a little more from the ratios $6/5$ and $5/3$ that it occurs in the latter in relation to the ratios $5/4$ and $8/5$. This depends on the fact that the former accept the ratios $7/6$ and $12/7$, which are somewhat half-way between consonances and dissonances, while the latter accept the ratios $9/7$ and $14/9$ that have to be counted among the dissonances, as I just stated. The four semi-diminished thirds and the four semi-diminished sixths can be seen in the table expressed both with the sharps and with the flats. If the tone of a articular composition requires sharps in the key signature, the ear ascribes the enharmonic alteration to the raising of the sound that corresponds to a key in which the alteration of the name has occurred. Such widening in the chords of the semi-augmented thirds and fifth renders the semi-augmented thirds cheeky and resentful and, by removing from the semi-diminished sixths in the chords of the semi-diminished fourth and sixth the tender character which is typical of the minor sixths, endows them with a little hint of courage. However, if the tone requires to be expressed with flats, the ear is convinced that the enharmonic alteration depends on the reduction of the sound of a key whose name has been changed. When this narrowing is considered in the mentioned accompaniments, it deprives the semi-augmented thirds of their cheerful character that suits the major thirds, turning them into sad and renders the semi-diminished sixths suitable to express said and tearful feelings.

As I did at paragraph twenty-four, I shall spare on this occasion as well the Readers the great tiresomeness produced from enumerating all the tones and accompaniments to which the thirds and

the sixths that we are discussing can belong naturally or artificially. I shall limit myself to reflecting on the four fundamental accompaniments of the semi-augmented third and fifth, each of whom is expressed both through sharps and flats. They are the following: B D # F # = C b E b G b, [-825-] F # A # C # = G b B b D b, C # E # G # = D b F A b and G # B # D # = A b C E b. I shall start from those expressed with the sharps and I shall consider the tones commonly called with the major third to which the four proud and resentful accompaniments. I said earlier ([signum] 25.) that the tone of A consists of the three accompaniments D F # A, A C # E and E G # B, all of which contain sharps. If we ascend in fifths, we encounter first the tone of E, which retains the accompaniments with the major third A C # and E G # A and adopts the accompaniment of the semi-augmented third B D # F # as well. It is followed by the tone B, which contains a single chord E G # B with the major third and two, B D # F # and F # A # C #, with the semi-augmented third. The tone of B is followed by two tones that are both generated by three accompaniments with the semi-augmented third, namely F #, consisting of the three chords B D # F #, F # A # C # and C # E # G #, and C#, consisting of the three chords F # A # C #, C # E # G # and G # B # D #. I inform the Reader that the tone of F # is some what more piquant because of its wider fifth G # D #, which occurs in the last accompaniment G # B # D #. The expert master of the chapel can imitate powerful emotions by means of the four described tones E, A, F # and C #, to whichever degree of strength these emotions belong, although he must employ them very sparingly because they prove very harsh and bitter to the ear.

I move on to consider our four accompaniments when they are expressed with flats. They assume a grave and emotional form and they belong to the tones called with the major third that I shall list. If we descend in fifths starting from the tone B b, which consists of the three accompaniments with the major third F A C, B b D F and E b G B b, the first tone to occur to us is based on the letter E b. It shares the two aforementioned accompaniments B b D F and E b G B b in common with the tone B b, and it also contains the accompaniment A b C E b of the semi-augmented third and fifth. We have noted earlier ([signum] 20.) that the tone of E b derives from the proportional consideration of the [-826-] fourth E b A b and of the semi-diminished seventh B b A b. After the tone of E b, we encounter the tone of A b, which is based on three fundamental chords. [One add. supra lin.] of them is E b G B b with the major third, while the other two are A b C E b and D b F A b with the semi-augmented third. The tone of A b is followed by the tones of D b and G b, both consisting of three accompaniments with the semi-augmented third, namely, A b C E b, D b F A b and G b B b D b in the first one, and D b F A b, G b B b D b and C b E b G b in the last one. The aforementioned four tones, two with the semi-augmented third and two mixed ones to express in music the more character of a more or less charged emotional seriousness.

I move from the tones with the major third, to employ my usual turn of phrase, to the ones with the minor third, and I consider in particular the accompaniment of the major third and fifth that is applied very often to their fifth note. The four accompaniments with the semi-augmented third marked with the flats belong to the fifth notes of the tones with the minor third that are not employed in counterpoint as they require double flats. The same does not occur in the case of the four accompaniments with the semi-augmented third marked with a sharp, namely, B D # F #, F # A # C #, C # E # G #, G # B # D # belonging to the fifth notes of the usual tones with the minor third E, B, F # and C #, which become more piquant and resentful for this reason as well.

[Riccati, The laws of counterpoint, 827; text: Serie di Quarte diminuite. Quinte superflue. C F b, G C b, D G b, A D b, E A b, B E b, F # B b, C # F, G # C, D # G, A # D, E # A, B # E, F 2# B, C 2# F #, G 2# C #, D 2# G #, 5/4, 9/7, +15868, - 106476, + 20374, - 101970, + 25078, - 97266, + 29386, - 92958, - 60092, - 64690, -101978, F b C, C b G, D b A, A b E, E b B, B b F #, F C #, C G #, G D #, D A #, A E #, E B #, B F 2#, F # C 2#, C # G 2#, G # D 2#, 8/5, 14/9, -15868, + 106476, - 20374, + 101970, - 25078, + 97266, - 29386, + 92958, + 60092, + 64690, +101978]

[signum] 28. The diminished fourths and the augmented fifths owe their introduction into music to the artificial notes of the mode with the minor third. The two [-828-] diminished fourth, of

which one is employed more often in counterpoint than the other one, occur between the seventh artificial note and the tenth natural one and between the fourth artificial one and the natural seventh. Consequently, the augmented fifths that added to the former form the octave occur between the third natural note and the seventh artificial one and between the seventh and the eleventh note, both of them artificial. The common keyboard instrument contain only four real diminished fourths, namely, B E b, F # B b, C # F and G # C, and as many augmented fifths, namely, E b B, B b F #, F C # and C G #. They accept the convenient ratios 9/7 and 14/9 with tolerable differences, among whom even the largest ones of ± 64598 . can be calculated demonstrably as smaller than those of the fifth A b E b = G # D # = $3/2 + 23345$. and of the minor third G # B = $6/5 - 38905$. in order that, when the diminished fourths are employed as major thirds and the augmented fifths as minor sixths, the former are not hear as too wide and the others too wide, in such a way that they would struggle to represent their own ratios 5/4 and 8/5. The remaining intervals, eight for each species of interval, which are employed on the commonly used instruments as diminished fourths and augmented fifths, are really major thirds and minor sixths that change the name of one key, abandon their original role and assume one that is alien to them. As the aforementioned interval alter their ratios 9/7 and 14/9, the first one by the reduction of an enharmonic diesis and the second by the enlargement of the same amount, which is contained between the values 92958. and 106476., they deserve to be called, strictly speaking, more-than-diminished fourths and more-than-augmented fifths.

If we start from the diminished fourth G # C and from augmented fifth C G # and we ascend in fifths, we find fourths and fifths that become more-than-diminished and more-than-augmented because of the enharmonic raising of a single sound, namely, the lower one in the former and the higher one in the latter. Conversely, when we descend [-829-] in fifths starting from the diminished fourth B E b and from the augmented fifth E b B, we encounter fourths and fifths made more-than-diminished and more-than-augmented by lowering enharmonically one sound, which is the higher one in the former and the lower one in the latter. We will learn from the sixth chapter, [signum] 12., that the diminished fourths and the augmented fifths are furnished with a character that goes from being fraught to being resentful. Such feeling of distress depends on the high sound of the diminished fourths and from the low sound of the augmented fifths; conversely, the feeling of anger is caused by the low sound of the diminished fourths and by the high sound of the augmented fifths. If the aforementioned enharmonic alterations derive from the rising of a sound, on account of this the more-than-diminished fourths and the more-than-augmented fifths become more daring and stinging than the diminished fourths and than the augmented fifths. In fact, if the aforementioned modifications originate from the lowering of one of the sounds, the opposite effects occur, since because of this the more-than-diminished fourths and the more-than-augmented fifths sound more fraught than the diminished fourths and than the augmented fifths. Nevertheless, the enharmonic alterations of our interval are mitigated by the fact that our interval accept the simple ratios 5/4, typical of the major third, and 8/5, typical of the minor sixth.

The four fourth diminished and the four augmented fifths belong to the following tones with the minor third: B E b, F # B b; E b B and B b F # to the tone C; F # B b, C # F, B b F # and F C # to the tone G; C # F, G # C, F C # and C G # to the tone D. These tones constitute a an ascending series of fifths. If this series is continued, we arrive at the tone A, which employs not only the diminished fourth G # C and the augmented fifth C G #, but also the more-than-diminished fourth D # G and the more-than-augmented fifth G D #. If we continue to rise in fifths, the tones noted herewith admit only more-than-diminished fourths and more-than-augmented fifths, since D # G, A # D; G D # and D A # belong to the tone E; A # D, E # A; D A # and A E # to the tone B; E # A, B # E, A E # and E # to the tone F # B # E, F 2# B, [-830-] E B # and B F 2# to the tone C # etcetera. Since the diminished fourths B E b and F # B b, the augmented fifths E b B and B b F # belong to the tone of C, if we descend by a fifth, the tone of F shall recognise as its own the diminished fourth B E b, the augmented fifth E b B and the fourth more-than-augmented A b E. If we continue to descend in fifths, the tones that I note herewith accept only more-than-diminished fourths and more-than-augmented fifths, since E A b, A D b; A b E and D b A belong to the tone B b; A D b, D G b and D b A, G b D belong to the tone E b; D G b, G C b, G b D and C b G belong to the tone A b

etcetera.

[-831-][Riccati, The laws of counterpoint, 831; text: Serie di Quarte. di Quinte. G b C b, D b G b, A b D b, E b A b, B b E b, F B b, C F, G C, D G, A D, E A, B E, F # B, C # F #, G # C #, D # G #, A # D #, E # A #, B # E #, 4/3, + 09521, + 05015, - 23345, C b G b, G b D b, D b A b, A b E b, E b B b, B b F, F C, C G, G D, D A, A E, E B, B F #, F # C #, C # G #, G # D #, D # A #, A # E #, E # B #, - 09521, - 05012, + 23345]

[-832-] [signum] 29. I have discussed expressly the twelve fifths and the twelve fourths that we find in the instruments commonly used and their unequal temperament in the sixth, seventh and eighth paragraph. I shall add now some considerations that shall prove useful to clarify further the different character of the tones. Starting from the fourths and fifths that do not need to change the name of any note or key to appear what they are, we have seen that they are eleven in all, six diatonic, three chromatic with the sharps and two chromatic with the flats. Although the chromatic fifths and fourths are more perfect than the diatonic ones, nevertheless the ear considers them to be less well-tempered because their excessive precision renders the tuning of the imperfect chromatic consonances worse. Now, since the imperfection of the aforementioned fourths and fifths derives from the fact that the sharps are a little too high or the flats a little too low, in the case of the fourths and of the fifths as well the heightening of the sharps contributes inspiring cheerful feelings, while the lowering of the flats contributes to produce tender emotions. The character of the twelfth fourth $E b A b = D \# G \# = 4/3 - 23345$. and of the one of the twelfth fifth $A b E b = G \# D \# = 3/2 + 23345$. is much clearer. Both of them accept to be written with flats or sharps, which derives from changing the name of one note instead of the name of the other. If the tone requires them to be written with flats, the ear believes that the notable alteration of the two consonances depends on the lowering of the sound A b, which renders them fraught and melancholic. When the tone requires them to be written with sharps, the same notes or keys produce a very different effect. We are convinced in that case that the imperfect tuning derives from raising the sound D #, which is enough for the aforementioned fourth and fifth to sound piquant and resentful to the ear.

From what has been said earlier (book 1., chapter 4., [signum] 9.) we deduce that, with the exception of the seventh note of the tone with the major third and of the second note of the tone with the minor third, the remaining six notes in both tones have a fifth that corresponds to them. Consequently, the fourth will apply [-833-] to six notes of both tones, with the sole exception of the fourth of the tone of the mode with the major third and of the second of the tone with the minor third. The six fifths and the six fourths corresponding to the white keys, which I am used to calling diatonic to be succinct, are accepted by the two tones sharing the same scale, namely C with the major third and A with the minor third. The number of the sharps or flats that a particular tone needs in its key signature, whose highest number is six, is the same as the number of the fourths and fifths called by me chromatic that belong to it. Said number includes the fourth $E b A b = D \# G \#$ and the fifth $A b E b = G \# D \#$. I restricted the number of the sharps and of the flats to six because, should one venture any further, some diatonic fourths and fifths masked with sharps and flats would find their way into the tones. One of these, for instance, is the fifth $E \# B \# = F C$. The unequal temperament of the twelve fifths contained in the instruments commonly used, among whom the sixth diatonic ones are narrow by 09521 and the five truly chromatic are narrow by 05015, while the twelfth fifth $A b E b = G \# D \#$ grows by 23345, the different number of the different species that is allotted to the different tones, the fact that that they are written with the sharps or with the flats, and, finally, their different position in relation to the notes of the tone are the elements through which, with the exception of the diatonic tones C with the major third and A with the minor third that are perfectly identical as to temperament of the fifths, the other tones differ all from each other precisely because of the aforementioned reason of the fifths. For instance, compare the two tones that accept the same scale, G with the major third and E with the minor third in relation to their fifths. Although both contain the same six fifths, five diatonic and one chromatic, nevertheless one can note the difference consisting in the fact that the chromatic fifth $B F \#$ is based on the third note

of the first tone and on the fifth note of the second. I could adduce many other examples that I consider redundant, as I am happy to close the present paragraph with the reflection that what I have said [-834-] about the fifths can be applied entirely to the fourths.

[Riccati, The laws of counterpoint, 834; text: F b B b, C b F, G b C, D b G, A b D, E b A, B b E, F A, C F #, G C #, D G #, A D #, E A #, B E #, F # B #, C # F 2#, G # C 2#, 7/5, 10/7, + 67062, - 20678, + 71568, - 16172, + 62556, - 25184, + 58050, - 29690, + 25184, + 20678, + 16172, + 29690, B b F b, F C b, C G b, G D b, D A b, A E b, E B b, B F, F #C, C # G, G # D, D # A, A # E, E # B, B # F #, F 2# C #, C 2# G #, 10/7, 7/5, - 67062, + 20678, - 71568, + 16172, - 62556, + 25184, -58050, + 29690]

[singum] 30. The major fourths and the minor fifths that added together constitute the octave, [-835-] have also the property that, since their difference consists of a single enharmonic diesis, one can be employed instead of the other one. The diatonic system were introduced into music by the diatonic system in the first place, since the natural fourth and seventh note of the mode with the major third, and the sixth and the ninth of the mode with the minor third form a major fourth, and, consequently, the seventh and eleventh natural notes of the mode with the major third and the second and the sixth of the mode with the minor third form a minor fifth. The major fourths and the minor fifths employed in counterpoint are provided by the artificial notes. In both modes the first note and the fourth artificial note form a major fourth, while the fourth artificial note and the octave form a minor fifth. Moreover, the mode with the minor third contains three other major fourths between the second artificial note and the fifth, between the third and the sixth artificial note and between the fourth and the seventh artificial note, as well as the minor fifths that added to the former constitute an octave and are located between the fifth and the ninth artificial note, between the sixth artificial note and the tenth and between the seventh artificial note and the eleventh one.

We find in the instrument commonly used sixth truly major fourths, one diatonic and five chromatic. The same applies to the minor fifths. These two intervals accept the suitable ratios 7/3 and 10/7 respectively with great precision. A smaller and barely audible difference is added to the major fourth F B ($F B = 7/5 + 16172$) and to the minor fifth B F ($B F = 10/7 - 16172$). Both belong to the diatonic system and are naturally typical of the tones C with the major third and A with the minor third. The major fourths and the minor fifths are affected by alterations that become greater according to whether they belong naturally to tones that are gradually further removed from the two aforementioned ones. Therefore, the greater difference 29690. by which the major fourth D G # is wider and which corresponds to the amount by which the minor fifth is narrower, is very moderate in relation to the ratios 7/5 and 10/7 that contain the [-836-] odd number 7., as they produce on the ear the same effect that would be produced by alteration of a perfect consonance, which alteration amounts to 12724. If the chromatic major fourths and minor fifths involve the use of sharps, since they are judged to be a little higher than they ought to, for these reasons such intervals shall prove considerably more daring than their respective diatonic ones. Conversely, if our chromatic intervals involve the use of flats, judged by the ear a little narrow, they shall make the major fourths and the corresponding minor fifths rather more emotional than the diatonic ones.

[signum] 31. The remaining six fourths, commonly called major fourths, deserve the name of semi-augmented, while, conversely, their respective fifths, commonly called minor fifths, require the name of semi-diminished. In fact, the former exceed the ratio 7/5 and the others the ratio 10/7 by an enharmonic element, which quantity is contained between 71568. and 58050. The semi-augmented fourths are very close to the ratio 10/7, while the semi-diminished fifths are very close to the ratio 7/5, since the greatest difference equals ± 29690 . The Reader surely realises that a curious sort of exchange is at work, since the semi-augmented fourths embrace the ratio 10/7 that is characteristic of the minor fifths, while the semi-diminished fifths embrace the ratio 7/5 that is characteristic of the major fourths.

After considering the semi-diminished fifths and the semi-augmented fourths in relation to the ratios 7/5 and 10/7, they are simply the difference between the major third 5/4 and the semi-

diminished seventh $7/5$ and the difference between the semi-diminished seventh $7/4$ and the major tenth $5/2$. It is easy to deduce from what I said at paragraph twenty-four and twenty-six that the minor fifth, which is the difference between the major third and the minor seventh, and the major fourth, which is the difference between the minor seventh and the major tenth, are used as representing the ratios $5/7$ and $10/7$.

If we consider again our intervals under their most considerable aspect, they require to become semi-diminished and [-837-] semi-augmented through the enharmonic reduction of a sound. In the case of the fifths this sound is the high one, while in the fourths it is the low one. Thus, while the major third of the consonant accompaniment remains untouched, the seventh turns from minor into semi-diminished, as it occurs in fact in the following chord E b G B b D b. Were the fifths and the fourths that we are discussing to become semi-diminished and semi-augmented by the enharmonic raising of the lower sound in the former, and of the higher sound in the latter, the major third would be damaged and it would be turned into a semi-augmented third, and this would be of no use at all in the seventh. The example I place below of the accompaniment B D # F # A of the semi-augmented third, fifth and minor seventh, in which the fifth D # A is made semi-diminished by the enharmonic alteration of the lower sound, shows clearly that what I say is true.

Moreover, very frequently certain circumstances occur in which the semi-diminished fifth and the semi-augmented fourth are employed as representing the minor fifth and the major fourth, or, in other words, the ratios $10/7$ and $7/5$. This occurs when the semi-diminished fifth and the semi-augmented fourth occur in the accompaniments, one of them fundamental and the other two derivative, commonly called of the minor third and minor fifth, of the minor third and major sixth and of the major fourth and major sixth, which are employed in music as if they were consonant. If sharps are involved in the intervals that we are discussing, one of them shall be larger than its usual size by an enharmonic element, which shall render the semi-augmented fourths cheeky because of the upper sound, and the semi-diminished ones because of the lower sound. If one or two flats occur in said intervals, the one that is larger than usual shall render the semi-augmented fourths sad and melancholic because of the lower sound, and the semi-diminished fifths such because of the higher sound.

I do not want to fail to inform the Reader that the semi-diminished [-838-] fifths consist of two thirds, one minor one and the other semi-diminished, which can be combined in two ways, either by placing the minor third in the bass and the semi-diminished one in the middle parts, or vice versa. In the first case, if the tone requires to be expressed with flats, the higher sound is deemed lowered, while, if the tone requires to be expressed with sharps, the lower and middle sound are deemed to be altered. In the second case, if flats are employed, the ear deems lowered the two higher sounds, while, if the tone requires to be written with sharps, the ear judges the bass to have been raised. The flats and the sharps that correspond to the key that have changed name shall illustrate all what I said clearly for the benefit of the Readers.

[Riccati, The laws of counterpoint, 838; text: Quinta [[più che]] [semi add. supra lin.] diminuita, al cui Basso corrisponde la Terza minore. semidiminuita, C, E b, G b. B #, D #, F #. F, A b, C b. E #, G #, B.]

It is easy to understand that the enharmonic enlargements and reductions of several sounds are employed to express the emotions that are the more extreme of their kind.

[signum] 32. I know very well that there shall be someone who believes that I have been looking for excessive and too subtle minutiae in my review of the keyboard instruments commonly employed. However, how could I have ever overlooked those small differences on which the notably different character of the various tones depends? These supposed irrelevant minutiae are reduced to two sorts, namely, to the enlargement of the sharps and to the reduction of the flats that are truly such, and to the enharmonic modification of the sounds related to the key or notes that change name. Albeit the differences of the first kind are much smaller than the ones of the second kind, nevertheless the various character of the tones [-839-] in which all the keys maintain their own

name derives from them. Who does not recognise the seriousness of the tone B b, the cheerfulness of the tone D both with the major third? Since one requires two flats in the key signature and the other one two sharps, one shall find that the alterations of the intervals that naturally belong to them to be equivalent. However, the different situation of the ratios altered by the same amount in relation to the notes of the tone and, in much greater measure, the contrary way of obtaining the same alteration by raising a note in the tone D, produces some difference that is recognised clearly by the ear, whose perceptiveness is highlighted admirably by the small amounts that we are discussing. When the ear judges a tone, it makes a high number of comparisons between one sound and another one, and, albeit the differences according to which the intervals are more or less altered with each other in different ways are small if considered separately and one by one, nevertheless their sum is considerable and constitute the basis of the judgement of the ear. I shall never agree to consider small amount the differences of the second sort. In fact, even if we consider only the consonances, the act of changing the name of a key or note introduces alteration that reach a comma and one fourth in size, as it occurs in three thirds that are called commonly minor. In fact, who could not distinguish the great difference between the tones E and F both with the minor third would be completely devoid of any aptitude to music. Were one to transport Pergolese's Stabat Mater from the tone F with the minor third, the original in which the composer wrote it, to the tone E, one would ruin that beautiful composition by depriving it of that fraught and tearful character because of which it is so well liked.

I shall conclude the present chapter by relating a fact that shall illustrate in practice the character of the different tones and how fruitful it can be in the practice of counterpoint. While Padre Calegari, whom I praised elsewhere, was master of the chapel in the basilica of Saint Anthony in Padua, made the choir sing an Introit composed by the famous composer [-840-] Costanzo Porta (a) [(a) Costanzo Porta, a Minor Conventual friar, was master of the chapel at Assisi and lived at the time of Palestrina. He was able to avoid many of the harsh features that abound in the compositions of that age, therefore his works are regarded very highly by the cognoscenti add. infra lineas] in the tone G with the minor third. As it proved too low, Padre Calegari transposed it to the tone of B b. It is necessary to say that such transposition was eminently suitable to the nature of the composition, which was the absolute favourite of Signor Tartini. When Padre Vallotti became master of the chapel, a few years later, he made the choir sing the same Introit in the tone of A with the minor third, in order to save singers and players the hardship of so many flats, which are required by the tone of B b. This was sufficient to render Signor Tartini unable to recognise it as the one that he had heard in Padre Calegari's time. He praised Padre Vallotti for the choice of composition, but he praised much more the composition that Padre Calegari used to have the choir sing. Padre Vallotti smiled and candidly said: "This morning's Introit composed by Costanzo Porta and the one that Padre Calegari used to have the choir sing are one and the same." Since Tartini repeated endlessly that this could not be, he took him by the hand and lead him to the harpsichord. First of all he played the Introit in the tone of G with the minor third and then in the tone A, he asked him if he thought he was hearing the Introit by Padre Calegari. Tartini answered that he did not. Finally, he chose the tone of B b, and, as soon as Signor Tartini heard a few bars of it, he said: "Now, this is Padre Calegari's Introit." This fact that I recounted confirmed all the more Padre Vallotti and Signor Tartini in their belief that the specific character of a tone contributes greatly to the beauty of a musical composition.

Fifth Chapter

On the expression of the sentiment of the words through music, with particular reference to the various passions of the soul

[signum] 1. Among the many prerogatives that adorn Music, the most important is its ability to imitate the meaning of the words and to awaken a variety of passions in the soul of the listener. If we listen to the ancient Greeks, they tell us of the prodigious successes caused by their music. If we free those tales from their hyperbolic and excessively miraculous character, we shall realise easily that certainly modern music, at the very least, cannot be inferior to the one of the ancients in the expression of the motions of the soul. The reason why singing is much more appreciated than

instrumental playing depends on this imitation, since the correspondence that it establishes between music and the sentiment of the words awakens a particular delight in the spirit of the listener. I have observed consistently that the most applauded compositions, whether sacred or secular, composed by good composers are the ones in which music is applied to the words with the greatest perfection. I am particularly keen to mention an excellent *Pange lingua* by Padre Calegari sung every year on the Tuesday of the Holy Week by decision of the master of the chapel in the church of Saint Anthony in Padua. The idea of vastness and of the ineffability of the mystery whose institution and praises are celebrated in the hymn is awoken in the soul when one hears it. It would be desirable that the masters of the chapel would always concentrate on the majesty, the seriousness and the devotion of the concepts that they have to accompany with music. This was the Padre maestro Vallotti's aim in his so lauded musical compositions, while I have often adorned these writings of mine with his name.

[signum] 2. Some very immediate imitations and little or no symbolic come spontaneously to the composer's mind. These require few instructions. I shall refer to them [-842-] according to the sequence of my discussion and the occasion requires. Therefore, on the matter of the internal motions of the soul, I say that they can be expressed in music in a principal way, namely, by ensuring to imitate with the musical sounds the alterations that the particular emotions produce in the natural voice. I distinguish human passions into strong and weak passions. I call strong passions the one that are accompanied by an extraordinary agitation of the spirits of the soul which exceeds its natural state. This category contains cheerfulness, courage, temerity, ire, fury and desperation. The other category of passions, which I call weak, renders the motion of the spirits of the soul more languid and less active. This second category contains delicacy of spirit, pusillanimity, sadness, the states of indolent desire and exhaustion. A tranquil state of mind, in which the spirit is furnished with the movements that are most conducive to the perfect performance of the functions of the soul, lays between those two extremes.

If we start from the simplest imitations, everyone knows that strong feelings make us speak louder and faster in relation to the sort of voice that we employ when we are feeling calm. Music benefits greatly and beyond what one would be prepared to believe from loudness and softness. I deem it almost redundant to note that, if music imitates sounds, the strength of the sounds that produce the imitation must be proportional to the sounds that are imitated. Any beginner can see that the loudness of thunder and the explosion of a bomb requires loud and thunderous sound, while, conversely, the pace of a limpid brook or of the branches waved by a soft wind requires soft and weak sounds. If we come to consider the feelings, the character of different passages, whether cheerful, sad or emotionally fraught, teaches the singer and the player who are endowed with good taste the appropriate degrees of loudness and softness required in the passages performed. A cheerful, decisive or resentful passage requires loudness, while, conversely, a tender, languid and melancholic passage [-843-] needs to be performed softly.

Speed, whether fast or slow, apart from the ability to express what is fast or slow, has such a closer relationship with feelings that the different degrees of speed at which the compositions are performed are indicated by the masters of counterpoint with the names of some of our different emotional states by writing at the beginning of the compositions indications such as *Allegro*, *Vivace*, *Moderato*, *Affettuoso*, *Grave* etcetera. One will never be able to inspire cheerfulness with a slow tempo or sadness with a fast one.

The prolonged stay on the same sound is employed to explain rest, duration, eternity, stability, constancy, quiet, sleep, admiration and ecstasy. Left aside the example of loudness and softness and of fast and slow pace, which are employed very frequently to symbolise the emotions belonging to both categories, I shall present some passages in which one can see that the sentiment of the words is imitated by the considerable length of a particular sound. This is how Signor Benedetto Marcello concludes the Psalm XXIII, where he expresses excellently the intensity of the desire of the saintly King David to reside continually in the place consecrated to God.

[Riccati, *The laws of counterpoint*, 843; text: *passar miei giorni e dimorar ogn'ora.*]

[-844-] In the following example, that I took from setting of the Psalm Salmo XXXIX. By the same composer, the trustworthiness of God's promises.

[Riccati, The laws of counterpoint, 844; text: e l'alta fede di tue promesse che fur mai sempre il saldo mio sostegno, et cetera]

I do not want to deprive the Reader of a third example. The aforementioned knight imitates equally well the ecstatic desire of the soul longing for heaven in the first versicle [-845-] of Psalm XIV.

[Riccati, The laws of counterpoint, 845; text: Allegro, O signor chi sarà mai che giunger possa colà dove avete posta la beata vostra sede, 3 #, 6, 4, 3. 8 3#, 7]

[signum] 3. The pitch of the voice can also prove useful in awakening in the mind the ideas of different characters and affections. Since the low voices belong to the male sex and the high voices to the female sex and since majesty and strength are distinctive of the man because of their make-up, while grace and softness are distinctive of the woman, consequently, since those ideas are connected reciprocally, the low sounds shall appear majestic and strong, while the high ones shall appear graceful and soft. Moreover, since a man is more powerful and courageous than a woman, and, therefore, is more prone to harming others and to inspire fear, composers employ low and masculine voices instead of high and feminine voices to produce a sense of fear. This matter is obvious and it is innate, I am about to say, since even little old women strive [-846-] to alter their voice when they tell children certain spooky tales that cause them much detriment. Darkness is closely connected with fear. If man, who is destined for light, cannot use his sight, the main among his senses, because of darkness runs the risk of being attacked without being able to defend himself. Therefore, when we are in the dark, everything makes us apprehensive and fearful. In that situation fantasy is perturbed easily and imagines horrible phantasms and terrible spectral apparitions. From these physical observations one can deduce easily the reason why low voices are employed when the text describes dark nights and ghosts of the dead that must sing with a low voice when they are assigned words. Depth is also symbolised with the use of low voices, since it is often associated with darkness and since it evokes fear in the spectators. For the opposite reason, high voices are employed to express sublime concepts. If, while I speak, I happen to describe the deep darkness of a valley and the steep peak of a mountain, I shall make the tone of my voice first lower and then higher naturally and without realising it. Therefore, without pursuing new observations, it is enough to inform that the lowest of the musical parts is called bass, while the two upper ones are called alto and soprano. These mere names clearly show the analogy that our mind creates between what is low in pitch and low in space and between what is high in pitch and in space. I draw the Reader's attention to how Signora Contessa Donna Teresa Agnesi, who is as excellent in counterpoint as Signora Contessa Donna Maria her sister is in mathematics and in the most elaborate calculations, inspires us the fantasy of a valley at the beginning of an aria of a very beautiful composition written in the style that we are used to calling *a cappella*.

[Riccati, The laws of counterpoint, 846; text: Dall'opaca valle impura, et cetera]

Since what is really or metaphorically ugly is often source of fear, it will require to be preferably expressed by a low voice. The following example is taken from the Psalm XV. by Signor Benedetto [-847-] Marcello, where the rejection of the saintly prophet towards the cult of the idols is expressed to perfection.

[Riccati, The laws of counterpoint, 847; text: il nome ancor dell'ampio culto immondo, #]

[signum] 4. Similarly, the different sorts of bars, whether binary or ternary, shall contribute

to imitation. Among those of the first sort, which are more serious than those of the other sort, the bar of four beats sounds more majestic than any other. The ternary bars sound cheerful and brilliant, and are express marvellously the feeling of joy, dance, and jubilation. The same occurs in the Dodecupla, Sestupla and in the Nonupla. Some masters are reluctant to employ them in church music (always provided that they are performed at a fast speed) because they appear to affect adversely the decorum of their function.

The choice between a time signature or another one of a particular subdivision of the beat contributes frequently to represent appropriately the meaning of the words. The strong beat that is self-sustaining symbolises very naturally tranquillity, constancy and a calm attitude. Conversely, the weak beat, which leans on to the immediately following strong beat is suited to indicate rush, movement and resolve. Nor should anyone claim that speedy pronunciation does not allow room to adapt strong and weak beats to the words. In fact, the accented syllables that are self-sustaining must be placed on a strong beat of the bar or on a strong subdivision of the beat, such as, as I explained elsewhere (book 1., chapter 8., [signum] 5., 9. and 10.) the first and the third beat or the first part of a subdivision in a bar of four beats [-848-] and the first and second beat of a bar of three beats or of a ternary subdivision of the beat. One does not need a long discussion to understand that, after assigning the strong beats to the accented syllables, I retain some freedom in relation to the syllables that are not accented. As long as the accented syllable begins on a strong beat, I can lengthen it by one or two beats and make the next adjacent unaccented syllable fall on a strong or on a weak beat. Even the syllable that precedes immediately the accented syllable can begin to be sung on a strong beat or on a weak one while preserving the correct placement of the accented syllable, as this depends on its duration. While the strong beat is reserved always for the second syllable of the words Dovunque, in the first of the examples added herewith the first syllable starts on a strong beat, while in the second one it starts on a weak beat. It seems to me that the second example shows better than the first one the preoccupation with turning around.

[Riccati, The laws of counterpoint, 848; text: Dovunque io volgami]

The syncopation and the tie that corresponds to it join together to beats, one weak and one strong (the first one of which must move towards the second one by its own nature) which represent very well the artificial union of two distinct entities. Therefore, composers have employed them to express real or metaphorical ties and to highlight their strength. In the following example, contained in the Psalm, XII. Signor Benedetto Marcello allows the listener to understand in an excellent way the violence suffered by an impenitent soul in detaching himself completely from God.

[-849-] [Riccati, The laws of counterpoint, 849; text: che stacca nel sonno mortale da te, et cetera]

The loss of balance of a fall or of a shaking action is expressed vividly by two notes, the first one followed by the dot and the second one equal in value to the [[beat]] [dot corr. supra lin.] itself. I take the following example from the same Psalm XII. by Signor Benedetto Marcello, who imitates always the meaning of the words in a marvellous way.

[Riccati, The laws of counterpoint, 849; text: Se scosso mi vedessero, et cetera]

[signum] 5. Not only the rests allow the voice to recuperate and the entries of the subjects and countersubjects [[help]] and of their answers to be more prominent, but they also help the expression of the feeling of the text. In the first place, they appear suitable to separate a concept from another one. Secondly, they are naturally suited [-850-] [to represent add. supra lin.] the idea of silence to the mind of the listener. I invite the Reader to consider the following two examples by Signor Marcello, the first one contained in his setting of Psalm XXXVIII., and the second one in his setting of Psalm XXIX.

[Riccati, The laws of counterpoint, 850; text: Adagio. tacer risolsi, 3 b, 3 [sqb], Forse un cenere freddo potrò lodarti, et cetera, 5 3 #, 7 5, 7 b 5, 6 5 b 3]

Thirdly, rests can indicate the action of waiting and expecting as well as any suspension or conclusion. Consider how well is presented to the listener the image of the destruction of the memory and worship of the heathen Gods in the following example provided by the Psalm XV. by Signor Benedetto. The resounding bass that stops abruptly indicates vividly that the memory of such celebrated gods shall disappear as fast as lightening.

[-851-] [Riccati, The laws of counterpoint, 851,1; text: Risoluto. ogni loro memoria si tolga dal mondo, et cetera]

The rests express in the following passages all those emotions that pervade us so completely that we lose the ability to speak. Doubt or suspension of judgement, confusion, admiration and incredulity belong to this category. I have taken the following example from Psalm XXXVII. by the same important composer whom I have lauded so many times.

[Riccati, The laws of counterpoint, 850,2; text: Largo. di profondo stupore ingombri e pieni, 4 #, 6, 5 3 #, 6 4, 5 3 #, 7 5, 5 # 4, 3 #]

[-852-] Finally, if there is a series of notes or rests of small and equal value that alternate regularly, they will produce an interrupted voice, whose interrupted quality will be even more prominent if the notes fall on the weak beat, since they will remain suspended because they lack a downbeat to sustain them. These interrupted voices can prove useful in different sorts of imitations. I invite the Readers to consider two examples taken from the usual collection of psalms. In the first one, from the Psalm VI., the interruption of the voice produced by profuse crying is represented perfectly. Observe that the notes in this example are sung on the weak beat, while the rests occupy the strong beat. The second example, from the Psalm XXXVII., represents the palpitation of the heart by resorting to two series of notes and rests, one placed in the singing part and the other one in the bass. These two series are organised in such a way that the note of the upper part coincides with a rest of the lower one and vice versa. The sounds of the alto and of the bass that hit the air alternatively represent very well the oscillations or the repeated beating of the heart.

[Riccati, The laws of counterpoint, 852; text: Andante, Dal lungo sospirar, gemer, Turbato e palpitante ho il cor in petto, et cetera, 5, 6, 7, 5 b, 3 b]

As I have encountered the following graceful example of two series of notes and rests ordered in such a way that a part leads and the other one follows, I thought it good to add it to the other two by transcribing it from the Psalm L., so that the Reader may enjoy the exquisite expression [-854-] of the words seguendo i passi miei.

[
Riccati, The laws of counterpoint, 853; text: i passi miei seguendo, et cetera]

[signum] 6. The sources of imitation ascertained so-far are softness and loudness, slow and fast pace, low and high pitch, the different sorts of bars, strong and weak beats, the ties and the rests. The Reader will have realised that I have not mentioned expressly yet two of the most important elements that constitute music, namely, harmony and melody. I shall discuss first some melodic expressions that depend on what has been explained in the previous paragraphs and do not require new principles.

I said earlier ([signum] 3.) that high voices represent sublime objects and low voices the lowly ones. One deduces easily from this that I shall employ sounds that move from the low register

to the high register to signify ascent and sounds that move from the high register to the low one to signify descent. This imitation is so appropriate that musicians call ascent the movement from the low register to the high one and, conversely, descent, the movement from the high register to the low one. Ascent is connected with rising, elevating and, metaphorically, with praising, glorifying and achieving a high and happy condition, which are all situations [-854-] that are expressed with melodies that move from the low register to the high register. Descent is connected with falling, sliding down, falling abruptly, submerge, throwing to the ground, lowering, oppressing, condemning, verbally abusing and with becoming depressed or losing a high status. It shall be possible to indicate each of those actions with notes that descend from the high to the low register. Albeit the two sorts of imitations that I described are encountered very frequently, nevertheless I shall present two examples of them, one for each sort. I have transcribed the first one from the Psalm XXXV. by Signor Benedetto Marcello and the second one by Monsignore Steffani's duet that begins Vorrei dire un non so che.

[Riccati, The laws of counterpoint, 854; text: Onde più non potranno alzar la fronte, et cetera, 6, 6 # 3, 3 [sqb], 3 b, 5 2, 6, 3 b, 5 4, 3, Vorrei dire un non so che mi tiene oppresso il cor, 3 #, 4]

[-855-] 7. If we compare two sounds, a low one and a high one, we say in musical speak that the high one 'is enlarged' above the lower one and the lower one 'is reduced' from the high one. Therefore, we shall be very correct in indicating enlargement and reduction by employing melodies that move respectively from the low register to the high one and from the high register to the low one. Enlargement and reduction are expressed frequently by ascending and descending respectively by a minor semitone, which hits the ear very prominently and allows us to form a very clear idea of said enlargement and reduction. Monsignore Steffani's duet Troppo cruda è la mia sorte provides me with the example placed herewith.

[Riccati, The laws of counterpoint, 855; text: quanto cresca il duolo, 5 3, 4 2, 6 5 3, 6 [sqb], 3, 3 [sqb], 9, 8, 5 4]

[-856-] In the following extract taken from Signor Benedetto Marcello's Psalm XI., the voice ascends by a major semitone at the word crescan and the beautiful device of repeating three times the word crescan in the higher alto produces the series of semitones G A b, A B b and B C. It is true that the repetition of the word crescan are separated by a few notes setting the words più numerosi, but it is also true that, when, for instance the word crescan is sung for the second time, the words interspersed do not make me forget the first time said word was sung, so that I am very well aware of the rising of the minor semitone A b A, which is all the more noticeable as it introduces a modulation, since through it the composition moves from the tone of E b to the tone of B b both with the major third. The same observations apply to the minor semitone B b B which occurs as the last note of the word crescan uttered for the second time and as the first word of the same word uttered for the third time. The lower alto repeats the word crescan twice, which produces the sequence of semitones D E b and E F <.> These repeated enlargements awaken the in the mind the idea that the increase of the of the number of those who are hones [and of their happiness add. supra lin.] must be continuous and ever increasing.

[Riccati, The laws of counterpoint, 856; text: Allegro assai, Crescan più numerosi, et cetera, 6 b, 7 b, 7 3 [sqb], 3 b]

[-857-] Just as progress is a sort of continuation and a repeated increasing of the started journey, thus the act of returning is a repeated reduction of the journey previously accomplished. Therefore, progress shall be with notes that move from the low to the high register, conversely, the act of returning shall be expressed with notes that return from the high register to the low one. I shall take as an example the settings of the Psalms employed earlier, as they contain every sort of

imitation. The Reader shall find the example written herewith at the end of the sixth Psalm.

[Riccati, The laws of counterpoint, 857,1; text: Largo, tornino indietro]

If one turns while walking or turning, one moves now forwards and now backwards. For this reason, such action shall be represented with notes that move sometimes upwards from the low to the high register and sometimes downwards from the high register to the low one. Consider the example contained in the Psalm XI, and note that, since the text talks of a circular movement that turns back onto itself, the melody as well returns to the note from which it started after it has completed its circular movement.

[Riccati, The laws of counterpoint, 857,2; text: Presto, Vanno girando a cerco, et cetera]

[-858-] Everything disappears and dies as a result of repeated reductions. Repeated reductions are illustrated by a series of sounds that descend from the high register to the low one. Therefore, the disappearing and the dying of an object shall be expressed vividly by the previous series. The first Psalm by Signor Benedetto Marcello suggests to me the following examples, where one part stops singing and then the other one and in this way as well the depiction of the dying and of the disappearing of the ways of the wicked.

[Riccati, The laws of counterpoint, 858; text: dileguansi, periscono]

An object disappears and its appearance is removed also when it is hidden and covered. Such concepts shall be imitated by melodies that move from the high to the low register. We shall find further confirmation of this when we recall what we said earlier (when we noted ([signum] 3.) that darkness is represented by low voices), when we observe that darkness hides and shields the objects from sight, and that we say metaphorically of something that is obscure that it is enveloped in thick darkness. Therefore, it shall be easy to draw the consequence that to unveil, to discover what something is and to have something in front of one's eyes are depicted appropriately with sounds that move from the low register to the high one.

[-859-] The first example that I present comes from the Psalm XXXIV. by Benedetto Marcello, a the composer that I praised many times. The second one is taken from the Psalm L.

[Riccati, The laws of counterpoint, 859; text: Tanto avvenga poich'essi a mia rovina tesero a torto insidiosi lacci e ingiustamente mi cuoprian d'oltraggi, et cetera, innanzi agli occhi ogn'ora]

[signum] 8. Distance and separation are expressed as a rule by [-860-] the masters of counterpoint with sounds that are far removed one from the other. The examples are as frequent as they can be. I transcribed the following one from a Cantata by Signore Scarlatti senior that begins with the words che principia Alfin m'ucciderete o miei pensieri.

[Riccati, The laws of counterpoint, 860,1; text: Pena ch'in lontananza ogni altra pena avanza sai qual è Clori mia? et cetera]

However, if one wants to imitate the action of the removing an object from an other one, since this distance is small to start with and becomes greater, thus it will be necessary to symbolise it with a series of note that become gradually more distant from the first one. We found this very well executed at the beginning of another cantata by Scarlatti himself, that I place under the eyes of the Readers<.>

[Riccati, The laws of counterpoint, 860,2; text: Già vicina è quell'ora ch'allontanarmi io deggio da te da te bell'Idol mio, et cetera]

In the following example Signor Benedetto Marcello [-861-] presents to the imagination the very distant boundaries to which the empire of the Messiah was going to extend. I transcribed it from the Psalm II.

[Riccati, The laws of counterpoint, 861,1; text: E non i monti o i mari ma la rimote estreme contrade della terra faran confine, et cetera]

[signum] 9. The simplicity and other similar virtues, such as innocence, purity, candour, peace, rectitude, truth, docility and facility are expressed with the most simple melodies. Such are the ones that can be observed in the recitative that I add herewith, which I transcribe from the Psalm I. Such melodies are of two sorts. The first sort one consists of harmonies sung as melodies, the second one consists of true melodies. The first ones are easy to tune because they are all consonant, while the second ones are also very simple. Proof of this can be found in the very frequent use in the bass part of the two passages A 3 # F # 6 [sqb] 3 and A 6 # 4 3 G 5 3, which [are add. supra lin.] the the only two true melodies contained in the example that I present.

[Riccati, The laws of counterpoint, 861,2; text: Sono esposte e son care al Signor nostro le vie per cui camminan gl'innocenti, 6, 6 4 2]

[-862-] The following excerpt from a recitative contained in the Psalm V. is very similar to the previous one. I did not want to omit it so that the Reader may observe the extreme ease that the singer finds in tuning the minor seventh G in relation to the fundamental bass A placed at the end of the first bar, to which one ascends from the consonant note E through the stepwise movements E F and F G.

[Riccati, The laws of counterpoint, 862; text: e fa che rette siano agli occhi tuoni le vie della mia vita, et cetera, 6, 5, 6 4 # 2, 6 5]

Since the unison holds the first place among the simple melodies, the masters employ it frequently to symbolise the aforementioned virtues by accompanying it with equally simple harmonies among the different parts themselves. I invite the Reader to consider the example by Signor Marcello taken from Psalm XI., where I note that the three bars in which both parts sing the same E b contain the unison both as a melody and as a harmony, because each part holds the note E b and because the two parts correspond to each other at the unison.

[-863-] [Riccati, The laws of counterpoint, 863; text: più non regna candor nè veritade]

The unison is eminently suitable to indicate the straight road, whether real or metaphorical. Since the unison does not tend towards the low register or towards the high one, thus a straight road does not bend towards the right or towards the left. Moreover, a road or any other horizontal object that does not rise or fall is expressed very well by the unison, as it does not fall towards the low register or rise towards the high register. I invite the Reader to observe as the straight road of virtue is represented in the Psalm XIII.

[-364-] [Riccati, The laws of counterpoint, 864; text: e tutti vede fuor del dritto calle gettar lor vita e lor fatiche, et cetera]

[signum] 10. What was said in the previous paragraph teaches us that composers must employ sophisticated and artificial melodies to illustrate shrewdness, tricks, expedients, plots, malice, turbulence, lies, fraud and deceit. If the topic concerns shrewdness, intelligence and honest expedients, the melodies shall be elaborate but also pleasant to the ear. On the contrary, if the

melodies need to imitate unjust plots, malice, turbulence, lies, fraud, deceit and other such vices, one must choose, among the elaborate melodies, those that sting the ear. The displeasure produced ensures the expression of the evil nature of the aforementioned vices, the damage that they cause society and the disapproval that they deserve. Lies, fraud and deceit shall be represented vividly by modulations that sound unexpected and trick the ear. Some composers employ the defective passages that I described in the seventh chapter of the first book to achieve this aim. In fact, the subject of several examples presented there deal with the subject of lies, fraud, deceit and betrayal. However, since the aforementioned [-365-] passages do not follow the good rules of counterpoint, I cannot approve their application, since music is rich in passages that follow the rules but thwart the expectations of the ear and that can be eminently suitable to that aforementioned sort of imitations. In the example that I present, which is borrowed from the Psalm XVI. the ear is convinced that the accompaniment B 6 3 must follow C 5 3. However, the famous composer thwarts such belief and follows B 6 3 with the harmony G # 5 3 and moves suddenly through it from the tone C with the major third to the tone A with the minor third to which the following passage G # 6 3 A 5 3 belongs. Now, while the becomes convinced that the melody can remain a while in the aforementioned tone, the composer moves unexpectedly to the chord D 5 3 # in order to conclude in an unforeseen manner on the tone G with the major third.

[Riccati, The laws of counterpoint, 865; text: che da un core escon sincero non da un labro ingannator, 6 3, #]

[signum] 11. When I dealt with the cadences (book 2., chapter 1.), I said that they are the last motion that one makes when completing the composition or some section of it, which movement must leave the ear satisfied and is equivalent to the conclusion of a sentence. Therefore, the completeness of the meaning of the words shall be expressed through cadences, whose degree of perfection and greater or smaller ability to provide conclusion I have discussed at length. The cadences shall be able to signify accomplishment, conclusion, establishment, decision and other such concepts after which the listener does not expect anything else. [-866-] It is worth repeating here what I pointed out earlier (book 2., chapter 1., [signum] 21.), namely, that the ear considers that a cadence has taken place only when the bass moves either from a fundamental accompaniment to another fundamental accompaniment or from a derivative one to a fundamental one. The example that I transcribe from the Psalm VII. contains a prayer of the psalmist to God so that he may decide a dispute in his favour and against his enemies.

Note that three cadences correspond to the word decidi, also repeated three times. Two such cadences are perfect (F 7 b 5 3 B b 5 3 and G 7 5 3 C 5 3), while one of them (A b 6 3 G 5 3 [sqb]) moves from the fourth to the fifth note of the tone C with the minor third in relation to the fundamental bass. In order to make more conclusive and decisive the perfect cadences, the composer thought it appropriate to add the minor seventh to the preceding accompaniment, which, as I noted elsewhere (book 3., chapter 2., [signum] 7.) renders necessary in a certain way the passage from the fifth note to the first one, which is typical of the perfect cadences.

[Riccati, The laws of counterpoint, 866; text: siedi e l'ardua lite e grande ivi decidi, et cetera, 7, 6 3]

Before I move any further, I do not want to fail to inform the reader that one modulates after a cadence, [-867-] such modulation, especially when one moves to a tone of a different nature from the preceding one, shall be useful to highlight the differences between the sentiments or the descriptions in the poem or in the prose. In the duet by Monsignore Steffani entitled Saldi marmi quoted herewith, after reporting Fille's words to her dead lover Fileno, the poet turns to tell the story of the loves of Fille and Fileno and of the death of the latter. Steffani expresses such variety of styles by passing from the tone B with the major third to the tone of G with the minor third.

[Riccati, The laws of counterpoint, 867; text: Così Fille dicea del suo perduto bene rivolta un giorno

a le bellezze estinte. Viss'ella di Fileno lunga stagione in fortunati amori, 6 3, 6 4 2, 5 3]

[signum] 12. The sense of the discourse shall be kept suspended by employing passages derived from the cadences that conclude on a derivative accompaniment of the third and sixth and of the fourth and sixths or by employing the fundamental passages that are not cadences or their derivative ones. In the example below the passage B b 5 3 A 6 3, ending on an accompaniment of the third and sixth, derives from the counter-cadence B b 5 3 F 5 3. The three following passages A 6 3 G 5 3 b, G 5 3 b D 6 b 3 and D 6 b 3 E b 6 b 3 originate from the fundamental passages [-868-] F 5 3 G 5 3 b, G 5 3 b B b 5 3, B b 5 3 C 5 3 b that are not counted among the cadences belonging to the tone B b with the major third on which the composition is based. A much greater sense of suspension is created when one employs the dissonant accompaniments, which the ear understands that cannot be employed as a full-stop since some sort of truly consonant accompaniment is required after them that may sweeten the harshness that they have produced on the ear. In the example that I adduce the ear shall never believe that the sense may end on the word bellezze, to the last two syllables of which are assigned the dissonant accompaniment E b 6 4 2. The suspended passages that we are discussing shall prove equally useful to express all the sentiments that leave the soul unsatisfied and require to be followed others, as for instance the sense of continuation of something, the need to search, the feeling of confusion and the doubt. I invite the Reader to enjoy the following recitative contained in the Psalm VIII. and to pay attention to the word confuso set to the three notes E b C and C. The first of them based on the note F # forms the diminished seventh, which is a dissonance, and the remaining two form a minor fifth, which in our case is a consonance by representation.

[Riccati, The laws of counterpoint, 868; text: Attonito e confuso fra sì gran meraviglia allora io dico]

[signum] 13. I find that the feeling of doubt and of searching are also imitated with the aid of frequent modulations from one tone to another one that leave the ear uncertain as to the key [-869-] onto which the composer may conclude the passage. As to the feeling of searching, the different keys express not only the uncertainty to find what is being sought, but also the variety of the places that are being searched. The first and very fine example is taken from the Psalm XII., while the second comes by a cantata by Signore Scarlatti senior beginning with the words Quanti affanni ad un core.

[Riccati, The laws of counterpoint, 869,1; text: dubbio ed incerto?, 3 #, 3 [sqb], 5 [sqb], 6 4 # 2, 6, 7]

The last repetition of the word dubbio is set to the inconclusive passage A 6 # 4 # 2 G 6 3, which ends on a derivative accompaniment of the third and sixth on which the singer must linger a little, as this sign indicates [corona]. This way of symbolising the feeling of doubt is connected to the method that I proposed in the previous paragraph.

[Riccati, The laws of counterpoint, 869,2; text: cercando Clori, 6, 4, 3 #, 5 3 [sqb], 5 3 #, 6 #, 3, et cetera]

[-870-] 14. Some objects that, considered by themselves, would prove difficult to imitate, can be expressed sometimes very easily by means of the circumstances that accompany them. At first sight one would not know how to express hope, but, were it for instance doubtful and ill-founded, these circumstances suggest the way to symbolise it with the devices that suggest doubt and deceit. The text of the Psalm XIX. tells of how the defeated enemies had placed their hope in their carts and horses. Now, since this hope was ill-founded, Signor Benedetto Marcello expresses it with the melodic passage D 5 3 # B 5 # 3 #, that is inconclusive and deceives the listener. It is

inconclusive because the passages of the third downwards or upwards do not count as cadences, while it deceives the listener because, since the composition is in the tone G with the major third, and singer the ear believes that the composer moves from D 5 3 # to G 5 3, such expectation is thwarted by having the accompaniment B 5 # 3 # follow the D 5 3 # and by moving suddenly from the tone G with the major third to the tone E with the minor third.

[Riccati, The laws of counterpoint, 870; text: riposta aveano ne' forti carri la speme, 4/2, 6 3, 6, 6 5, 3 #]

[-871-] If one was asked to imitate lightening, one could note that it moves quickly and in a tortuous way. Since I have explained earlier ([signum] 9.) that straight progress is represented with the unison, it follows that progress that does not follow in a straight line shall be expressed with sounds that move now from the high to the low register and now from the low to the high register. One shall be able to depict lightening with this sort of sounds that move in rapid succession. Signor Benedetto Marcello employed this device in the following passage of the Psalm VII.

[Riccati, The laws of counterpoint, 871; text: vi farà strisciar su gli occhi la sua spada balenante, et cetera, 4, 3[aqb], 6 5, 6 5 3, 6, 4 b 2, 4 2, 5, 7]

[signum]. The consideration of the impression that are produced by the motions of the soul and of the feelings that they inspire is one of the main circumstances that one can note in them. Therefore, one can see that, if we are able to express the different sort of emotions, we shall also be able to imitate any object that arises them in our imagination. Why the most pleasant melodies and harmonies are employed in music in bucolic subjects? Certainly for no other reason than such topics produce in us pleasant feelings. Pastoral life is perceived as comfortable, simple and innocent and it is conducted in pleasant surroundings that please the ear to a high degree. The animals of which shepherds take care are docile and useful, as they provide wool to make garments and milk, which is such a fine food. Moreover, poetry, instrumental playing and singing are associated with shepherds, all things that are exceedingly pleasing. Therefore, as I proceed to seek out the sources of the imitations of the emotions, first of all I divide them into pleasant and unpleasant. Mixed emotions are half-way between those two extremes, as they partake of either to a varying degree. It would be long-winded and redundant to list the pleasant, unpleasant and mixed feelings. It is sufficient to refer to one's inner sentient faculty to ascertain if a particular emotion belongs [-872-] to one sort rather than to another one. For instance, everyone shall agree that the pleasure that one derives from acquiring the object of one's affections is a pleasant feeling, while the hope to acquire it again is a mixed emotion, since it is always accompanied by the uncertainty of being able to do so.

Pleasant emotions are evoked in music by employing the most exquisite melodies and harmonies. I have already taught (book 1., chapter 1., 2., and 3.) that the most pleasant melodies are stepwise motions and consonant motions upwards and downwards, and that the most pleasant harmonies consist of aequisonances, perfect consonances and imperfect ones. Piquant melodic motions, consonances by representation and dissonances can be added to evoke pleasant emotions, as long as they are chosen among the milder ones and they are employed merely as a condiment, with the same aim as the one of a cook mixes with food salt, acid, bitter and severe flavours, namely, to render the composition tastier. Harsh-sounding melodies must be employed sparingly and the dissonances must be heard in passing, with the exception perhaps of the minor seventh, which is so mild that the composition can linger on it for a considerable time without losing its pleasant and graceful character. Among the piquant melodies and the consonances by representation, the less harsh ones are the minor fifth and the major fourth since they derive from the diatonic system, as long as one applies the prescribed rules and precautions in handling them. Among the dissonances that bear the same name, the minor seventh, the major ninth, the eleventh or fourth and the thirteenth or major sixth deserve the highest consideration. The major eleventh or

major fourth and the diminished eleventh or diminished fourth, as consonances of an extremely harsh character, need to be avoided if one aims to express pleasant emotions. To that aim, one shall employ the most elegant passages, and, if one of them is not considered among the most perfect when employed from a root position to another root position, it should be used in the forms in which it sounds more pleasant. Moreover, the best modulations from one tone to another one [-874-] must be practised in the most graceful ways, so that one may derive from them very considerable pleasure. The artificial fourths of both modes and the second of the mode with the minor third must be employed sparingly and carefully, ensuring, finally, that one employs tones that are well tuned to each other and in which the keys preserve their proper name.

[signum] 16. The simplicity and happiness of pastoral life are expressed most excellently in the first example that I present, which is taken from an aria by the Signora Contessa Donna Teresa Agnesi. All the melodic passages employed in the singing part move by step or by consonant intervals except for a single one, C # G of the minor fifth upwards, which nevertheless is very easy to tune, as it is part of a harmony employed in a melodic fashion. The Reader will observe that the accompaniment of the minor third and diminished fifth, which is the most elegant of the ones that are consonant by representation, is employed three times in it. The dissonant accompaniment of the major fourth and sixth is used once, but the third and the fifth are removed from it to curb its excessive harshness. The minor seventh is heard three times, in one case added to the consonant accompaniment of the major third and fifth, and to the accompaniment of the major third and minor fifth, which is consonant by representation, in the other two cases.

Signora Contessa introduces all of these consonances in passing and only as a condiment to enhance the harmonies of composition. This has to be said with even greater truth with regard to the appoggiaturas that, among the many used, can be considered as consonances. The perfect passages contained in the proposed example are very familiar and the only modulation from one tone to another one is the most simple one that moves from the principal tone to the subordinate one based on the fifth note.

[-875-] [Riccati, The laws of counterpoint, 875; text: Innocenti pastorelle quanto invidia il vostro fato di guidar l'amata agnella dalla fonte al monte al prato è vostr'unico pensier, 6 4, 5 3, 7 5, 6 3, 5 3, 3 #, 7, 6]

[-876-] The second example is taken from the beginning of a duet by Monsignore Steffani. He expresses the sweetness of the ties of love by employing the devices that I described earlier. The melodic motions are all by step or consonant, only one consonant accompaniment by representation (B 6 3 #) is used, and only two dissonances, the minor and the major seventh, are heard tied and prepared with the pure aim to express such amorous ties. The passages adopted are some of the most common.

[Riccati, The laws of counterpoint, 876; text: Placidissime catene, et cetera, 6 4, 5 3, 5, 6, 7, 5, 3]

The third example is richer in dissonances, but they are as tasty as they can be. I have taken it from the third Psalm by Signor Benedetto Marcello, in which the composer expresses with the his usual skill, the happiness of the psalmist when his prayer is answered by God.

[Riccati, The laws of counterpoint, 877; text: Adagio. Già di mie preci il grido al mio Signor alzai ed ei dal sacro monte in cui dimora mi porse orecchio e m'esaudì chlemente. 7 4, 5 3, 6 4, 6 5 3, 11 9, 10 8, 6, 7, 6 4 2, 6 3]

[signum] 17. We shall achieve the goal to awaken unpleasant emotions in the listener's soul by introducing more often melodies and harmonies that sound resentful and piquant into our composition. The melodies that we mention contain the minor semitone, the augmented second and the diminished seventh, the diminished third and the augmented sixth, the diminished fourth and the

augmented fifth and the major fourth and the minor fifth. The melodies that sting the ear are divided into consonant by representation and dissonant. One shall employ the first sort by employing the four fundamental consonant accompaniments by [-878-] representation, namely, those of the minor third and minor fifth, those of the diminished third and minor fifth, those of the major third and augmented fifth or their derivative ones. As to the more or less harsh nature of said accompaniments, I have explained it to the Reader in the appropriate places. Consequently, when we employ the aforementioned melodies and harmonies, we shall adopt on many occasions the two artificial notes that strike the ear more powerfully, namely, the fourth of both modes and the second of the mode with the minor third.

The dissonances shall render the composition harsher (even if the mildest of their sort are selected) when they are introduced frequently and are not resolved so quickly that the ear cannot taste their bitter flavour. More and more unpleasant emotions shall be evoked in the mind if the most biting consonances of each sort are employed in the way described just now. Such dissonances are the diminished seventh and the major one, the minor ninth, especially when it is added to the consonant accompaniment with the major third with which it forms a diminished seventh, the diminished eleventh or diminished fourth, the major eleventh or major fourth and finally the minor thirteenth or minor sixth, especially when it is added to the consonant accompaniment with the major third, with which it forms an eleventh or a diminished fourth. When the dissonances are added to the consonant accompaniment, it is common practice to omit for the most part the sounds of the aforementioned accompaniment that form a descending second with the dissonance and on which the dissonance resolves. Now, if we want the harshness of the dissonances to be more prominent, we have to privilege the sounds of the consonant accompaniment that form with them a second or a seventh upwards or downwards as well as the ones that form with them some altered interval of the augmented second or of the diminished seventh, of the diminished fourth or of the augmented fifth and of the major fourth or of the minor fifth. Similarly, their bitterness shall be increased when more than one dissonance is employed at the same time, especially the ones that form a seventh or a second, which occurs in the dissonance of the sixth and of the seventh or in the case of some altered interval, as in the major seventh and the eleventh, major ninth and minor thirteenth, [-879-] which form with each other the interval of a minor fifth. Employing deliberately and without preparation the minor and diminished seventh [(and sometimes even the major one) add. supra lin.], preparing with them or through consonances by representation the dissonances of the ninth, eleventh and thirteenth, resolving the dissonances on the aforementioned seventh or in a consonance by representation and, finally, seeking and applying all the devices that contribute to render the dissonances more audible shall benefit our aim to awaken sad emotions inside of us.

I invite the Reader to employ with discretion the fundamental and derivative passages as well as the less modulations from one tone to another one that are less elegant and to choose the less well-tempered tones that express best the varied character of negative emotions according to what yet remains for me to explain. I invite the Reader to note that the aim of music is to produce pleasure. Therefore, one has to give way to such harsh elements with the precaution that is conducive to the character of music and that is more in tune with good taste and with the approval of those who are more expert of it.

[signum] 14. The examples that I add herewith shall show the Readers practically what I explained in the previous paragraph. The first example is provided to me by the duet by Monsignore Steffani entitled Placidissime catene. The melodic movements, except for a single G C #, a minor fifth downwards, which is nevertheless easy to tune, are among the most exquisite, as they are stepwise movements or consonances. However, the accompaniments are all truly consonant, except for a single E G C # derived from the fundamental C # E G, which derivative accompaniment is the most perfect of the consonant accompaniments by representation. One can also observe the great elegance in the fundamental and derivative passages, which please the ear to a high degree. Nevertheless, the famous composer expresses excellently the sentiment of the words Vivo in doglie, e moro in pene by employing frequent dissonances, which, although they are the mildest of their sort, make an impression on the ear because they are held for one quarter of a slow bar. In

addition to this, while said dissonances are heard, the singing parts form always with each other intervals of the seventh, ninth or [-880-] augmented fifth, which renders the dissonances themselves more prominent.

[Riccati, The laws of counterpoint, 880,1; text: Adagio. Vivo in doglie moro in pene, et cetera, 5 3, 5 4, 3 #, 9, 7, 6, 7 3 #, 8, 4]

In the following example taken from another duet by Monsignore Steffani we can see that the minor ninth is added to a consonant accompaniment with the major third, which third, however, is not heard. Moreover, the hellish pain felt by the lover in his heart is expressed with two harsh melodic passages, G # C of the fourth upwards and C D # of the diminished seventh downwards.

[Riccati, The laws of counterpoint, 880,2; text: Son lontano da chi adoro e un inferno io provo in sen, et cetera, 9. 8. 7. 6 #. 4 # 2, 6]

[-881-] The inverted seventh placed in the bass at the beginning of the eighth bar contributes to the expression of the aforementioned sentiment, with which the singing part forms a major fourth A D #, aided by the melodic passage of the minor semitone G G # contained in the same bar.

In order to express the damage of the human soul, Signor Benedetto Marcello employs in the Psalm XIII. Several of the devices that I suggested rather abundantly, since he is preoccupied with expressing the feeling of utter rejection. We are presented with two resentful melodic passages, one of the major fourth upwards, F B, and the other one of the minor fifth downwards, E b A. As to the harmony, apart from the first bar, all of the other ones contain at least one, but mostly two or up to three dissonances. If I limit myself to making some observation on the most prominent ones, in the first bar we meet firstly a major ninth added to a consonant accompaniment with the minor third, while the two singing parts form a minor second G A b formed by the ninth G and by the minor tenth A b. This is followed by a minor ninth added to a consonant accompaniment with the major third, where the two singing basses makes us hear the minor ninth A b and the major tenth B, which form the augmented second A b B between them. In the fifth bar, the dissonant sound E b is introduced deliberately without preparation, which forms a minor seventh with the fundamental bass F, which in turn prepares the eleventh that follows immediately. All the dissonances contained in the example become more audible to the ear because they are made to last one fourth of the bar and the composition has to be performed at a slow tempo. Finally, I note [-882-] that the semi-diminished note A b corresponds to the word *guasto*. As that note lasts for three fourths of a bar, it allows the ear to understand clearly that said tuning is faulty in relation to said sound.

[Riccati, The laws of counterpoint, 882 text: Largo, è guasto il cord'ognuno e d'abbominio oggetti, et cetera]

Should one be curious to see the third or major tenth and the dissonance of the minor thirteenth, they should consider the example placed herewith borrowed from the end of a duet by Monsignore Steffani beginning with the words Vorrei dire un non so che. The aforementioned dissonance is rendered more unpleasant by the addition of the major tenth with which it forms the diminished fourth. It is employed by the excellent composer to express the ardour felt by a lover who is apart from the object of his love. I also invite the Reader to observe that the painful separation is symbolised by the harsh melodic passage C D # of the diminished seventh downwards.

[-883-] [Riccati, The laws of counterpoint, 883; text: perchè gelo vicino ardo lontano, et cetera, 4, 3 #, 6 3 #, 5 #, 6, 5, 7, 5 # 4, 3]

I conclude this long paragraph with a fifth example taken from the beginning of a duet of the many times lauded Monsignore Steffani, which expresses the words Occhi perchè piangete? to perfection.

As to the melody, he employs many minor semitones all descending for reasons that I shall explain in the following sixth chapter. Moving on to the harmony and overlooking the dissonances that are most familiar, note that the eleventh or major fourth is employed three times in the fourth, eighth and twelfth bar. At bar eight, the two singing arts form a major third and the augmented fourth with the bass and the major second with each other. [-884-] At bar twelve, said parts sing the major fourth and the fifth against the bass and the minor second against each other. The passage B 5 # 3 # D 5 3 [sqb] of the major third contained in the eleventh bar deserve particular attention, as it considered to be one of those that require to be employed with great care. It leads from the tone E to the tone A, both of them with the minor third, and, since the accompaniment D 5 3 [sqb] belongs to the tone D with the minor third, the passage then leads to the tone A 5 3 # of said tone. The modulation described above from the tone E to the tone A does not deserve a place among the most elegant ones and our expert composer employs it only to express the tears. See what I wrote earlier of the passage B 5 # 3 # F [sqb] 5 3 [sqb] (book 2., chapter 7., [signum] 10.)

[Riccati, The laws of counterpoint, 884; text: Occhi perchè piangete? et cetera, 4, 3 #, 5 b, 3 b, 3, 6, 5 # 4, 4 3, 5 3 b, 8 3 #, 7, 5 4]

[signum] 19. If we follow a middle path between the two already described that inspire in the listener pleasant and unpleasant emotions, we shall achieve the aim to evoke mixed emotions. All we have to do is to mix in our composition, both in relation to the melody and to the harmony more bitterness than pleasant feelings require and more sweetness that are employed in expressing unpleasant emotions. I shall deem it sufficient to provide a single example taken from the sixth Psalm by Signor Benedetto Marcello, in which the mixed feeling of hope and fear felt by the psalmist who prays to be helped by God. I find only a harsh melodic passage employed twice, namely D A b of the minor fifth upwards, which moves within the accompaniment B b D F A b of the major third, fifth and minor seventh from the major third D to the minor seventh A b. Such consists of the notes of the harmony employed as a melody and it is easy to tune because of the mild nature of the minor seventh. One can see that many dissonances are put into practice so that hardly a bar is free of them, while the third and the fourth bar contain up to four each. At the same time this most noble author adopted every device to make the composition emotionally charged, but not bitter. First of all, apart from the minor seventh, which is the most perfect dissonance and it is heard twice lasting for a quarter of the slow bar, and apart from the fourth that in a single case is held for a similar time, all the other dissonances only last for one eighth of a bar and they not let the ear recognise them calmly. Moreover, the dissonances adopted are the easiest to handle of their kind. [-886-] The minor seventh is employed twice, the major ninth once, the fourth eight times, twice the major sixth and the minor sixth only once, although it is added to a perfect consonant accompaniment with the minor third, with which it forms a fourth. The *basso continuo* and the two singing parts touch on the accompaniment C E b A b of the minor third and minor sixth. The minor sixth in this case must be regarded as a dissonance. This occurs in the middle of the fourth bar. At the beginning of the third and four bar Signor Marcello adds the two consonances of the fourth and major sixth E b and G to the consonant accompaniment with the major third B b D F. These consonances have the attractive property of forming a major third between them. Moreover, so that they may prove more pleasant, he omits the sounds of the consonant accompaniment D and F, allowing them to be heard in the resolution. That minor seventh B b A b, which is employed twice unprepared and serves as a preparation to the fourth in the third bar and to the minor sixth in the fourth bar, is very apt to evoke compassion. Similarly, the modulation from the principal tone E b with the major third to the subordinate tone F with the minor third, which is employed with great sensitivity at the end of the composition, contributes to the aforementioned aim. The versicle concludes with the cadence D b 6 3 C 5 3 [sqb] belonging to the tone aforesaid tone of F, which moves from the fourth note B b to the fifth note C in relation to the fundamental bass. Now, the flat of the note D b and the [sqb] of the note E, which sound new and unexpected to the ear used to listening to the tone E with the major third, express the pitiful question quando aspetti mio Dio di

darmi aita? to perfection. I pledge to explain the reason of such an effect in the following chapters in which I shall attempt to clarify why the tone E b with the major third sounds emotional and the tone F with the minor third inspires compassion, a quality because of which they have been chosen in the present instance by our most excellent composer.

[-887-] [Riccati, The laws of counterpoint, 887; text: quando aspetti mio Dio di darmi aita? 5 4, 3, 5 9, 6 b 8, 4 b, 6 4, 5 3, 5 b, 7 b, 6 b, 5, 4, 3 b, 5 b, 3 [sqb]]

[signum] 20. I return to consider the classification of the human passions that I discussed at the very beginning of the present chapter ([signum] 2.), where I divided them into strong emotions, which need to be accompanied by an extraordinary agitation of the spirits of the soul, which goes beyond its normal state, and into weak emotions, which render the movement of the aforementioned spirits more languid and less active than usual. The first kind of emotions make it such that our voice rises from the low to the high pitch and sometimes descends from the high to the low register while we speak employing larger and more frequent leaps than the ones employed occasionally when we are calm. [-888-] The second kind of emotions produce the opposite effect, since they do not allow there to be very considerable alterations in the voice as to its high or low pitch. Therefore, in order to represent strong passions, one shall introduce long sequences of movements by step and by leap both upwards and downwards, which have to be all the more numerous and wide, the greater the emotion that is portrayed. We shall be able to symbolise weak emotions with through the opposite device, namely, avoiding at least frequent leaps and particularly the wide ones, as well as those that move from a weak beat of the bar or of a subdivision of the beat to a strong beat or to a strong subdivision of it, since they are the most noticeable. One shall have to avoid long sequences of stepwise motions that lead the voice too far towards the low or towards the high register.

[signum] 21. Moving on to the examples, the Psalms by Signor Benedetto Marcello are endowed with a richness of extremely lively representations of both categories of passions considered just now. I chose one from the Psalm XIII. because it contains the representation of two emotions, one strong and one weak. First of all the zeal of the saintly prophet against the wicked, who laugh at the just because they placed their hope in God, is expressed by employing large and repeated leaps. Then, as the composer wants to represent God's compassion, which is a soft passion of the human soul ascribed by our imagination to that immutable Being who is incapable of emotions, the style changes and the leaps in the sung parts are abandoned except for the small one of the minor third upwards. Long sequences of notes that produce an excessive change of tone in the voice by moving by step are also not allowed.

[Riccati, The laws of counterpoint, 888; text: O scellerata perfida gente voi vi ridete di noi che posta abbiamo in Dio la nostra speme per questo appunto ch'egli pietoso si sta col giusto, et cetera, 7]

[-889-] God's compassion is represented in music also in the second example transcribed from the fourth Psalm by that many times commended knight. The singing parts move by small motions, the largest of whom is the leap of a fourth downwards that is barely heard because moves from a strong beat to a weak one. As I am presented with this opportunity, I note that the words Oh così sempre abbi di me pietade express David's desire accompanied by strong trust and a little worry. This desire belongs to those emotions that partake much more of what is pleasant than of what is unpleasant. Therefore, according to what I explained in the previous paragraphs, Signor Marcello introduces several dissonances ordered in such a way that their sole function is to garnish the sweetness of the consonances with a little bitterness.

[-890-] [Riccati, The laws of counterpoint, 890; text: Oh così sempre abbi di me pietade, et cetera, 9 7, 8 6, 6 4, 5 3, 7 5]

I am provided with the third example by an most beautiful aria by Signora Contessa Donna

Teresa Agnesi, which evokes in the imagination the lively picture of the suffering of a desperate lover. Observe the large number of leaps introduced in the singing part and in the violins to express the fury of the feeling of the words Ah non son io che parlo. These are followed then by the idea portrayed in the words è il barbaro dolore, che mi divide il core, which contains a mixture of emotional drive and tearfulness. Compassion is inspired in the listener by employing the notes B b, E b reduced by a minor semitone, while the strong expression of the barbarity of pain and of a broken heart is depicted by employing the note C # raised by a minor semitone. The minor seventh A G, which occupies the whole of bar sixteen, proves also very emotional. I shall provide a reasoned explanation of such effects very soon. Meanwhile, I move on to the circumstances that concern our topic and I say that the aforementioned mixture of emotions requires that one should employ some leaps, but not too frequently. While the soprano sings the words è il barbaro dolore, it does not leap at all. Conversely, when it expresses the feeling of the words che mi divide il core, [-891-] it produces four leaps that correspond to the [[word divide]] word divide and express the violence of the breaking of the heart. Other leaps, that I shall consider presently, can be seen in the instrumental parts.

I also invite the Reader to reflect that, since pain is an unpleasant emotion, Signora Contessa employs several resentful melodies and harmonies in conformity to what I taught in the seventeenth paragraph. I invite the Reader to observe the passage BB b C # of the augmented second upwards in both violins. All the instruments, which play at the unison in the fifth and ninth bar, employ the melodic movement G C # of the minor fifth downwards. Moreover, the singing part at bar eleven and twelve lets us hear the melodies G C # and C # G of the major fourth upwards and downwards.

As far as the harmonies are concerned, I invite the Reader to note that the consonant accompaniment by representation E b G C # of the major third and augmented sixth, derived from the fundamental one C # E b G of the diminished third and minor fifth to which the seventh B b is added, is employed twice. In order to render more audible the bitterness of said accompaniment, it is made to last one bar and a half and the listener is made to hear all the harsh harmonies that it contains. The bass and the second violin form the augmented sixth B b C #, while the soprano and the viola form the major fourth G C #. The minor seventh is introduced the minor seventh unprepared and it is held for the entire aforementioned bar.

The example ends with the words che delirar mi fa, whose sentiment is represented by Signora Contessa (in conformity with the rules that I provided in the second paragraph and in the twentieth) with fast notes and with four consecutive leaps. I invite the Reader to be happy with this extract, although the aria is so excellent that it would deserve to be transcribed in full.

[-892-] [Riccati, The laws of counterpoint, 892; text: Allegro. Ah non son io che parlo è il barbaro dolore che mi divide il core, 6, 6 5, 3 b, 6 # 5 b 3, 7 3 #, et cetera.]

[-893-] [signum] 22. Sleep must also be considered among the weak feelings that calm the motion of the spirits of the soul. I said in the previous paragraph that sleep is represented by lingering at length on the same note. I add now that the same effect shall be achieved then the voice produces small movements. I take as an example the following beginning of a duet by Monsignore Steffani, where the melodic passages of the singing parts are almost always of the unison or of the second; only in three cases they include a minor third. The bass itself, which by its own nature is fond of leaps, proceeds mostly by step. It is worth observing that the excellent composer allows the parts to employ only very few notes. The first soprano begins and ends on A, while its lowest note is G, which lays at the distance of a second from A and its highest note is D, which lays at the distance of a fourth from A. The second soprano starts on G and descends to E at most, which is at the of a minor third from G, while it rises at most to C, which is a fourth removed from G. However, since he is portraying a calm sleep, Monsignor Steffani has set out to allow only eminently pleasant melodies, as we can find only a single harsh movement consisting of a minor semitone in the bass. The introduction of some consonant accompaniment by representation or dissonant contributes to make the harmony more tasty and full of flavour.

[Riccati, The laws of counterpoint, 894; text: Sonno placido e pietoso sovra me dispiega l'ali, 4 3, 5 3 #, 6 4 # 2, 3, 4 2, 6 5, 7, 6, 3 #, 5 3, 6 #, 9, 8, 6 5, et cetera]

Sixth Chapter

Continuation of the discussion on expressing the sentiment of the words by means of music, with particular reference to strong and weak feelings.

[signum] 1. The subject matter of the present chapter is provided by another very important circumstance that must be noted in strong and weak feelings, which, when it is imitated in music, produces marvellous effects.

Apart from the fact that strong feelings make us speak louder and faster than usual and weak feelings softer and more slowly, as I noted earlier (chapter 5. [signum] 2.), it also occurs that the former elevate the ordinary tone of our voice and the latter lower it. If at first I inject a little air into an organ pipe, if I increase its speed and power, I obtain not only a louder sound, but one that is also considerably higher in pitch. The entire opposite occurs when I reduce the speed and the strength of the air. The mentioned experiment illustrates the link between higher power, faster speed and higher pitch, which are all effects caused concomitantly by the violent agitation of the spirits of the soul. It also shows the links between reduced power, reduced speed and lower sound, which are all effects [-895-] jointly produced by the slow motion of the same spirits which accompany intrinsically the weaker emotions.

However, the aforementioned doctrine teaches me that the mere passage from low pitch to high pitch or vice versa, without adopting any altered notes (I shall explain later that these altered notes are) contributes to express a degree of strength or weakness of emotion. The question, which is placed among the rhetoric devices, contains in itself a kind of energy by virtue of which it is expressed by a melodic passage that moves from the low to the high register. I shall present two examples taken from two cantatas for solo voice and instruments, one by Signora Contessa Donna Tereza Agnesi and the other one by Padre Maestro Vallotti. The first example lets us hear a question excellently expressed in a tone with the major third, while the fundamental bass executes the perfect cadence G 5 3 C 5 3. all the emphasis of the question depends from the movement D E of the major third upwards, so that, if one substituted instead of it the passage F E of the minor second downwards, one would not find any trace of a question any longer.

[Riccati, The laws of counterpoint, 895; text: Chi v' insegnò quest'arte d'infierire così? et cetera]

The question contained in the second example is more common in music, as it originates as to its melody and fundamental harmony from the cadence that moves from the fourth to the fifth note in the mode with the minor third. Even [-896-] in our case The substitution of the passage B C # of the major second upwards with the other one B A # of the minor third downwards would cause great detriment to the composition. However, such detriment would be not be total, since the artificial sound A # contains some strength, as I shall strive to clarify in the appropriate context.

[Riccati, The laws of counterpoint, 896; text: Ma ancor seguite o figli nelle angoscie primiere? et cetera, 6, 3 #]

Apart from the character of energy, a question implies the element of suspension, because something else is required to occur after the question. Hence, the cadence from the fourth to the fifth note is employed often in order to illustrate a question in music, because it is less fulfilling than the other ones. The feeling of suspension is increased if the composer ensures that the bass touches the fundamental note of the accompaniment on which the question ends. The singing part must not form a unison or an octave with it, but it must sing the third or the fifth. The fifth is sung in the second example, which produces a high degree of suspension, as the chord of the fourth and sixth corresponds to that sound, which chord the hear understands clearly as a derivative of the

fundamental chord of the third and fifth. In the first example the third of the last accompaniment, which contains the chord of the third and sixth, which is a derivative chord and one recognised as such, but not as clearly as the one of the fourth and sixth. Since a perfect cadence is employed in that example, it would have been necessary to allow some element of suspension in order to imitate the character of the question that must not leave the ear entirely satisfied. [-897-] Although the considerations that I have just expressed may appear alien to the topic discussed at present, I deemed it appropriate not to overlook them, since they came into the discussion.

[signum] 2. However, just as strong and weak feelings alter the ordinary tone of our voice, thus music takes greater pleasure in employing altered notes in order to imitate them vividly. If the alteration consists in raising the sound, it contributes to the expression of stronger emotions, while, if it consists in lowering it, it helps to express weaker ones. Such alterations are divided into two categories. The first one contains the notes altered by a minor semitone, which are involved in counterpoint and obey its laws. The second categories embraces the alterations that depend on the unequal tuning of the keyboard instruments commonly used, which I discussed in detail in the fourth chapter.

Starting from the first category, it is divided into various species that all need separate consideration. First of all, certain notes deserve to be called altered, although they belong naturally to the tone on which the composition is based. In order to uncover this apparent secret, I observe that the best state of man consists in tranquillity not dominated by weak or by strong passions. Now, such a perfect state of music is represented by means of the melodico-harmonic system that is the best of all, which is a property that competes to the mode with the major third as it originates from the combination of two systems, the melodic one and the harmonic one, that are endowed with the highest degree of perfection. The mode of the major third is based on the major second, the major third, the fourth, the fifth, the major sixth, the major seventh, the octave, the major ninth, the major tenth etcetera, which intervals, since they are determined by the most perfect origin of our mode, appear as perfect. There are another six modes apart from the mode with the major third, namely, the mode with the minor third and five secondary and derivative modes that I discussed at length in the fourth chapter of the first book. When the scales of these six modes are compared with the scales [-898-] of the mode with the major third, if a note is lower in pitch in comparison with the corresponding note of the tone with the major third, it evokes weak emotions, while, conversely, if it is higher in pitch, it awakens strong emotions. I believe it appropriate to present to the Readers the scales of our seven modes, employing the diatonic ones as a model for all the others that are similar to them, and noting as lower or higher in pitch those sounds that can be described as altered in the way described just now.

[Riccati, The laws of counterpoint, 898; text: Modo per Terza maggiore che rappresenta lo stato tranquillo dell'Uomo. 5 3, 6 4, 6 3, derivato fondato sulla quinta corda del, quarta, minore, crescente, calante, settima, seconda]

[-899-] Since the seven modes listed above share the same scale, the same sounds are involved in all of them, while a particular note is considered as belonging to a given mode as it is associated to an accompaniment determined by the origin of the mode itself.

I took care to note these accompaniments with the usual numbers. The note E, for instance, when it belongs to the tone C with the major third needs the accompaniment of the minor third and minor sixth, while, as belonging to the tone A with the minor third, [-900-] can accept one or the other of the two chords of the minor third and fifth or of the minor fourth and sixth. In the scale of the derivative tone B I have marked five sounds as lower in pitch. They accept this property as long as the base B is not altered. The first, fourth and fifth note occupy the first place in a tone not only in respect of the melody whose system they constitute, but also with regard to the harmony, since they form the base of the three accompaniments of the third and fifth that constitute the tone. If I consider the three aforementioned notes in the derivative tone B and if I presume perfect the first note B, I find also that the note E is equally perfect, while the fifth note F is lower with regard to the

latter one. In fact, if one wanted the note F to be perfect, the two notes A and E would result higher. Now, the union of the two notes B and E makes a stronger impression on the ear than the sole note F, which, consequently, is considered to be lower. When I said (book 1., chapter 4., [signum] 16.) that the diatonic tone B rejects equally to be turned into the tone B with the minor third or into the tone B with the major third, I meant to say simply that both these changes require the alteration of the notes F # and C # in the first case, and B b E b in the second. However, if one considers the importance of the notes that are altered, the raising to F # and C # of the fifth and second note is less considerable than the lowering to B b and E b of the first and fourth note. Therefore the transformation into the tone B with the minor third is more natural than the one into the tone B b with the major third. The five sounds indicated as lower deserve such name in relation to the simplest transformation, because the three notes D, G and A are lower in the tone B b as well, while the two notes E and R, since they are altered by the sharp in the aforementioned tone, are recognised as lower in our own.

[signum] 4. The notes that are lower, perfect or higher determine the character of the different melodies and harmonies, whether strong, calm or emotionally fraught. For instance the same passage B C of the minor third can appear as cheerful, emotional or resentful. It will prove cheerful when it moves from [-901-] the seventh to the octave of the tone C with the major third, emotional when it moves from the second to the third note of the tone A with the minor third, which is lowered in said tone. Finally, it will prove angry or resentful when it moves from the fourth note, which is raised to the fifth note of the derivative tone based on the fourth tone F of the tone C with the major third.

If I now turn to the harmony, I observe that a tender and emotional quality is usually attributed to the minor third. This is in agreement with the factual truth, as long as it is considered in its principal role, namely, as occurring in the perfect consonant accompaniment with the minor third and in its derivative accompaniments. In fact, in such instance the higher note of the two that constitute the minor third is lowered. In different circumstances, when the minor thirds are placed above the major third in the perfect consonant accompaniments that generate the mode with the major third and also take place in their derivative modes, they shall be heard as consisting of notes that do not suffer any alteration. Such is the minor third E G, which forms with the bass the accompaniment E G C of the minor third and minor sixth derived from the perfect accompaniment C E G based on the first note of the tone C with the major third. However, do we want a minor third that expresses resentment and, I am about to say, anger? It is sufficient that we think of the derivative mode based on the fifth note of the mode with the major third, as, for instance, the adjacent tone F. The fourth note B of said tone is raised, therefore, because of this, the minor third B D that is formed with the base of the accompaniment B D F belonging to that note sounds resentful. Similar observations can be made with regard to the major third. Nevertheless, one must observe that the major third G B itself proves tranquil in the accompaniment C B G belonging to the fifth note of the tone C with the major third. It proves emotional in the accompaniment G B belonging to the [-902-] fifth note of the tone A with the minor third. In the first case no alteration of the notes G and B intervenes, while in the second case one can see that the note G is lowered.

[signum] 5. In our discussion on the dissonances we have seen (book 3., chapter 1., [signum] 1.) that they must be at the services of the consonances, so that, once the scale of the mode has been established by uniting the two systems, the melodic and the harmonic one, the dissonances that must be taken from the aforementioned scale are also determined. Now, if the melodies that move from one consonance to another one and the consonant harmonies that correspond to the base of the mode with the major third appear as perfect and represent the tranquil state of man, this will apply all the more to the dissonances that are sounded against the same base, as they sound mild to the ear because they depend on the scale of the best mode.

The major seventh, the major ninth, the eleventh or fourth, the thirteenth or major sixth correspond all to the first note of the mode with the major third and all of them sound perfect and, when they are employed as a garnish or in passing, they symbolise the calm of the human soul. The major ninth deserves the aforementioned property for two reasons, because the best scale establishes it as

such and because it is simpler than the minor ninth. As to the sevenths, it is clear that the minor one, which is employed as representing the ratio $7/4$, is more perfect than the major one that embraces the ratio $15/8$. Nevertheless, since the dissonances must be considered principally as at the service of the consonances, the minor seventh sounds altered and proves emotionally fraught or resentful according to whether the higher note is lowered or the lower one is raised. The minor seventh always proves sentimental when it is added to a perfect consonant accompaniment with the major third or with the minor third, because the base of said chord, which is not altered, makes the dissonant added sound sound diminished. However, the accompaniment with the major third is more apt than the one with the minor third to [-903-] express the tender character of the minor third. This emotion is produced by the comparison of the major third with the aforesaid seventh, which form a fifth with each other. In order to stay with the diatonic tones, which are the model of all the other ones that are similar to them, we find in them four major sevenths added to four truly consonant accompaniments, one with the major third G 5 3 and three with the minor third, A 7 5 3, E 7 5 3 and D 7 5 3. The masters of counterpoint employ them very widely, especially the first one, which corresponds to the note G of the tone C with the major third. Similarly, the accompaniment E 5 3 # through the seventh artificial note G # is applied very frequently to the fifth note E of the tone of A with the minor third. However, I shall discuss this in greater detail when I shall discuss the artificial raising and lowering of notes. Our diatonic tones contain another minor seventh which is coupled with the consonant accompaniment by representation B D F of the minor third and diminished fifth. If this accompaniment is considered as based on the fourth note of the derivative tone F, which note is raised, the minor seventh B A, or the complex of sounds that constitute it, proves resentful because of the aforementioned raising of the note B. Conversely, if the aforementioned accompaniment is based on the fifth perfect note B of the derivative tone E, the mentioned seventh sounds tender to the ear.

The sevenths discussed so-far are fundamental dissonances, namely, they are dissonances added to fundamental tones of the third and fifth. However, one often encounters sevenths that are formed with the bass of some derivative accompaniment. In order to judge the character of these sevenths, it is necessary to observe the fundamental chord on which they are base and take note diligently of the perfect, lowered and raised sounds that occur in it. If we are presented with the chord E G C D of the third, sixth and seventh, all of them minor, and if we consider that it originates from the fundamental chord [-904-] C E G D of the major third, fifth and major ninth containing no altered sound, we shall draw easily the conclusion that in our case the minor seventh E D is mild. We shall not have to judge the major seventh F E, which is based on the accompaniment F G B E of the major second, major third and major seventh, in the same way, since said accompaniment depends on the fundamental one E G B F of the minor third, fifth and minor ninth containing the two lowered sounds G and F. On such an occasion, the major seventh shall sound somewhat emotional because of the lowered sound F.

[signum] 6. Something remains to be said on the fundamental dissonances of the eleventh and thirteenth, or of the fourth and of the sixth. The dissonances of the fourth and of the sixth are employed as representing the ratios $11/8$ and $13/8$. The ratio $11/8$ is half way between two ratios, namely, $4/3$, belonging to the fourth and exceeded by it according to the ratio $33/32$, and $7/5$, which represents that major fourth, which ratio exceeds $11/8$ by $56/55$. In a similar way, the ratio $13/8$ is contained between the two consonances $5/3$ of the major sixth and $8/5$ of the minor third, as it is smaller than the first one by $40/39$ and it exceeds the second one by $65/64$. Now, albeit the proportions $11/8$ and $13/8$ of the dissonances of the fourth and of the sixth are further removed, respectively, the former from the fourth $4/3$ than the major fourth $7/5$, and the latter more from the major sixth $5/3$ than from the minor sixth $8/5$, nevertheless, since the fourth and the major sixth correspond to the base of the most perfect move with the major third and since the dissonance depend on the mode in which they are contained, the ear listens to them as if they were perfect, while it judges the major fourth and the minor sixth as altered.

In the diatonic tones one finds only one major fourth, F B, which shall contribute to express weak or strong feelings according to whether the lower sound F is lowered or the higher sound B is

raised. For instance, if one prepares the aforementioned fourth with the passage E 5 3 F 4 5 3, which moves from the fourth to the fifth note in the derivative tone B, the excellence [-905-] of the previous accompaniment E 5 3, which contains the two perfect sounds E and B of the aforementioned melodic system, makes us believe that the third sound of the melody, F, is lowered, and for this reason it renders emotionally fraught the fourth B F.

The diatonic tones themselves contain four major sixths and three minor sixths. The major sixth as a dissonance shall prove tranquil, as long as the ear does not believe that the base of the perfect accompaniment to which said dissonance has been added has been lowered. If one adds the major sixth D to the perfect accompaniment F 5 3, the harmony F D shall prove tranquil when said chord belongs to the diatonic tones C and F, while it shall prove emotionally charged then F 5 3 belongs to the diatonic tone B, in which the fifth note F is lowered. If the major sixth B is added to the consonant accompaniment with the minor third D 5 3, the minor third D and the major sixth B shall form a major fourth that shall highlight the sad character of the sound F.

The minor sixth as a dissonance shall be heard as sentimental or resentful, according to whether the ear deems the higher sound to be reduced or the lower one to be raised. I take as an example the sixth B G: it shall produce a tender impression if the composition is based on the diatonic tones E and B in which the sound B is perfect, but it shall sound resentful if the composition is based on the diatonic tone F, in which the hearer knows that the note B is raised.

[signum] 7. From what has been said so far about the natural notes of the tone on which the composition is based and that deserve to be called altered we can derive several important consequences. Firstly, even if the tone is with the major third, it will be able to express tender feelings, if the minor seventh based on the fifth note of the tone is employed appropriately. Moreover, it is necessary to rely on the emotional consonant and dissonant notes of the derivative tones, which depend in such a way on the two perfect ones with the major third and with the minor third that, as I explained earlier (book 1., chapter 4., [signum] 20.) the transfer [-906-] from the [[former]] latter to the former is not considered as a modulation. I discussed the passages that are specific of the derivative modes and those that move from one mode to another one in the second chapter of the second book. I refer the Reader to them. The examples shall clarify how the mode with the major third can be associated with tender feelings by employing the devices described earlier. The first example is suggested to be by Monsignore Steffani's duet entitled *Saldi marmi*. I place the fundamental bass under the *basso continuo*, so that one may be distinguished more easily the minor seventh added to the accompaniment of the fifth note of the mode and the emotional consonant and dissonant harmonies borrowed from the derivative modes.

[Riccati, The laws of counterpoint, 906; text: Deggio al nuovo desire opporre il vostro gelo o pur morire, et cetera, Basso fondamentale, 7 b, 4, 3, 7, 3 b, 9 8, 10 9, 7 5, 5 4, 4 3]

[-910-] Two tones with the major third are employed, namely, first of all B b and then F. The minor seventh added to the perfect accompaniment of the fifth note of the mode with the major third is heard seven times, while fifteen chords of the minor seventh and fifths are borrowed from the derivative modes, to one of which, G 5 3 b at the beginning of the third bar, the minor seventh is added. The accompaniment E 5 b 3 of the minor third and minor fifth, based on the fifth note E of the tone A derived from the tone F with the major third, is employed at bar ten. In said derivative tone, which is similar to the diatonic tone E, the second chord B b is lowered. Therefore, the minor fifth in the chord E 5 b 3 sounds emotional. In order that weak emotions may be expressed fully, it is necessary that the part that sings about some tender feeling touches the lowered notes on which the [-908-] emotional character of the harmonies and melodies depends. Note that at bar sixth the sad sounds F, first minor third of the accompaniment G 5 3 b and then minor seventh added to the accompaniment C 5 3 based on the fifth note of the tone F with the major third on which the composition is based, are attached to the word morire. The same sounds correspond to the same bar at bar ten, with this sole difference that the sound B b is employed first as the minor fifth of E and then as the minor seventh added to the chord C 5 3.

I borrow the second example from a duet by Signor Sassone, which begins with the words Non temer non son più amante. In the excerpt that I present, which belongs to the tone F with the major third, the sound B b is employed as minor third and then as minor seventh of the fundamental accompaniments G 5 3 b and C 7 b 5 3 to express the word sospiri.

[Riccati, The laws of counterpoint, 908; text: dunque addio ma tu sospiri, Basso fondamentale. 6 4, 5 3, 7 5 3 [sqb], et cetera]

I transcribed the third example from the Psalm L. by Signor Benedetto Marcello. The tones B b, E b and A b employed by this eminent composer are all with the major third, since even the last bar can be ascribed to the tone A b. Fourteen of the twenty fundamental chords that constitute our example deserve to be considered as emotionally charged, either because they are minor chords or because they contain the addition of the minor seventh or of any other tearful dissonance, such as the minor sixth or minor thirteen that one sees added to the accompaniment F 5 3 b [-909-] in the last bar. Observe in the singing bass, at the end of the fifth and at the beginning of the sixth bar, the passage of the fifth D A b of the minor fifth upwards that moves from D, major third of B b, to A b, minor third of F, whose sad character is rendered more prominent by the aforesaid piquant melodic passage.

[Riccati, The laws of counterpoint, 909; text: Adagio, Violetta, Basso fondamentale, il sacrificio sono sol di Dio degno ma dolente uno spirito contrito e umiliato, et cetera, 7, 3 b, 7 b, 7 3 b, 7 b 3 [sqb], 4 b 3, 7 3 [sqb], 11 5 3 b, 10 b, 4, 6 b 4, 5 3 b]

[-910-] [signum] 8. The observation with which I concluded the seventh paragraph provides me with the opportunity to another more important consideration that I should not have overlooked. If we consider the passage of the minor fifth upwards or of the major fourth downwards from the seventh, on which the accompaniment of the minor third and of the minor sixth is based, to the fourth, which requires the accompaniment of the major third and fifth, in the mode with the major third, albeit said passage moves from one sound to another one, none of which accepts any sort of alteration, nevertheless, the altered nature of the passage itself shall make the ear believe at first that the fourth has been lowered, so that it shall sound emotional. The contrary motion from the fourth to the seventh note of the aforementioned mode shall persuade the ear that the seventh note is raised, and for this reason it shall sound resentful and will be involved in the representation of strong emotions. Since my current aim is to illustrate how tender passions may be expressed in the mode with the major third, I present an example of the passage A 6 3 E b 5 3, which moves from the seventh note to the fourth one of the tone B b with the major third, through which one makes the note E b sound emotionally charged. It is taken from the fourth versicle of the Stabat Mater by Signor Giambattista Pergolese.

[-911-] [Riccati, The laws of counterpoint, 911; text: et tremebat cum videbat nati poenas incliti, et cetera, 6, 5 3, 6 5, 7 5, 6 4, 6 3, 5 4, 3 [sqb], 3]

When a natural altered note occurs in the minor fifth and in the major fourth, after the first note has been struck and the ear has become used to it a little, if it is raised, the other note shall be heard as a little emotional; if it is lowered, the other note shall be heard as a little resentful. If we apply the passage F 6 3 B 6 5 3 of the minor fifth downwards which belongs to the diatonic sounds G and D derived from F, minor third of the perfect accompaniment D 5 3, and B, major third of the perfect accompaniment G 5 3, in which passage the preceding note F is lowered, we shall find a certain strength in the following note B, which nevertheless is perfect. The same effect shall occur when the minor fifth and the major fourth, which contain an altered note, shall be used harmonically. The passage C 5 3 B 7 5 3 belongs to the diatonic derivative tone F, [-912-] in which the fourth note B is raised and the first note F is perfect. Nevertheless the note F shall assume a rather emotional

character in the accompaniment B 5 3 because of the juxtaposition of the fourth note B, which is raised. Conversely, in the accompaniment G 7 5 3, which belongs to the diatonic tones C, G, and D, the minor seventh F, which is lowered, shall make the major third B appear animated. In order to ascertain fully the truth of this, let us compare the two chords G 7 5 3, F 7 5 3. We shall hear that in the first one the major third is more sparkling than in the second one. This depends solely on the difference between the added sevenths, the first of whom is minor, while the second one is major.

[signum] 9. The mode with the minor third is extremely apt to express weak emotions. If its notes are considered in their main role, namely, as they are determined by the two systems, the harmonic and the melodic one, we find that three of them are lowered and emotionally fraught. They are the third, the sixth and the seventh. If we turn to consider the dissonances that can be added to the three accompaniments that generate the mode, half of them shall have to be classed as tender. Of these, the minor seventh and the minor thirteenth is assigned to the first note, the minor seventh to the fourth and the minor seventh, the minor ninth and the minor thirteenth to the fifth note. Moreover, we can borrow the notes that express tenderness of feeling, both consonant and dissonant, from the derivative modes, as said mode contain them in abundance. The following first versicle from a Miserere for two voices without instruments written by Padre Maestro Vallotti will present us with all those devices applied in practice. I write the fundamental bass under the *basso continuo* for the reasons explained elsewhere.

[Riccati, The laws of counterpoint, 912; text: Miserere mei Deus secundum magnam misericordiam tuam. Basso fondamentale. 11 9, 7 5, 6 4, 5 3 #, 9, 10 8, 9 7, 7]

[-913-] The third and sixth note, both emotionally charged and belonging to the tone D with the minor third, in which the Miserere is written, are particularly prominent in the first bar. That sad movement A B b from the fifth to the sixth note in the higher contralto produces an excellent effect of which the ear is very aware. The fundamental passage D 3 E 5 b 3 from the first to the second note, which is contained in the second bar, belongs, rigorously speaking, to the derivative tone A, similar to the diatonic tone E, of which the second contralto touches the tearful note A, first as minor third of D and then as minor ninth of E. The accompaniment A 7 5 3 # follows, which is made tender by the addition of the minor seventh sung by the higher contralto. The dissonant chords G # 7 5 and A 6 4 are employed in the three bars that follow, apart from the perfect accompaniment with the minor third D 5 3.

[-914-] As to the accompaniment of the minor fifth and diminished seventh G # D F, note that, had Padre Vallotti employed the natural fourth note G instead of the artificial one G #, the sound D would form a perfect fourth with G and would sound calm, while the sound F would form a minor seventh with G and would sound emotionally charged. Now, the raising of the fourth note, which renders it of a resentful character, makes it such (according to what I said in the previous eighth paragraph on the natural raised notes, which I have to complete with what I have left to say on the artificial notes) that the sound D, which forms a minor fifth with G #, sounds rather emotional, while the sound F, which forms a diminished seventh with it, is heard as melancholic and lamenting. The two aforementioned sounds that contribute to express weak emotions are sung by the voices, which, in order to reflect the sentiment of the words have to inspire compassion. As to the dissonant chord A D F of the fourth and minor sixths, I have to point out only that the minor sixth of minor thirteenth F, a sad sound, is sung by the first contralto. Bar six, seven and eight and part of bar nine belong the tone F with the major third, which shall be suit said emotions if those devices that I explained not long ago are applied. As to the rest of the versicle, which belongs to the principal tone of D with the minor third, I invite the Reader to pursue an examination similar to the one of the first five bars. Thus, one shall discover that, apart from the accompaniment A 5 3 # employed twice because of the cadences, all the others are of a tender nature. Two of these, C # 7 5 and G # 7 5, if the minor fifth and diminished seventh, are tearful in nature. Their bases, C # and G #, which express resentment, are placed in the *basso continuo*, while the minor fifth and the diminished seventh, which are emotional notes, are assigned to the singing parts.

I add a second example with which Monsignore Steffani completes an excellent and passionate duet of his. It begins with the words E così mi compatite?

[-915-] [Riccati, The laws of counterpoint, 915; text: E così mi consolate, basso fondamentale 3 #, 9, 8, 9 7 3 #, 10 8 5 #, 7, 7 3, 8 5, 6 4, 5 3 #, 8 3 #]

I leave many consideration to the efforts of the Reader, but I address two main points of greater importance. The passage E F at bar four from the fifth to the sixth note in the soprano part proves very tender. The sixth note is held for three beats as a consonance and it becomes a minor ninth at the beginning of the fifth bar, where it joins the minor seventh on the accompaniment E 5 3 #. In the same fifth bar the two accompaniments C 10 8 5 7 # and F 7 5 3 follow the accompaniment E 9 7 4 #. As to the first one, the artificial sound G #, which is implied, albeit not expressed by the parts, renders emotional the sound C, with which it forms an augmented fifth. After the sound C has been heard as lowered, the sound F, which forms a fourth with it and it is the base of the following accompaniment F 7 5 2, is also considered lowered. Therefore, one feels tenderness in the major sevenths C D and F E because of the lowered notes C and F. It is possible to note that, were the tone of [-916-] the example with the major third, the sounds C and F would be affected by the sharp sign, and, therefore, the sevenths C # B and F # E would be minor. One can realise more clearly that the fundamental sound F of the major seventh F E, employed at the beginning of the seventh bar, is lowered since this is indicated manifestly by the passage E 5 3 # F 7 5 3 which belongs to the diatonic tone B, in which the note F is lowered. In the same bar the contralto performs the movement G # C, from the seventh artificial note to the tenth of the tone A with the minor third. The seventh artificial note G # highlights further the tenderness of the tenth note C.

[signum] 10. After indicating the way to imitate weak emotions through the natural notes of the tones, I shall say something on the expression of strong and vehement emotions. The only rising natural note that sounds resentful for that reason, is the fourth of the derivative mode that is based on the fourth note of the mode with the major third or on the sixth note of the mode with the minor third. The mode of F, among the diatonic ones, is the one to which we are referring, and its fourth rising note B accepts the accompaniment of the minor third and minor fifth. Since the chord B 5 3 is shared among the three diatonic tones B, E and F, it shall be recognised as belonging to this last tone of F thanks to the application of the passages that belong to it. The two fundamental passages B 5 3 C 5 3, C 5 3 B 5 3 belong solely to the tone of F. In relation to the tone C with the major third, which employs them frequently, they move from the seventh note to the eighth and from the eighth to the seventh. The equally fundamental passage F 5 3 B 5 3, which moves from the fourth note to the seventh in the tone of C, shall be ascribed to the tone F, rather than to the tone B, which share it, when B 5 3 is followed by C 5 3 or by some other accompaniment that leads to C 5 3. Therefore, when the passages from the seventh to the eighth note, from the eighth to the seventh, and when the passage from the fourth note to the seventh is employed in the manner described, this note shall sound resentful, and, placed on the lips of the singer who has to awaken some vehement passions, it shall produce the require effect.

As to the tone A with the minor third, that I have always considered as a model for all the others, the aforementioned passages are G # 5 3 A 5 3, A 5 3 G # 5 3 and D 5 3 G # 5 3. [-917-] However, I shall discuss soon, when I deal with the artificial modes, the energy of the seventh artificial note of the mode with the minor third, especially in this type of context.

Going back to the natural sounds that can be useful to represent strong emotions, I invite the Reader to remember what I said earlier ([signum] 8.) about the lower sound of the minor fifth, and how, when it is employed melodically and harmonically, it is more or less apt to represent strong feelings, although said sound is not raised and although the other sound, which together with the aforementioned one forms the minor fifths and the major fourth is perfect or even lower. I will not repeat what I explained in the quoted passage, but I limit myself to clarify them further with the aid of a few examples. The first example is suggested to me by a duet by Signor Pergolese written in the tone A with the major third, which begins with the words Ne' giorni tuoi felici ricordati di me.

[-918-] [Riccati, The laws of counterpoint, 918; text: anima mia perchè così mi dici o Dio ah che tacendo mi trafiggi il cor, Basso fondamentale, 3 [sqb], 7 3 #, 4, 3 #, 7 3 #, 7 3 [sqb], 3, et cetera]

The energy of the question expressed by the word *perchè* is explained in the third and fourth bar through two fundamental passages D # 5 3 # moving from the seventh to the eighth note of the tones with the major third E and D. Note that the seventh emphatic notes D # and C # are sung by [-919-] the part that utters the word *perchè*. Our good composer represents also very well the harshness of the transfixed heart. First the second soprano sings the sound D, then, at the beginning of the sixth bar, the first soprano enters at the fourth higher G #. Here we find ourselves in the situation described in the eighth paragraph, where, considered the notes D and G # in relation to the base E, the minor seventh D makes the major third G # appear raised. The sound G # is rendered even more vehement by the memory of the sound F touched by the *basso continuo* at the end of the previous fifth bar, with which G # forms the altered interval of the augmented second. The accompaniment F 6 3, derived from the fundamental one D 5 3 is employed through a passing modulation to the tone A with the minor third. The chord E 5 3 # in relation to the preceding chord D 5 3 belongs artificially to the tone A with the minor third, while in relation to the following chord A 5 3 # belongs naturally to the tone A with the major third. However, we shall discuss the artificial accompaniments in detail in the eight paragraphs between the twelfth to the nineteenth included.

Therefore, I move on to the second example taken from a duet from Signor Sassone entitled *Tu vuoi ch'io viva o cara*. In this duet the cruelty of the gods is represented by a leap of the major fourth upwards, whose lower sound is the minor seventh and the higher sound is the major third of the fundamental accompaniment, which is based on the fifth note of the tone with the major third on which the composition is based.

[Riccati, The laws of counterpoint, 919; text: Quando finisce o Dei la vostra crudeltà, et cetera, Basso fondamentale, 7 5, 6 4, 5 3, 5 3 #, 7, 7 [sqb]]

[-920-] I omitted the parts of the violins because they are at the unison with the singing parts.

[signum] 11. I must mention a rich source of imitation of our weak and strong emotions that springs from modulating from one tone to another one which do not share the same scale and from putting immediately into practice the natural notes (I shall discuss the artificial ones later) that the new tone alters with the minor semitone. If the alteration consists in raising the pitch, it shall contribute to representing strong emotions, while, if it consists in lowering the pitch, it shall contribute to representing weak emotions. This aim shall be achieved sufficiently even when the altered sounds in the new tone are perfect, namely, when they are not raised or lowered, and particularly when the comparison with some other sounds makes them look raised or lowered, or when they are truly such. In all of the aforementioned instances the imitation shall be even more lively if one employs one after the other two sounds that differ by a minor semitone and belong to two different tones, so that the tone is changed by ascending or descending by minor semitone. If we resort to the examples without wasting time, the first one is contained in a very beautiful cantata for solo voice and instruments by Signora Contessa Donna Teresa Agnesi. The extract that I present is of an emotional character expressing a mixture of anxiety and preoccupation.

[-921-] [Riccati, The laws of counterpoint, 921; text: deh rispondi per pietà dove sei ascolta o Dio, Forte, Piano, Basso fondamentale, Col, 3 [sqb], 7 b, 5 3 [sqb], 7 b 5 3 b, 7 5 b 3 #, 3 [sqb]]

The first bar belongs to the tone F, while the second one belongs to the tone C, both with the major third. At the beginning of the second bar one hears promptly the sound B, which is the one that had to be raised by a minor semitone in order to move from the tone F to the tone C. [-922-] Although the mentioned sound is perfect in relation to the tone C, in which it is employed as the major third of the fundamental chord G 5 3, nevertheless, since it sounds new to the ear, it expresses very well

the vehemence with which the words per pietà are resumed to be uttered. In the third bar, which returns to the tone F the note B b sounds emotionally charged for two reasons, firstly, because it originates from lowering the note B by a minor semitone in the modulation from C to F, and secondly, because it is lower in this tone as well, since it is employed as a minor seventh added to the fundamental accompaniment C 5 3. The modulation moves again to the tone C with the major third in the fourth bar and then moves to the tone C with the minor third in the two following bars. The third note E b sounded by the violins and by the singing part as well as the sixth note A b played by the *basso continuo* are heard as sadder than they normally would sound because of the memory still fresh of the tone C with the major third in which the notes E and A belonging to it are employed, which highlight more prominently the alterations of the notes E b and A b. The two bars mentioned last set the words dove sei ascolta o Dio, which contain a passionate question. The emotional character, as I said, derives from the sounds E b and A b, while the energy that the question requires comes from the artificial notes, namely, the fourth F # and the seventh B that are raised by a semitone above their corresponding natural notes.

I take the second example from the versicle of the Miserere by Signore Sassone, which sets the words Domine labia mea aperies, et os meum annuntiabit laudem tuam. Although at first sight the sentiment of this versicle appears to be cheerful and happy, one may consider that the joy of praising the Lord for the forgiveness of grave sins does not belong to the present but to the future, and that who feels currently profound pain and contrition hopes to achieve it in the future with a desire mixed with hope and fear. Signor Sassone satisfies artfully all the aforementioned conditions. However, I consider what relates to our discussion and I observe that in the fourth bar the music moves from the tone G with the minor third to the tone E b with the major third, whose natural scales differ because the scale of the first tone contains

[-923-] [Riccati, The laws of counterpoint, 923; text: laudem tuam, Basso fondamentale. 6 4, 5 3, 7 5 3 #, 9 4, 8 3, 6 4, 5 b 3, 7 b, 9 4 b, 7 5 3 [sqb], 7 [sqb]]

[-924-] the note A, while the one of the second one contains the note A b. Now, such note is sung immediately by the first contralto, while it is played by the second violin, and it proves extremely charged with emotion, not only for the memory of the sound A, but also for the comparison with the fundamental sound D, with which it forms a minor fifth. The very first contralto, which lets us hear the note A b in passing still in the fifth bar, then rests on the note A throughout the sixth bar. Therefore, after abandoning the tone E b with the major third, which requires the flat in front of the note A, the modulation moves to the tone of B b, also with the major third. Here we find a new reason why the note A proves energetic and apt to express the present trust and the future happiness of the penitent David. The minor seventh E b played by the first violin and by the second contralto renders more brilliant the sound A, which is the major third of F according to what I said earlier. ([signum] 8.)

The third example shall prove useful to demonstrate the admirable effect produced by modulating by means of the minor semitone. I transcribed it from the lauded cantata entitled Fra voi taciti boschi by Signora Contessa Agnesi. Oh, how sad the passage B b of the minor semitone downwards, by which the music modulates from the tone C to the tone G both with the major third, does sound!

[Riccati, The laws of counterpoint, 924; text: ah si fra tante pene lasciatemi morir, et cetera, Basso fondamentale, 5 4, 3 [sqb], 7 3 b, 7 5 3, 4 9, 3 8, 7]

[-925-] [signum] 12. Now that we have discussed sufficiently the natural notes of the modes as to the expression of weak and strong emotions, I approach the subject of the artificial notes. I have shown elsewhere (book 2., chapter 1. and 2.) the origin, the number and the quality of the artificial notes, which are the sixth and the seventh of the mode with the minor third and the fourth of both modes. All of these notes are raised above the natural ones by the minor semitone and

contribute to the representation of strong emotions. This sequence is concluded by the second artificial note of the mode with the minor third, which is a semitone lower than the corresponding natural note and is eminently capable to express tender feelings.

Harmony and melody are the tools employed by the ear to recognise the artificial notes and their being raised or lowered. In order for a note to manifest itself as artificial it is not sufficient that certain melodic or harmonic comparisons show it as raised or lowered when some totally similar comparisons can be established between purely natural notes. For instance, the fundamental melodic passage C 5 3 F # 5 3 of the major fourth upwards and of the minor fifth downwards that moves from the first note C to the fourth artificial F # of the tone C with the major third, shall show the note F # as raised. Nevertheless, [-926-] since said passage can be ascribed to the tone G with the major third, in which it moves from the fourth note to the seventh, we need something more in order to assign it to the tone C with the major third and to believe that the note F # is artificial. Similarly, the passage A 5 3 D b 3 belongs artificially to the tone A with the minor third, and naturally to the tone D with the minor third, while in both instances the comparison between E, fifth of A, makes us consider as lowered the note B b, minor sixth of D. If we move on to providing some example of the harmony, the accompaniments E 7 5 3 # and G # 5 3 are shared by the two tones A with the major third and A with the minor third. In both cases the sound G # is considered spirited through its comparison with the sound D, which forms a minor fifth with it. Therefore, it is necessary to investigate the foundations of which the ear avails himself in order to discern the artificial notes, which appear more resentful when they are raised and more emotionally fraught if they are lowered more than the natural notes are perceived to be in similar cases.

I shall start from the sixth and seventh of the mode with the minor third and I say that their recognition depends on the three natural lowered notes, namely, the third, sixth and seventh, which lead us to form a clear and special idea of the aforementioned mode. As to the sixth artificial note, I add that it must be preceded or followed by the artificial seventh, thanks to which it is admitted in the mode with the minor third. In short, it is the sum of different preceding and consequent accompaniments that indicate to the ear the tone on which the composition is based and that allow it to distinguish the artificial sounds for what they are. The seventh artificial note forms an augmented fifth, an augmented second and a minor semitone with the three natural lowered third, sixth and seventh respectively. If the natural notes third and sixth are transposed to the higher octave, they form a diminished fourth and a diminished seventh with the seventh artificial note. As to the sixth artificial note, I must only note at present that it lies at the distance of a semitone from the natural one. The aforementioned intervals that are altered in comparison with their nature [-927-] and whose origin depends on the chromatic system, appear more striking to the ear than the major fourth and the minor fifth, ratios that were introduced into music by the diatonic system. Now, if the major fourth and the minor fifth, diatonic intervals, make the seventh natural note of the mode with the major third appear raised, it is certain that the augmented fifth and the diminished fourth, the augmented second, the diminished seventh and the minor semitone, which are chromatic intervals shall render even more resentful and raised the artificial sixth and seventh notes of the mode with the minor third. If the comparison with the three natural lowered notes renders the artificial sixth and seventh note more powerful, it follows that these last notes, heard as powerful, increase the emotional charge of the lowered natural notes. It is sufficient for the natural lowered sounds to be contained in the preceding accompaniments to ensure the artificial sixth and seventh note are heard as resentful. This effect shall be enhanced if one part moves from the lowered sound to one of the aforementioned artificial notes. The energy of our notes shall reach its climax when they are joined by one or more lowered notes in the same accompaniment. Similarly, in order to highlight the emotional character of the three natural notes that have been mentioned so often, it is sufficient that the preceding accompaniments contain the artificial notes that are raised by a minor semitone above the natural ones. The lowered notes shall be heard as more emotionally fraught if one part moves from an artificial raised sound to one of the lowered parts that shall obtain in the end a profoundly tender and passionate character when they are added to the artificial raised notes in the same accompaniment.

It is easy to draw the conclusion that the mode with the minor third, which contains four lowered notes, three natural ones and an artificial one, as well as three artificial raised note, is very apt to [-928-] express a mixture of emotions, some tender and some vehement. When we feel afflicted, we take to certain emotional releases that consist of exaggerations, imprecations, demonstrations of impatience, agitation and desperation. When one is depicting a sad matter, it is occurs commonly to represent the strength of the pain, the cruelty of men and gods and a thousand other connected emotions characterised by vehemence and energy. Similarly, it often happens that, while two parts sing together, one must evoke weak passions and the other one strong feelings. A good composer aims to employ at the right time and place the lowered and raised sounds, by assigning to a particular part the notes of one sort or of the other required by the different situations.

In order to illustrate even further what I explained in the present paragraph, I shall provide several examples in the following one, which shall be aimed mainly at illustrating the imitations of the feelings that we are discussing, which are accomplished by means of the artificial sixth and seventh note of the minor with the minor third.

[signum] 13. The first example borrowed from the sixth Psalm by Signor Benedetto Marcello represents David who offers humble prayers to the Highest so that he may free him from the well deserved punishment that he received for his sins.

[Riccati, The laws of counterpoint, 928; text: nè mova a fulmini aspro gastigo eguale al mio sì grave error, solo, tutti, Basso fondamentale. 4, 3 [sqb], 3 b, 5 b, 5 b 4, 3, 3 #, 5 [sqb], et cetera]

[-929-] The illness that affects the prophet king, the repentance from his sins and the humility of his prayer require a sad and emotional treatment. On the contrary, the circumstance of him being struck down by the lightening of punishment and because of the gravity of his sin require a resolute and vehement treatment. In the first four bars the bass descends with a series of semitones from the octave to the fifth note of the tone F with the minor third. Two of these semitones, E E b, from the seventh artificial note to the seventh natural one, and D D b, from the sixth artificial note to seventh the natural one, are minor semitones. The preceding artificial notes make the following natural notes, which are perceived clearly as lowered, sound eminently sad. A sequence of semitones similar to the one described earlier from the octave to the fifth note of the tone C with the minor third is contained in the fifth, sixth and seventh bar. A similar effect is produced by the movement of the minor semitone B B b in the contralto from the seventh artificial note to the natural one of the tone just mentioned. The contralto itself proceeds from the third to the fourth bar with the passage A b E of the diminished fourth downwards to express the word fulminarmi. The third lowered note A b of the tone F with the minor third, which is compared with the artificial seventh E by means of the aforementioned passages introduced into music only by the chromatic system, makes the note E assume a resentful character. [-930-] Finally, Signor Marcello illustrates the severity of the sin committed by David, by ascending by step in the mode with the minor third from the fourth note to the octave and by employing the artificially raised notes of the sixth and of the seventh. The bass and the contralto adopt the aforementioned movements respectively in the tone G with the minor third and in the similar tone C. I invite the Reader to consider the eighth, ninth, tenth and eleventh bar.

An excellent aria beginning with the words Afflitta e misera langue quest'anima and composed by Signora Contessa Teresa Agnesi provides me with the second example. The word afflitta cannot be set to music in a better way. The emotional charge of the sixth natural note A b of the tone C with the minor third is highlighted in two ways, namely, melodically and harmonically. It is highlighted melodically by the motion of the augmented second downwards B A b from the seventh artificial note to the sixth natural note of the aforementioned tone. It is highlighted harmonically by placing in the bass the sound B, which forms the diminished seventh with A b. If the note A b, minor seventh of B b, appears tender, it is clear that, if one substitutes the natural note B b with the artificial one B, the lowering of the note A b shall appear much more prominent and it shall sound desolate. Moreover, the word misera is also expressed very well.

[Riccati, The laws of counterpoint, 930; text: Basso fondamentale, Col, Afflitta e misera langue quest'anima il cor che palpita non può resistere, Basso fondamentale, 3 [sqb], 7 5, 7 5 3 [sqb], 6 4, 5 3 [sqb], et cetera]

[-931-] The last syllable of the aforementioned word is set on the note F, minor seventh of G, which is rendered more melancholic by comparison with the seventh artificial note B placed in the *basso continuo*, which forms with it a minor fifth. This is followed by the feeling expressed by the words langue quest'anima. I note here, since I have the chance to do so in advance, that Signora Contessa, in order to represent the languishing soul more vividly, places the fourth artificial note F # in the bass, with which note the first and the third notes sung by the soprano form the minor fifth and the diminished seventh. Now, the raising of the artificial note F # softens the note C, albeit it is perfect, and renders the lower note E b very feeble. I do not want to overlook to reflect on the imitation of the words il cor che palpita realised perfectly by the bass and by the first violin, which play alternatively and produce the feeling of a sort of palpitation. The example concludes with the words non può resistere. In order to symbolise the strength of the pain that the heart cannot bear, the first device employed is to allow the listener to hear the sixth lowered note A b, [-932-] and then the raised seventh artificial note B after one rest. The memory of the descending sound A b highlights fully the raising of the artificial sound B, which exceeds A b by an augmented second, an altered interval that has been introduced into counterpoint by the chromatic system. The comparison with the note F sounded by the bass render even more resentful the note B, which forms a major fourth with F.

The third example, transcribed from the Stabat Mater by Signor Pergolese contains passages of the augmented second downwards, from the seventh artificial note to the sixth natural one, and from the augmented second upwards, from the sixth natural note to the seventh artificial note of the mode with the minor third. Two movements of the augmented second downwards A G b, which belongs to the tone B b with the minor third, and B A b, which belongs to the tone C with the minor third pass from the first to the second bar and from the third to the fourth one. In both cases, in order to allow it to inspire a greater degree of compassion, the bass contains the seventh artificial note placed under the sixth note sung by the soprano, which form a diminished seventh. The only passage of the augmented second upwards D b E, which belongs to the tone F with the minor third, can be seen in the sixth bar. The seventh artificial note E sounds resentful to the ear for two reasons: firstly, because it is preceded immediately by the sixth natural note D b, and secondly, because the bass contains the sound B b, which forms a major fourth. The aforementioned passage D b E corresponds to the words dum emisit spiritum, and it expresses perfectly those last efforts of someone who is about to die in the act of exhaling the last breath. It remains for us to consider the fifth bar, which suits the word desolatum eminently well. The notes B b, minor seventh of C and A b, diminished seventh of B, fourth artificial note, prove, the first one emotional, and second one pitiful and utterly desolate.

[-933-] [Riccati, The laws of counterpoint, 933; text: vidit suum dulcem natum morientem desolatum dum emisit spiritum, Colla Parte, et cetera, 3 b, 4, 3 [sqb], 7 b, 3 [sqb], 7 5, 4 9, 3 b 8, 7 5, 3 8, 7, 5 b]

The fourth example that I add comes from a Psalm of the Terce [Memor esto add. supra lin.] for eight voices by Padre Francescantonio Calegari. I add it here with the main aim to show the seventh artificial note of the mode with the minor third harmonically added in various ways with the natural third, [-934-] the natural fourth and the natural sixth, and so that one may enjoy the effect that it produces, which consists in softening always the fourth note, albeit it is encountered as perfect, and in rendering the third and the sixth note more emotionally charged. The versicle becomes very expressive thanks to these devices, as well as utterly capable to symbolise divine compassion. The part that I present here

[Riccati, The laws of counterpoint, 934; text: misericordia tua Domine plena est terra, et cetera, Basso fondamentale, 3 #, 8 3 #, 7, 10 9, 8, 7 b 6 4 5 3, 5 3, 7 b 5 3, 4 9 5 3, 3 8, 6 4 5 3 #, 5 3 #, 4 2, 3 [sqb], 4 7 5 3, 8 5 3 b, 6 4 5 3 #]

[-935-] belongs to the tone A with the minor third. At the end of the first whole bar and of the sixth bar the tenor sings the fourth note D as a minor seventh added to the accompaniment E 5 3 #, whose character is highlighted further by the seventh artificial note G #, major third of E sung by the contralto. The third and fourth bar represent vividly the compassion felt by God towards our misery. The consonant accompaniment by representation G # 5 3, of the minor third and minor fifth, occupies entirely the two aforementioned bars. The minor fifth D, fourth note of the tone sounds emotional to the ear because of the comparison with the seventh artificial note G #, base of the accompaniment. The soprano is the part that sings that sound D. Moreover, the diminished seventh F, sixth note of the tone, is added to the accompaniment G # 5 3, which seventh is sung by the tenor throughout the two aforementioned bars and proves very emotional, as we have said above. Moreover, at the beginning of the third bar the composer adds the dissonances of the diminished fourth and of the minor sixth created by the sounds C and E, third and fifth note of the tone, to our accompaniment G # 5 3. The third note C in this occasion inspires more compassion than it normally does because of its correspondence with the seventh artificial note G #, with which it forms a diminished fourth, which is a chromatic interval. At bar six and eight the dissonances A and C, fourth and minor sixth are added to the perfect accompaniment E 5 3 # based on the fifth note. The minor third C, which is a soft dissonance by its own nature, is rendered more emotional by the comparison with the seventh artificial note G #, major third of the accompaniment E 5 3 #, which is sung by the bass at bar six and at bar eight by the tenor.

[signum] 14. From the sixth and seventh artificial note of the mode with the minor third, I move on to discuss the fourth artificial note of both modes, [-936-] considering in this case as well only the weak emotions. Our note shall be distinguished as artificial fourth when it is included in accompaniments belonging to the tone that indicate to the ear that it is employed as accidental, so to speak, and by virtue of the cadences which end on the fifth note. The fourth natural note, whether preceding or following the artificial one shall be a great indicator of it shared by the two modes. Moreover, as to the mode with the minor third, a firm rule is represented by the union with the sixth natural note, in the accompaniments of the diminished third and minor fifth, of the major third and of the minor fifth, as well as in their derivative ones in which the fourth artificial note is certainly included. When the fourth and sixth and fourth artificial notes are added together, once the first is noticed, the second one shall be noticed as well. I have already discussed the means employed by the ear to discern the sixth artificial note of the mode with the minor third.

If the fourth natural and artificial notes, which form with each other a minor semitone, and the first and and fourth artificial note, which form a major fourth, or the artificial fourth and the octave, which form a minor fifth, are compared in both tones, it shall follow that the the natural fourth note, the first or the octave enhance the vehemence of the fourth artificial note, and, conversely, the fourth artificial note shall add an emotionally charged quality to the natural first, and fourth note, or shall render it more prominent. I have expressed myself in this way, because the natural notes first and fourth can be truly lowered on certain occasions, for instance, when they are considered as minor seventh of the second and fifth note, in which case the contrast of the fourth artificial note enhances their softness.

The relationships of the fourth [-937-] artificial note with the three natural lowered notes, third, sixth and seventh, in the mode with the minor third deserve the highest consideration. The third note and the fourth artificial note form an augmented second; the fourth artificial note and the tenth, which is aequisonant to the third, form the diminished seventh. The comparison between the artificial fourth or eleventh with the natural sixth produces the diminished third and the augmented sixth, intervals that are employed harmonically only between those notes. Finally, the addition of the artificial fourth with the natural seventh and the addition of the natural seventh with the artificial

eleventh produce a diminished fourth and an augmented sixth respectively. The aforementioned intervals, introduced into music by the chromatic system, achieve the effect to enhance on one hand the energy of the artificial fourth, and on the other the weakness of the lowered natural notes, namely, the third, sixth and seventh. I deem it redundant to discuss the different degrees of combination that highlight the fourth artificial note to a varied degree, since what I said earlier on the matter of the artificial sixth and seventh note can be applied to the present instance.

[signum] 15. Descending from theory to practice, I transcribe the first example from the second half of a duet by Signore Sassone entitled Ah se di te mi privi. We are presented immediately with the fourth artificial note F # of the tone C with the minor third, which note softens the first note C and renders the note E b more sad, as these notes, added harmonically to F #, constitute the accompaniment F # 7 b 5 of the minor fifth and diminished seventh. I note that the sound E b proves even more emotional because the composition was based earlier on the tone C with the major third. At the end of bar seven, Signor employs the fourth artificial F # of the tone C with the major third, which reaches the ear more resentfully because of the memory still preserved of the fourth [-938-] natural note contained in the two previous bars. It is worth noting the beautiful expression of the notes quanto si può soffrir. The image of a long suffering is represented by the second soprano with the sound C and by the first soprano with the sound D, both of which are held at length.

[Riccati, The laws of counterpoint, 938; text: ho già sofferto or mai. quanto si può soffrir, Basso fondamentale, 7 5 b, 6 b 4, 5 3, 7 b 5, 5 3, 3 [sqb], 7 5 3, 3 #, 5 3, 7]

[-939-] Moreover, since suffering is all the more painful the longer it is, the consonant sound C is introduced first, and then, it is made to become a minor seventh added to the perfect accompaniment with the minor third D 5 3 with the added artifice of having the two singing arts form the second major C D, which is an interval very suited to symbolise the torment caused by the protracted suffering. Finally, said minor seventh C is made to be heard more raw through the comparison with the fourth artificial F # placed in the *basso continuo* at the end of bar seven. In the following bar, the first soprano sings almost immediately the fourth natural note F, minor seventh of G, which sounds very tender because of the memory still fresh of the fourth artificial note F #. In the example adduced earlier, the fourth artificial note of the mode with the minor third assigns a pathetic character to the first note and enhances it with the third note, with which it constitutes the accompaniment of the minor fifth and diminished seventh. The example that I propose, taken from the Stabat mater by Signore Pergolese, contains the aforementioned accompaniment laid out in such a way that the first and third note enhance the energy of the fourth artificial note. The exclamation of wonder in the words O quam tristis requires a portion of vehemence, which also contributes to indicate the immensity of the pain. The soprano, which sings the fourth artificial note C # of the tone G with the minor third, satisfies this first intention. Meanwhile the third note B b, which is naturally emotional, is allotted to the contralto and it is employed to express the word tristis to perfection. The aforementioned two notes that form an augmented second, namely, B b and C #, influence one another in such a way that the B b enhances the emphasis of C #, while C # enhances the sadness of B b. The first note G played by the *basso continuo* highlights clearly the raising of the artificial note C #, which forms the major fourth G C #. The following words are et afflicta, to which the fourth note C sung by the soprano corresponds perfectly. This note is perceived as sad [-940-] for three reasons, namely, because it is preceded by the fourth artificial note C #, albeit not immediately, because it is employed as minor seventh of D, and because it acts as fifth of F #. When the words et afflicta are repeated later on, the accompaniment C # 7 b 5 of the minor fifth and diminished seventh is employed

[Riccati, The laws of counterpoint, 940; text: O quam tristis et afflicta fuit illa benedicta mater unigeniti, Unisono, Basso fondamentale, 7 5, 5 3 #, 7 5 3, 4 9 3 8, 6 4, 5 [sqb] 3, 8 5 3, et cetera]

[-941-] in a similar way to the one adopted by Signor Sassone in the first example in employing the similar chord F # 7 b 5. The piece continues with the sentiment of the words fuit illa benedicta mater Unigeniti, which is expressed by adding music of a sentimental character. The passage from the accompaniment D 5 3 #, which belongs to the tone G with the minor third, in which the word afflicta ends, to the accompaniment B b 5 3, which belongs to the tone B with the major third, to which the word fuit is set, proves tender because the note F # is contained in the preceding accompaniment D 5 3 #, the natural note F is contained in the following accompaniment B b 5 3. I mentioned this because the occasion invited me to do so. Now I return to the thread of the discussion and I note that, after the said natural sound F, the soprano sings the fourth note E b of the tone B b with the major third firstly as minor third of C, and then as minor seventh of F. After we have heard the note E b, always emotionally charged in character, the bass sounds the fourth artificial note E in the following bar, which note becomes resentful because of the memory of E b. Therefore, such greater vehemence has the effect of softening the sounds placed in the singing parts, B b and D, which join the note E in the accompaniment E 7 5 b of the minor fifth and minor seventh. At bar seven the soprano sings again the fourth natural note E b as minor seventh of F, which note E b is rendered more tender by the memory of the note E.

Signor Benedetto Marcello, in his excellent cantata entitled Se la speranza o Dio imitates, as the great composer that he is, the ardent desire of the sailor, tired of the hardship of the sea, who longs to reach the shore through a melodic passage G C E of the minor fifth downwards, which moves from the eighth note to the artificial fourth in the tone G with the minor third. The eighth note G takes the role of the fifth in the accompaniment C 7 b 5 3 b, while the fourth artificial note C # is employed as the artificial note C # in the accompaniment A 7 3 #. Now, not only the sound G sung by the soprano, but also the other three, namely, E b, C and B b, contained in the previous accompaniment C 7 b 5 3 b and played by the [-942-] *basso continuo*, highlight the raising of the fourth artificial note C #, major third of the following accompaniment A 7 3 #. The same effect is also achieved

[Riccati, The laws of counterpoint, 942; text: muor di desio nel mar infido vicino al lido canto il nocchier, et cetera, Basso fondamentale. 5 3 b, 7, 3 #, 8 3 #, 3 b]

by the sounds B b and G played by the bass while the singer sings the note C #. B b is merely a note of the melody and it is allowed because it occurs on the weak part of the subdivision of the beat. The same cannot be said with regard to G, which is a note of the harmony, as it belongs to the accompaniment A 7 3 # as a minor seventh. In the following bar, C and E b, which are natural notes, prove emotionally charged because of the memory not yet erased of the artificial note C #.

The fourth natural note of either mode shall prove even more expressive if it follows the artificial one immediately or if it is employed also as a lowered sound. All of these conditions occur in the example that I place herewith contained in the second aria of the cantata Sfortunati miei sospiri of the most noble composer lauded just now. After bar eight, there are two bars that are joined together. At the beginning of them, the soprano sings the fourth natural note E b of the tone B b with the major third, at first [-943-] as a fundamental sound of the accompaniment E b 5 3, and then as minor seventh added to the accompaniment F 5 3, whose major third A, with which E b forms a minor fifth, is placed in the *basso continuo*. After the fourth natural note E b has been softened in such a way, the singer moves on, through the single note D, to the fourth artificial note E, which sounds resentful to the ear not only because it is preceded so closely by the natural note E b, but also because because the *basso continuo* sounds the first note B b, which is the minor seventh added to the accompaniment C 5 3, with which note E forms a major fourth. After the preparations described above which have the function of highlighting as fully the vehemence of the fourth artificial note,

[Riccati, The laws of counterpoint, 943; text: io tel perdono se vuoi lasciarmi, Basso fondamentale,

7 b, 6 3, 5, 7 b 5, 9 8, 5 b 3, 7, 7 5 3, 8 3 [sqb], 7 b 3, 7 3 [sqb]]

[-944-] E, Signor Marcello makes it follow immediately by the natural note E b used as a minor seventh added to the accompaniment F 5 3, while he places in the *basso continuo* the major third A, above which E b follows immediately E, given that the intervening semiquaver F has the function of a mere vocal ornament which helps the singer to tune more easily the minor semitone E E b. Let us now consider the words crudel se vuoi lasciarmi, io tel perdono. We shall notice easily that the two notes E and E b, to which the word perdono is set, are employed with the aim of expressing through the former note, which is the vehement, the violence that the lover applies against himself when he forgives the object of his affection for deliberately abandoning him, and of inspiring through the latter note, which is mournful, the feeling of compassion in the beloved. At bar fourteen, at the word lasciarmi, the soprano sings again the artificial note E accompanied in the bass with the first note B b, which forms a major fourth with the note E. These devices are employed to represent the cruelty of being abandoned undeservedly.

I take the last example from the second art of a very emotional aria by Signora Contessa Donna Teresa Agnesi beginning with the words In questo estremo addio. The sentiment of the words dove trovar del mio più tormentoso affanno, destino più tiranno, più barbaro dolor? is complex in nature. A pitiful state of pain and sorrow requires, on one hand, an emotionally fraught handling, but, on the other hand, the protracted question and the images of the tyranny of destiny and of the barbarity of pain require vehement means of expression. Therefore, after choosing the tone of G, which is naturally soft, as it is with the minor third, the fourth artificial note C # is used frequently. We have seen earlier ([signum] 1.) that the cadence most suited to set a question is the one that moves from the fourth note to the fifth one in the mode with the minor third. Now, the cadence from the fourth artificial note to the fifth one is employed as many as five times in our example, in order to emphasise its efficacy. I shall examine it orderly and I shall note in particular what relates to the fourth [-945-] artificial note. In the first bar the *basso continuo* moves from the natural note C to the artificial one C #, which, compared to the preceding note C, sounds very resentful and suitable to represent the word tiranno. The artificial note C # softens the

[Riccati, The laws of counterpoint, 945; text: Presto, destino più tiranno barbaro dolor dove trovar del mio tormentoso affanno. Basso fondamentale. 7 5 3, 8 5 3 #, 7, 5, 3#, 5 3 b, 7 5 3 b]

[-946-] first note G sung by the soprano, with which it forms a minor fifth, while it renders the third note B b, played by the first violin, mournful, as it forms a diminished seventh with the artificial note C #. Signora Contessa moves from the first to the second bar with the cadence C # 7 5 D 5 3 # from the fourth artificial note to the fifth one, which cadence is one of the five ones mentioned a little earlier. Observe that at the beginning of this second bar the soprano hits in quick succession the artificial notes F #, the seventh, and C #, the fourth, both of which are raised above the natural ones, and thus the word tiranno is expressed in admirable fashion. The cadence E b 6 # 3 D 5 3 #, derived from the fundamental one C # 5 3 b D 5 3 #, which moves from the fourth artificial note to the fifth one, begins in the third bar. The aforementioned cadence terminates at bar five, since Signora Donna Teresa requires the singer to linger considerably on said cadence, as it is indicated by the signs [signum], in order to express the end of a complete phrase and a fully accomplished question. In the accompaniment E b 6 # 3 the sixth note E b and the first one G highlight the raising of the fourth artificial note C #, while, conversely, the raising of this note makes the first note G look soft and the sixth note E b mournful. As to the singing part, we must pay attention to [-947-] two melodic movements: G C #, of the major fourth upwards, which highlights the vehemence of the fourth artificial note C #, and C # G of the major fourth downwards, which ascribes to the first note G an emotional character. The example ends with three similar passages to which correspond the three following verses:

Dove trovar de mio

Più tormentoso affanno,

Destino più tiranno?

Each of those passages contains a cadence that, in relation to the fundamental bass, moves from the fourth artificial note C # to the fifth one D. I shall consider now the two melodic movements played at the same time by one of the violins, B b C #, and by the *basso continuo*, C # E b. In the first one third note B b, which proceeds from the fourth artificial note C #, enhances its power, while in the second one the preceding artificial note C # ascribes a compassionate character to the sixth note E b. The opposite effect of the two mentioned melodies is eminently apt to the mixture of feelings contained in the three verses quoted above.

[signum] 16. It remains for us to say something on the second artificial note of the mode with the minor third that is a semitone lower than the natural one. It shall be recognised as an artificial second note if it is preceded or followed by specific accompaniments of the tone that let the ear understand the fact that it has been lowered artificially. Since it has been introduced into music by the cadence that moves from the second to the fifth note, this cadence shall be as a sure rule to discern the second artificial note. In the aforementioned cadence one part moves from the second artificial note, or from the ninth, base of the previous accompaniment, to the seventh artificial note, which is the major third of the following accompaniment either immediately with a leap of the diminished third downwards or in an indirect way by passing through the eighth note and by dividing the mentioned diminished third into two minor seconds. Thus, the artificial second note and the artificial seventh note are compared together very distinctly, as the second note makes the seventh look very [-948-] daring, while the seventh makes the second sound very emotionally charged. Other comparisons, albeit less distinct, can be made between the second artificial note and the natural fifth and second note contained in the second accompaniment of our cadence. The second artificial note and the second natural note differ by the minor semitone, while the relations of the major fourth and of the minor fifth occur between the fifth and the artificial ninth. These juxtapositions also increase the tenderness of the artificial second note and the brilliancy of the accompaniment based on the fifth note. An accompaniment based on the fourth artificial note, to which the sixth artificial note can also belong, intervenes sometimes between the two accompaniments that constitute the cadence that we are describing. I invite the Reader to observe the two series of accompaniments in the tone A with the minor third, namely, D 8 6 b 3, D 7 5 3 #, E 5 3 # and D 8 6 b 3, D # 5 3 #, E 5 3 #. Although on these occasions the melody moves from the ninth artificial note to the eighth note, which is the minor fifth of the intervening accompaniment, and from this note to the seventh artificial note, nevertheless some comparison can be established between the second artificial note or the ninth contained in the previous accompaniment and the artificial fourth and sixth notes contained in the consequent accompaniment. The second artificial note, or the ninth, and the sixth artificial note form an augmented fifth or a diminished fourth, while the artificial second, or ninth, and the artificial fourth form the interval of an augmented third or of a diminished sixth. These last intervals are entirely excluded not only from the harmony, as I illustrated at the appropriate time (book 2., chapter 4., [signum] 7., 8. and 9.) but also from the melody, so that one part can never move from the second artificial note, or from the ninth, to the fourth artificial note or vice versa. The described juxtapositions, however imperfect, of the second artificial note, or ninth, with the fourth and sixth artificial notes obtain the effect of enhancing to a very high degree the emotional character of the former and resentful one of the latter two.

I shall not attempt to discuss of the comparisons between the [-949-] second artificial note and the sounds contained in the accompaniment that can precede the one which is based on the second artificial note. What has been said so-far provide a secure method to judge the impression that the aforementioned juxtapositions would make on the ear.

[signum] 17. The second part of the aria Crudel se vuoi lasciarmi by Signor Benedetto Marcello, which was mentioned by me a little earlier, begins with the extract quoted in the following example. Since the eight bars contained from bar eight to bar fifteen belong to the tone C with the major third, the sound D b, sung by the soprano at bar eleven and twelve is heard clearly as an artificial second.

[Riccati, The laws of counterpoint, 949; text: Basso fondamentale, ma poi se la mia fede merta qualche mercede prima d'abbandonarmi dammi la morte in dono, 7, 4, 3, 7 3 #, 7 b, 8 3 #, 3 [sqb], 3 b, 7, 5 b 3, 8, 7, 9 3 b,]

[-950-] the fifth note G is contained in the accompaniment C 5 3 b that precedes the accompaniment D b 5 b 3, in which the artificial second, or ninth note occurs. The ninth artificial note D b sung by the soprano forms a minor fifth with the fifth note G played in the *basso continuo*. The noted juxtaposition, however obscure, has the power to produce a great impression of the ear, because it is held for one bar and one third and it is eminently suitable to the expression of the pitiful feeling of the words dammi la morte in dono. The singer moves from the ninth artificial note to the equally artificial seventh B by means of the eighth note C, which is employed as major seventh added to the accompaniment D b 5 b 3. I invite the Reader to observe that the pitiful character of the sound D b depends very much on the sound B, and that the resentful character of the latter is enhanced considerably by the memory of the former. Said effect is aided notably by the relationship of the sound A b, played with the *basso continuo*, with the note B, which relationship corresponds to the interval of an augmented second. Moreover, pitiful emotions are the predominant ones in the passage just described. In fact, these three notes, the artificial second D b, the third E b and the sixth A b, contained in the series of accompaniments C 5 3 b, D b 5 b 3 and G 5 3 [sqb] are truly lowered, since they are a semitone lower from their corresponding notes of the tone with the major third, while the sole note B, the artificial seventh, is indeed higher than the natural note B b by a minor semitone, but this only reduces it to its perfect state, since, when it is raised in this way, it is similar to the seventh natural note of the mode with the major third.

The second example contains the same progression of the chords C 5 3 b, D b 5 b 3 and G 5 3 [sqb]. It is taken from the masterly duet by Monsignore Steffani entitled Che volete o crude pene. I do not want to omit an observation, although it does not refer to the second artificial note which we are discussing at present. I invite the listener to enjoy how vividly the two B b of the tenth bar express the word languè. The end of the eighth bar and the beginning of the ninth belong certainly to the tone C with the minor third. The remaining part of the ninth bar [-951-] can also be assigned to the same tone, and, as the ear is convinced of this and believes that F # is the fourth artificial note, it is persuaded to be hearing in the tenth bar the seventh artificial note B. At this point the composer thwarts ably the expectation of the ear by placing the sound B b in the singing part, which, as it is a semitone lower than the sound B that the ear expected, proves eminently languid. The following harmonies and melodies show manifestly that at bar nine a modulation occurs by means of the passage A b 5 b 3 D 5 [sqb] 3 #, in which the preceding accompaniment belongs to the tone C and the following one to the tone G, both with the minor third. Monsignore Steffani has found the appropriate device for a passage that modulates from one to the other of the aforementioned tones, which is very rarely found in composition. As to the similar passage B b 5 3 E 5 b 3 #, which modulates from the tone D to the tone A, both of the minor, I produced of it an example earlier on (book 2., chapter 7., [signum] 12.).

[Riccati, The laws of counterpoint, 951; text: Che volete o crude pene del mio sen che langue e more, et cetera, Basso fondamentale. 5 4, 3 [sqb], 8 3, 7, 3 b, 7 b, 8 5 [sqb] 3 #, 3 #, 4 3, 3, 7 3 #, 5 3]

[-952-] The second half of the last aria of the cantata Lontananza, e gelosia by Signor Benedetto Marcello provides me with the third example. I believe that one shall struggle to find an aria more expressive than this one. I remember that it was praised to the highest degree by Signor Girolamo Ascanio Giustiniani, a nobleman and a senator of the Republic of Venice, who wrote the text of the *Parafrasi sopra i primi cinquanta Salmi* set to music by the knight Signor Benedetto Marcello, who coupled a profound theoretical, practical and contrapuntal knowledge with the most choice adornments worthy of his birth and of his rare talent. In the aforementioned aria, a lover complains to have been abandoned by his beloved

Dorinda, of whom he would never have expected such thing. Therefore, it would have been appropriate to add to an emotionally expressive style the emphasis required by the narration of a fact that, albeit it occurred, it seems impossible. Signor Marcello fulfils these requirements of these circumstances to the highest degree of excellence. As to the artificial note of the second, or ninth, at bar seven the music moves immediately

[Riccati, The laws of counterpoint, 952; text: e pure per mio duol ora non è più mia quella no, Basso fondamentale. 3 #, 7, 5, 5 # 3 #, 5 3, 3 #, 7, 7 3 #, 6 4, 5 3 #]

[-953-] from the ninth C to the seventh A #, both of which are artificial sounds of the tone B with the minor third. Thus, the juxtaposition of the sounds C and A # appears very clear, and it allows the sadness of the previous sound and the vehemence of the following one to be extremely prominent.

I add a fourth example taken from the previously lauded Miserere for two voices by Padre Maestro Vallotti. I present it here with the only aim to show the Reader the sequence of the fundamental chords E b 8 5 b 3, G # 7 5, A 5 3 # in the seventh and eighth bar. The two extreme accompaniments

[Riccati, The laws of counterpoint, 953; text: et exultabunt ossa humiliata, Basso fondamentale. 7, 3 #, 7 3 #, 7 5, 6 4, 5 3 #]

[-954-] constitute the cadence that, in relation to the fundamental bass, which is not employed in the counterpoint, moves from the artificial second E b to the fifth A in the tone D with the minor third. Padre Vallotti places an intermediate accompaniment between the two mentioned earlier, namely, G # 7 5, which is based on the fourth artificial note G #. This note, which is played by the *basso continuo*, makes the second artificial note E b sung by the second contralto more emotionally charged than usual, while, conversely, the note E b makes the note G # appear more resentful than the ordinary. The noted enhanced energy of the artificial note G # softens extraordinarily, according to their different character, the two sounds D and F sung by the two contraltos, which sounds form a minor fifth and a diminished seventh with G #. The aforementioned emotionally charged notes E b, D and F are very apt to express the meaning of the word humiliata.

[signum] 18. I said earlier ([signum] 11.) that the modulations from one tone to another one is very useful in depicting weak and strong emotions, if one employs promptly the natural notes that the new tone alters with the application of the minor semitone. I must render my statement more general immediately, by stating that the prompt use of the natural or artificial notes of the new tone that differ from the natural or artificial notes of the preceding tone by a semitone contribute greatly to the expression of the aforementioned emotions. I have mentioned in the passage quoted above [-955-] the natural notes of the consequent tone that are lowered or raised by a minor semitone compared with the natural notes of the preceding tone. Therefore, I shall limit myself to dealing with that concerns in some way the artificial notes. I observe that there are two given combinations, since the difference of a minor semitone can be contained between the natural note of the preceding tone and the artificial one of the consequent tone or between the artificial note of the preceding tone and the natural tone of the consequent tone. If the mentioned difference in relation to the new tone consists in raising a note, it shall be involved in the expression of strong feelings, while, if it consists in lowering a note, it shall be employed in the expression of weak emotions. The expression of said emotions shall be accomplished adequately even when the sounds modified in the following tone are perfect, or neither raised or lowered, while their expression shall be much more effective when the juxtaposition of some sound makes them appear raised or lowered or when they are truly such. In all of the aforementioned circumstances the imitation shall be deemed to be all the more lively if two sounds that differ by the minor semitone and belong to two different tones are heard in rapid succession, so that the modulation occurs by means of ascending and descending by a minor semitone.

In fact, if more than one natural or artificial note of the new tone in succession or at the

same time, which notes differ from the natural or artificial ones of the preceding tone by a minor semitone, one shall be able to imitate the most extraordinary emotions of both categories, namely, the weak and the strong ones, and also those that consist of both, according to whether the alterations make the sounds be raised or lowered or be partly raised and partly lowered at the same time.

[signum] 19. The doctrine explained above needs to be illustrated with examples. I take the first one from the second half of the duet by Signor Sassone entitled Ah se di te mi privi, from which the first example of the fifteenth paragraph is also borrowed. When one moves from the first whole bar to the second one, the tone of A is changed to the tone of E, both with the minor third. The first soprano and the first violin let the listener hear soon, at the beginning of the second bar, the second artificial note D # of the tone E, which is raised by a sharp above the fourth natural note D of the preceding note A. Although the sound D # considered in itself is perfect, when it is used as [-956-] major third of the perfect accompaniment B 5 # 3 #, which is based on the fifth note B, it is made to appear raised by the melodic leap A D # of the major fourth upwards, performed by the first soprano and by the first violin, and by the dissonances C, minor ninth, and A,

[Riccati, The laws of counterpoint, 956; text: ho già sofferto ormai quanto si può soffrir, et cetera, Basso fondamentale, 3 #, 9, 8, 5 # 3 #, 7, 7 5, 8 5 # 3 #]

minor seventh, added to the accompaniment B 5 # 3 # and performed by the second soprano and by the second violin, with whom the note D # forms an augmented second and a major fourth. Our artificial sound D # expresses perfectly the hyperbole contained in the two following verses:

ho già sofferto ormai
quanto si può soffrir.

I borrow the second example from the cantata by Signor Benedetto Marcello entitled Lungi, lungi speranze. The loss of the flame of love is depicted perfectly through a series of semitones, one major and the other one minor in alternation, descending from the high to the low register, while the growth of the ardour of disdain is expressed through an opposite sequence of semitones rising from

[Riccati, The laws of counterpoint, 957; text: Sento già che nel mezzo al mio core va mancando la fiamma d'amore e s'avanza di sdegno l'ardor, Basso fondamentale, 3 #, 3 b, 7, 8 3 #, 7, et cetera]

[-958-] the low to the high register. The first six bars belong to the tone F # with the minor third, while at bar seven there is a modulation to the tone B with with the minor third. The modulation is indicated by the use of the seventh artificial note A # sung by the bass at the beginning of that same bar. A # is raised above the third natural note of the previous tone by a minor semitone. The artificial note A # is employed with the main aim to make the natural note A, which follows it immediately, sound lowered, and in order to express the word mancando by means of descending through the minor semitone A # A. The tone B with the minor third continues at bar eight, where the singer touches on the artificial sixth note G # and the natural sixth note G. The *basso continuo*, which at the beginning of bar nine sounds the note D #, artificial seventh of the tone E with the minor third, indicates clearly the passage of the modulation to the aforementioned tone. Said artificial note D #, which exceeds the third natural note D of the preceding tone by a minor semitone, is employed in order for the seventh natural note D, which follows it immediately, to be heard as lowered even further. In this case as well the semitone D D # is employed as a means to express the sentiment of the words va mancando la fiamma d'amore. At bar ten Signor Marcello returns to the tone B with the minor third, whose artificial notes A #, seventh, and G #, sixth, are allotted to the singing bass. These notes are raised by a sharp above the fourth natural note A and above the third natural note G of the previous tone E with the minor third. Their being raised and the repeated ascending passage G #, A #, B are perfectly apt to express the words e s'avanza di sdegno l'ardor.

[-959-] I invite the Reader to note at bar sixteen, seventeen, eighteen and nineteen, the sequence of

notes B C C # D D # E, which ascend by a semitone and set the verse just now adduced e s'avanza di sdegno l'ardor. The passage C # D belongs certainly to the tone D with the major third, as it is derived from the perfect cadence of said tone A 7 5 3 # D 5 3 #. The motion D # E can be ascribed to the tone A with the major third, if we consider D # as the fourth artificial note of the aforementioned tone and the motion D # E as derived from the species of cadence B 7 5 # 3 #, E 5 3 #, which moves in the tone of A from the second note to the fifth. Under this hypothesis, when the modulation occurs, the composer lets the listener hear immediately the note D #, fourth artificial note of the new tone, which is a minor semitone higher than the note D, fourth note of the following tone and first one of the previous one. The natural sound D, which precedes the artificial sound D # immediately, highlights the energy of the latter and allows the music to represent the sentiment of the words in the best way.

The third example is suggested to me by an excellent response beginning with the words Tuam coronam spineam veneramur Domine for eight full voices and instruments by Padre Maestro Francescantonio Vallotti. In the first bar the composer modulates from the tone G to the tone C with the minor third through the passage D 5 3 # F 6 4 2 derived from the fundamental one D 5 3 # G 7 5 3. The following accompaniment, F 6 4 2, contains two alterations. First of all it employs the fourth natural note F of the tone of C, which is lowered by a semitone than F #, seventh artificial note of the tone G contained in the previous accompaniment D 5 3 #. Then, the composer employs the seventh artificial note B of the aforementioned tone C, which exceeds by a minor semitone the third natural note B b of the tone G with the minor third. The two described contrary alterations, one which lowers one sound and the other one that raises another one, produce a marvellous effect. The sound F sounds very compassionate as it is compared with the melodically with the sound F # and harmonically

[-960-] [Riccati, The laws of counterpoint, 960; text: Larghetto. qui passus es pro nobis, Basso fondamentale. 5 3 #, 7 [sqb] 5 [sqb] 3 [sqb], 7, 3 [sqb], et cetera]

with the sound B, which forms an augmented fourth with it and because it is sung by the bass as minor seventh added to the perfect accompaniment G 5 3. Conversely, the artificial sound B sounds full of resentment, not only because of the memory of the sound B b that was heard as minor third of the chord G 5 3 b throughout the last half of the previous bar, but also because it forms a major fourth with the natural sound F. The two notes F and B, sung by the extreme parts, bass and soprano, which are the most noted by the ear, symbolise [-961-] respectively the compassion that the suffering of the Redeemer deserves and the cruelty of the suffering itself to a high degree of perfection.

I transcribe the last example from the second part of the aria In questo estremo addio by Signora Contessa Donna Teresa Agnesi, whom I praised elsewhere.

[Riccati, The laws of counterpoint, 961; text: Col Basso, più barbaro dolor, Basso fondamentale, 3 #, 7 [sqb] 5, 5 4, 7 b [5 [sqb], 3 [sqb], 7 b 5, 6 4, 5 3 #, et cetera]

[-962-] We can observe in this passage three modulations as well, each of whom is accompanied by two alterations belonging to the category of those that we are discussing at the moment. Moving from the first to the second bar, the music modulates from the tone G to the tone D, both with the minor third. The accompaniment D 5 3 #, belonging to the tone G, contains the seventh artificial note F, while the one immediately following, G # 7 [sqb] 5, belonging to the tone D, contains the third natural note F, which is lowered by a minor semitone in comparison with the artificial note F #. Moreover, the accompaniment G # 7 [sqb] 5 is based on the fourth artificial note G # of the tone D, which is raised by a sharp above the natural note G of the preceding tone. The notes F and G # sound respectively sentimental and resentful because of their lowering and raising, which was noted earlier. Their opposite character is highlighted more distinctly because of the interval G # F of the diminished that they form with each other, which is an altered interval by its own nature. This observation must be applied also to the minor fifth G # D, formed between the bass and the soprano, which sings in unison with the first violin. The note D highlights the raising of the note G ', while, conversely, the note G # shows the note D as lowered and sad. The composition returns to the tone G with the minor third by means of the passage A 5 3 # F # 7 b 5 [sqb], which moves from the third to the fourth bar. The soprano and the first violin modulate by descending through the semitone C # D, which is the same as saying that they move from C #, seventh artificial note of the tone D with the minor third, to C, fourth natural note of the similar tone G. The second violin moves also from the tone D to the tone G, which I have already mentioned, by descending through the semitone E E b. However, this lowering of the note E has to be considered among those considered earlier ([signum] 11.) because it leads from one note to another one both of whom are natural, namely, from the second natural note E of the tone D to the sixth note of E b, which is also natural, of the tone G, both with the minor third. However, the seventh artificial note F # of the tone G with the minor third played by the bass, which is raised above the natural note F of the similar preceding tone D, belongs to our current investigation. As to the opposite character of the sound F #, which was raised, of the sound C, and of the sound E b, which was lowered, that constitute the accompaniment F # 7 b 5 of the minor fifth and diminished seventh, similar considerations apply to the ones that I made already [-963-] in relation to the sounds G #, D, F and to the accompaniment G # 7 [sqb] 5. I simply add that, since the aforementioned movements of the minor semitone C # C, E E b notify the ear of the lowering of the notes C and E b, the latter sound more emotionally charged than the notes D and F. The accompaniment G 5 3 [sqb], which follows immediately the accompaniment F # 7 b 5, belongs to the tone C with the minor third. The passage G 5 3 [sqb] C # 7 b 5 then follows, through which the modulation returns to the tone G with the minor third. Said passage is similar to the passage D 5 3 # G # 7 5, which I considered in the first place. The only alteration, which I shall call accidental, consists in the movement of the higher parts. In the passage G 5 3 [sqb] C # 7 b 5 the minor semitone B B b sung by the soprano and by the first violin deserves to be highlighted, as it contributes considerably to the mournful character of the third note B b of the tone G with the minor third. It is easy to draw the conclusion that the opposite alterations considered by me in the example place above, some of them involved in raising the pitch of a note and others in lowering it, suit admirably the mixture of contrasting emotions, some strong, others weak, contained in the verses

Dove trovar del mio

Più tormentoso affanno,

Destino più tiranno,

Più barbaro dolor?

[signum] 20. Apart from the alterations considered up to this point, which determine the sounds that belong to the tone to which the composition belongs naturally or artificially, music sometimes avails herself of some others that allow us to hear sounds that do not belong to the tone, not even as artificial sounds. Since these sounds cannot occur in the harmony, it is necessary, therefore, that they should be employed on secondary parts of the beat on which the bass and the other parts that constitute the harmony are silent or, at least, they are not repeated. Alternatively, they can be employed as simple appoggiaturas, when they need to be employed on the strong beat.

If these altered sounds are admitted into our musical compositions with the aforementioned precautions, they will produce an excellent effect and they will contribute to represent strong or weak emotions according to whether the alteration lowers or rises the note to which it is assigned<.>

[-964-] The first aria of the cantata for solo voice and instruments entitled *Fra voi taciti boschi* by Signora Contessa Donna Teresa Agnesi provides me with two very elegant passages. In the first one we note that the altered note G #, raised by a minor semitone above the natural note G of the tone F with the major third to which the adduced passage belongs, occurs on a weak subdivision of the beat on which the bass is not repeated. Two major semitones occur in each of the seven scales of the two modes with the major and minor third and of their five derivative ones. The difference between the scales depends on the different position of these two major semitones. Any pair of adjacent notes chosen at will forms a major semitone in two of the aforementioned scales. If we take the diatonic modes as model for all the others, the first and second note in the derivative tones B and E form a major semitone, the second and third note in the tone A with the minor third and in the derivative tone D form a major semitone, and so on. Now, the function of the altered notes that we are discussing is to introduce artificially the major semitone found naturally between two corresponding notes of a particular mode between two adjacent notes of another mode.

[Riccati, The laws of counterpoint, 964; text: Col Basso, rispondi per piet , Basso fondamentale. 7]

[-965-] In the example placed above the second raised note G # is used to ascend to the third note A by means of the movement of the major semitone G # A and to imitate artificially in the mode with the major third the natural semitone that is found between the second and third note of the mode with the minor third. As to the expression of the emotions, the passage B B G # of the diminished third downwards deserves to be considered. The note B b, minor seventh added to the accompaniment C 5 3 is rendered more emotionally charged by its comparison to the resentful note G #. Equally, the latter note becomes more vehement when compared to the sentimental note B b. The passage B b G # is eminently appropriate in order to express the words *per piet *, which contain a feeling of efficacy and compassion mixed together.

The ritornello, which is played by the instruments after the end of the second part of the aria and before the repeat of the first section, is of the same character. Here is the second of the two elegant passages that I promised earlier.

[Riccati, The laws of counterpoint, 965; text: Basso fondamentale. 7 b, 3 [sqb], 7, 6, et cetera]

I invite the Reader to observe that the altered seconds C # and D # of the tones B b and C with the major third are employed in the first and second bar on the strong beat, which corresponds to the percussion of the note in the bass and in the viola, but I observe that they are used otherwise as simple appoggiaturas. They are introduced thanks to the semitones C # D and D # E, which imitate the natural semitone that is located between the second and the third note of the mode with the minor third. [-966-] The accompaniment B b 5 3, which occupies the whole second half of the first bar must be considered as belonging firstly to the tone B b with the major third and secondly as belonging to the similar tone F. We have seen earlier (book 2., chapter 3., [signum] 5.) that the passages that are similar to B b 5 3 G 5 3 [sqb], through which the music moves from the first to the second bar, are respectful of the rules in as much as they consist of two accompaniments belonging to tones that preserve a mutual subordination, such as the tones F and C with the major third. The accompaniment B b 5 3 has to be assigned to the first of them, while the accompaniment G 5 3 [sqb] must be assigned to the second one. Therefore, the sound F #, laced at the end of the first bar on a weak subdivision of the beat is merely the first altered note of the tone F with the major third employed to ascend by the semitone F # G, which represents the natural semitones that in the derivative modes similar to the diatonic ones B and E move from the first to the second note.

I invite those who would like to see the sounds that we are discussing introduced on a weak

subdivision of the beat and then continued as appoggiaturas onto the following second beat of a tripartite bar to observe the beginning of the cantata Occhi miei già che non lice by Signor Benedetto Marcello, which I place herewith. The natural semitone B C at bar eight, which moves from the second to the third note of the tone A with the minor third, is imitated in the diatonic derivative tones G, F and E by means of the notes A #, G #, F # through the notes raised with the sharp. In order to obtain clear evidence of this, it is sufficient to cast an eye on the similar passages contained within bar eight, bar nine, bar ten and bar eleven, which belong respectively to the aforementioned sequence of diatonic tones A, G, F and E. The aforesaid altered notes A # G # F # represent vividly the vehemence of the passion that leads the lover to his death.

[-967-] [Riccati, The laws of counterpoint, 967; text: basso fondamentale, 7, 3 #, Occhi miei già che non lice dir col labbro o Dio mi moro voi parlate almen per me, et cetera]

The same effect of expressing a violent and painful passion is obtained through the first and second raised notes of the tone E b with the major third, which are employed masterfully by Signora Contessa Agnesi in the following fourth example taken from the aria In questo estremo addio. These notes fall on a weak division of the beat on which no parts of the accompaniment begin to play.

[-968-] [Riccati, The laws of counterpoint, 968,1; text: In questo estremo addio, Basso fondamentale, 7 6, 5, 5 4, 3, et cetera]

I add the fifth and sixth example which are borrowed from the second aria of the cantata Fra voi taciti boschi by Signora Contessa Donna Teresa Agnesi, whom I have praised several times. In the fifth example the fifth natural note F of the tone B b with the major third is employed firstly on a strong beat,

[Riccati, The laws of counterpoint, 968,2; text: Col Basso, fra tante pene, et cetera, Basso fondamentale, 7]

[-969-] while the same note, but altered to F #, is introduced into the adjacent weak subdivision of the beat. The noted raising of the note F to F # symbolises perfectly the idea of the many sorrows that have become too numerous and painful to bear contained in the words Ah si fra tante pene lasciatemi morir.

The violin parts contribute to the imitation of the same sentiment

[Riccati, The laws of counterpoint, 969; text: Piano, Unisono, Col Basso, Misera in tanti guai par che m'uccida ognora, Basso fondamentale. 7 5 #, 3 #, 7, 4 9, 3 8,]

[-970-] in the sixth example, which is taken from the beginning of the second part of the aforementioned aria. I invite the Reader to observe the fourth and the ninth bar, which contain the same passage, namely, F # G G #. The first natural note G of the tone with the minor third that takes its name from it, which is laced on the second beat of a tripartite bar, is followed immediately on the third beat of the same bar itself, which is a weak beat, by the first note raised to G #. I must not omit to point out that the *basso continuo*, the singing part and the viola are silent before bar four and before bar nine, while the violins play at the unison.

Seventh Chapter

On the expression of weak and strong emotions through the unequal tuning of the keyboard instruments commonly used.

[signum] 1. It remains for us to clarify what part the alterations of certain sounds, which originate from the unequal tuning of the keyboard instruments commonly used, as I established it and explained it in the fourth chapter, in expressing weak and strong emotions. Such alterations are

divided into two categories. One category comprehends the alterations that lower the sound and that are connected to the expression of [-971-] weak emotions. The other category contains the alterations that raise the sound and that are connected to the expression of strong emotions. The notes preceded by a flat are lower than they would be even if they also shared the temperament of the [[high]] notes that correspond to the white keys. Here it is necessary to distinguish the altered sounds into two categories. Some are employed in their proper role and maintain their specific name. Others change their name and assume a role that is alien to their nature.

Starting from the first category, we have seen in the chapter quoted above that, of the five black keys of the organs and harpsichords, three are called F #, C # and G #, while two are called B b and E b. If the aforementioned keys are employed in this way, the three sharps sound a little higher and the two flats are a little lower than they should, if they were perfect. F # is raised and B b is lowered by a quantity that is equivalent to one twelfth of the comma. The raising of C # and the lowering of E b corresponds to twice of that quantity, while the note G # is raised by three times that quantity, compared to what it should be. If an interval is of a calm nature, the aforementioned raising of it render it spirited, cheerful and a little resentful. For instance, the major third that occurs in the accompaniment of the major third and fifth and in its derivative ones is considered a calm interval. I pointed out in the aforementioned fourth chapter, [signum] 25. that the major thirds D F # and A C # in the accompaniments D F # A and A C # E sound cheerful and that the major third E G # of the accompaniment E G # B sounds rather daring and piquant. I have also pointed out that the properties of the aforementioned accompaniments are extended to the tones with the major third which comprehend those accompaniments. Therefore, a spirited and energetic character is ascribed to the tone G, a cheerful character is attributed to the tone D, and a cheerful but rather cheeky character is specific of the tone A, all these tones being with the major third.

Hence, Monsignore Steffani employed very successfully the tone D with the major third to imitate the cheerfulness of the thoughts that are free and unimpeded by the troubles of love. I invite the Reader to consider the following example, which is taken from the beginning of the duet Non ve ne state a ridere.
Se sciolti ve ne andate o miei pensieri by that esteemed composer.

[-972-] [Riccati, The laws of counterpoint, 972,1; text: Non ve ne state a ridere, et cetera, 3 #]

Similarly, the tone A with the major third, chosen by said Monsignore Steffani as the tone of one of his prized duets, suits perfectly the words Ho scherzato in verità.

[Riccati, The laws of counterpoint, 972,2; text: Ho scherzato in verità, et cetera, 3 #, 5 3, 6 4, 6, 5, 6 #]

[-973-] [signum] 2. Moreover, if an interval is tender, the three raised sounds F #, C # and G # reduce that quality in it. For instance, the minor third has a tender character in the perfect accompaniment of the minor third and fifth and in its derivative ones. Now, the minor thirds F # A, C # E and G # B in the accompaniments F # A C #, C # E G # and G # B D # sound gradually less sentimental than the diatonic ones because of the aforementioned raising of their bases F #, C # and G #. I said earlier (chapter 6., [signum] 4.) that the major third proves emotionally charged in the perfect accompaniment of the minor third and fifth, in which it occurs above the minor third. Therefore, the three sounds F #, C #, G # shall temper the softness of the major thirds D F #, A C # and E G # in the [-974-] chords B D F #, F # A C # and C # E G #, as well as in their derivative ones. This is the reason why the tones with the minor third E, B, F # and C # become gradually and orderly ever less emotionally charged and more resentful as I noted earlier (chapter 4. [signum] 23.) when I dealt with the unequal tuning of the keyboard instruments commonly used. I invite the Reader to observe that only the chromatic accompaniment B D F # (I shall call it chromatic to be succinct) is involved in the tone of E together with the two diatonic accompaniments E G B and A C E. The two

accompaniments B D F # and F # A C # together with the diatonic accompaniment E G B, are involved in the origin of the tone B. The tone F # consists of the three chromatic chords B D F #, F # A C # and C # A G #, while, finally, the tone C # is generated by the three chords F # A C #, C # E G # and G # B D #. The gradual reduction of the tenderness and the gradual increase of the energy in the aforementioned tones depends on two elements, namely, on the larger number of the chromatic accompaniments that partake in their production and on the ever larger number of the sharps that find their way into those accompaniments. The sound D #, which is employed with a name that does not belong to it and is raised enharmonically, belongs to the accompaniment G # B D #. I shall discuss further on the use of such alterations in relation to weak and strong emotions.

For now, it is appropriate for me to observe that the tones with the minor third E, B, F # and C # prove eminently suitable to express certain feelings that are a mixture of sentimental and resentful. When Signor Benedetto Marcello has to depict the pitiful condition, the worries and the uncontrollable desires of a jealous lover, he chooses appropriately the tone B with the minor third.

[Riccati, The laws of counterpoint, 974; text: Adagio, Deh lasciatemi un momento larve torbide e gelose, et cetera]

[-975-] Monsignore Steffani avail himself of the same tone to suit the music to the feeling expressed by the words

Nulla più mi ricrea, tutto m'affanna:

La lontananza è una crudel tiranna.

The elegant passage that I present is contained in the excellent duet entitled Lungi dall'idol mio.

[Riccati, The laws of counterpoint, 975; text: Nulla più mi ricrea, tutte m'affanna, et cetera]

[-976-] The masterly cantata entitled Lungi, lungi speranze by Signor Benedetto Marcello provides me with a third example. Our eminent composer employs the tone F # with the minor third to illustrate more closely the pain and the disdain of a lover betrayed by Filli.

[Riccati, The laws of counterpoint, 976; text: Lungi, lungi speranze pi tradiste abbastanza il cor vi sdegna, et cetera]

[signum] 3. What is left for us to say is that, if the three raised sounds F #, C # and G # occur in chords that are similar to the ones that sound resentful in the diatonic system, they increase their energy and their vehemence. I illuminate this general statement through a particular example. We have seen that the accompaniment B D F of the minor third and minor fifth based on the note B of the derivative diatonic tone F sounds resentful because the note B, which is the foundation of that accompaniment, is raised in that tone. If the composition is based on the tone A with the major third, in that case G # B D, based on the fourth note G # of the tone D derivative of the tone A with the major third, will be found to be the accompaniment similar to B D F. If the accompaniment G # B D is employed in this role, we shall perceive it [-977-] as more resentful than the accompaniment B D F because the base G # of the former is rather higher in relation to the upper sounds B and D than the base B of the latter in relation to the upper sounds D and F.

[signum] 4. Although the sounds F #, C # and G # contain an energy that grows progressively, they can be useful in some way to express weaker emotions because they ensure through the comparison with them that other notes are perceived as lowered, and therefore softer or more sentimental, than the other notes. I invite the Reader to recall what I said several times in the preceding sixth chapter in relation to the comparison between two sounds that form an altered interval. In the eighth paragraph of the aforementioned chapter I took as an example the accompaniment B D F of the minor third and minor fifth, which is based on the fourth note of the diatonic derivative mode F. Although in the mentioned accompaniment the note F is perfectly tuned, nevertheless it assumes a rather emotional character when it is compared to the fourth note B, which

is raised. I consider again the accompaniment G # B D, which is based on the tone D, derivative of the tone A with the major third, which is similar to the accompaniment B D F. Since the tuning of G # is a little higher in relation to D in the first accompaniment than the tuning of B in relation to F in the second accompaniment, it follows that the sound D, minor fifth of G #, proves softer on the ear than the sound D, minor fifth of B. In the aforementioned paragraph I informed the Reader that in the accompaniment G B D F which belongs to the diatonic tones C, G and D, the minor seventh F, which is a lowered sound, ensures that the major third B sounds energetic. I add now that the major third B enhances the emotional character of the minor seventh F. We shall have practical evidence of this if we compare the two chords G B D F and G B b D F. In the first of them we shall here the minor seventh F softer than in the second, and this originates only from the third, which is major in the former and minor in the latter chord. Now, if in another accompaniment of the major third, fifth and minor seventh, for instance E G # B D, which belongs to [-978-] the tone A with the major third and to its derivative E and B, the major third E G # shall be raised more than the major third G B because of the upper sound G #, as it occurs in reality, the fact that G # is raised by a larger amount shall cause the minor seventh E D to sound even more tender.

In the following extract taken from the cantata Fra voi taciti boschi, which I praised on several occasion, by Signora Contessa Donna Teresa Agnesi, I invite the Reader to consider the fourth bar from the first one in the example. The sound D, minor seventh added to the accompaniment E G # B, which is employed here and in the following bar, sounds sentimental not only because that is the character of the seventh, as it is a semitone lower than D #, which is the note played by the second violin at the end of the previous third bar, but also because

[Riccati, The laws of counterpoint, 978; text: viscere del mio sen anima mia]

[-979-] it is compared melodically and harmonically with the raised sound G #. The melodic comparison is very distinct, since the first the first violin moves directly from G # to D at the beginning of the fourth bar. The ears compares it very clearly also in relation to the harmony, since the bass and the first violin, with which the singer is in unison for a considerable part, sound the notes G # and D for the duration of one and a half bar.

[signum] 5. After considering the three sharps F #, C # and G #, I move on to discuss the two flats B b and E b, which sound a little lower than they would if they followed the temperament of the notes corresponding to the white keys. As the lowering of a sound suits the expression of weak emotions, it shall follow that our flats will introduce a degree of seriousness in the movements of the melody and in the quieter chords, making them veer somewhat towards the emotional. The tenderness of those melodies and harmonies that are already tender shall be enhanced, while the vehemence of those melodies and harmonies that are already vehement shall be reduced.

I shall illustrate the three observations made above with a few examples. We have noted earlier (chapter 4., [signum] 25.) that the major thirds B b D and E b G in the perfect accompaniments B b D F and E b G B b will prove more serious and less cheerful than the ones that occur in the accompaniments with the major third F A C, C E G and G B D, which consist merely of white keys, and that this property is shared by all the tones with the major third that concur to the formation of those accompaniments. Therefore, the major tones F and B b prove less tranquil than the tone C, because [-980-] they are based on two accompaniments containing a flat, namely B b D F in the case of the former and E b G B b in the case of the latter.

At the beginning of Monsignore Steffani's duet entitled Saldi marmi, Filli, remembering her dead lover Fileno, shows herself as wavering and uncertain as to whether she should die or correspond a new lover. The address to the tomb of her dead lover produces a serious and emotional impression. Since the tone B b with the major third is very appropriate for it, it was elected for this reason by the famous composer.

[Riccati, The laws of counterpoint, 980; text: Saldi marmi che coprite del mio ben l'ignuda salma]

I invite the Reader to remember what I said earlier (book 4., chapter 4., [signum] 23.) on the effect produced by the flats in the minor thirds considered in the fundamental accompaniment of the minor third and fifth. Such flats render them more tender than the one that consist of two white keys and ensure that the tone G with the minor third, which consists of the three accompaniments C E b, G B b D and D F A sounds rather more emotionally fraught than the tone A, which is based on the three accompaniments D F A, A C E and E G B without any alteration of flats or sharps.

In order to express the emotional and humble lamentations of the saintly king David, Signor Benedetto Marcello employs the tone of G with the minor third. If we were to transpose a major semitone lower in the tone F # with the major third the passage that I present, which is placed at the beginning of the third Psalm, we would incur the error of adding an element of resentment in a composition that has to inspire only resignation and humility.

[Riccati, The laws of counterpoint, 981; text: Largo, O dio perchè cotanto è mai cresciuto lo stuol di quei che in mille pene amare mi fan passar così infelici i giorni, et cetera, b]

I come now to the third property mentioned above, namely, that the flats ensure a level of tenderness as well as energy in certain melodic passages and accompaniments. I compare the chord A C E b with the corresponding chord B D F of the minor third and minor fifth mentioned several times and I presume that both of them are based on the fourth raised notes B and A of the tones F and E b, of which, the first one derives from the tone C, and the latter one from the tone B b with the major third. The bases B and A, both of which sound resentful, accept in both cases the same tuning which is shared by all the white keys. However, the same does not hold true in the case of the minor fifths F and E b, the second of which is a little lower as to the base A than the first one F in relation to the base B. The noted lowering of the note E b introduces a degree of tenderness in the resentful accompaniment A C E, which is presumed to be based on the fourth note A of the tone E b derived from the tone B b with the major third.

[signum] 6. Just as the three sharps F #, C # and G # can contribute indirectly to the depiction of weak feelings by ensuring that other sounds appear to be lowered because they themselves sound raised, so that those other sounds acquire an emotional character or increase the one that they already possess, thus, conversely, the two flats, B b and E b, are suited to foster the expression of strong feelings because, they ensure, thanks to the fact that they are themselves lowered, that other sounds are considered to be raised [-983-] and acquire in this way a vehement character that they did not possess or that they enhance the one that is already their own. I start by considering the melodic movement B b C # of the augmented second upwards, which moves from the second natural note B b to the fourth artificial note C # of the tone C with the minor third, as well as the harmonic relation of the augmented sixth E b C # created by the two notes E b, natural sixth, and C #, artificial eleventh, of the aforementioned tone. Even if the third note B b and the sixth E b were to accept the participation of the white keys, they would highlight nevertheless the energy of the fourth and eleventh artificial note C #, because the third and sixth note of the mode with the minor third sound emotionally charged and the two intervals B b C # and E b C # sound altered. However, since B b and E b are a little lower than what the aforementioned temperament requires, one discovers that precisely for this reason the vehemence of the note C # shall be highlighted further.

I chose deliberately the melodic passage B b C # and the harmonic interval E b C #, because they are employed by Signora Contessa Agnesi in the following example borrowed from the second part of the last aria of the very beautiful cantata for solo voice and instruments entitled Fra voi taciti boschi. In order that the meaning of the words may be understood more clearly, I write herewith the verses of the aforementioned second half.

Misera in tanti guai
par che m'uccida ognora,
E non m'uccide mai
Il barbaro martir.

I invite the Reader to cast an eye on the seventh bar, where we can see that the barbarity of pain is symbolised perfectly by means of the augmented fourth, or eleventh, C # of the tone G with the minor third. The artificial note itself is enhanced considerably thanks to the two natural notes altered with the flat, namely the third note B b, which precedes it immediately, and the sixth note E b, which forms an augmented sixth with the aforesaid eleventh note C #.

[-984-] [Riccati, The laws of counterpoint, 984; text: e non m'uccide mai il barbaro martir]

[signum] 7. Everything that I said on the effects produced by small alterations of the genuine sharps and flats in relation to the imitation of strong and weak feelings can be applied proportionally to the much larger alterations produced by the sharps and by the flats that occur to us in a form that does not belong to them and that, strictly speaking, are major semitones considered upwards and downwards employed as sharps or flats. True sharps and true flats sound respectively higher and lower than what would be required by the temperament that is applied to the diatonic system in our commonly used instruments. Such differences amount at least [-985-] to one twelfth of the comma and at most to three twelfths of a comma, or one fourth of it. The alterations of the presumed sharps and flats that we are discussing prove much more considerable. A b is three fourths or nine twelfths of a comma lower. D # and D b are raised and lowered respectively by ten twelfths of a comma. A # and G b then are raised and lowered respectively by eleven twelfths of a comma, E # and B # are raised by a whole comma and C b and F b are lowered by a whole comma. The sequence concludes with the three double sharps, F 2#, C 2# and G 2# that sound higher than they should by eight seventh of a comma if they are compared with the corresponding sounds of the participation accepted by the diatonic notes on the instruments commonly used.

[signum] 8. Starting from the seven sharps, for of which are simple, namely, D #, A #, E # and B #, and three are double sharps, namely, F 2#, C 2#, G 2#, I say that, because of them, a cheerful interval becomes harsh and resentful, and sometimes irate, while the fifth G # D # becomes resentful. The character of the thirds B D #, F # A #, C # E # and G # B #, commonly called major and contained in the accompaniments of the major third and fifth B D # F #, F # A # C #, C # E # G # and G # B # D # and in those derived from them, becomes more mischievous. I stated earlier (chapter 4., [signum] 27.) that the tones commonly called with the major third E, b, F # and C # are progressively more suited to the imitation of the most violent motions of the soul. In fact, the accompaniment B D # F # is involved in the formation of the tone E, the two accompaniments B D # F # and F # A # C # contribute to the formation of the tone B, the three accompaniments B D # F #, F # A # C # and C # E # G # generate the tone F #, while these other three, F # A # C #, C # E # G # and G # B # D # concur to form the tone C #, whose last accompaniment contains the raised and resentful fifth G # D #'# occurs. Our four accompaniments, that, rigorously speaking, should be called of the semi-augmented third and fifth, share their nature also with the tones with the minor third that employ them artificially. The accompaniments B D # F #, F # A # C #, C # E # G # and [-986-] G # B # D # belong ordinarily to the fifth of the tones with the minor third E, B, F #, C #, when the seventh artificial note is employed.

When Signor Benedetto Marcello wanted to elect a tone that would suit his second Psalm Quare fremuerunt gentes, he chose the tone E with the major third. In the first versicle, whose beginning I present to the Reader, the subject consists of the revolts against the Highest and his Christ. Such an enormous act of revolt requires the most violent means of expression. Therefore, since there are two similar subordinate tones that are related to the tone E,

[Riccati, The laws of counterpoint, 986; text: Risoluto, Tutti Donde cotanto fremito ed a qual fine s'ergono macchine tante in aria da nazioni e popoli, fermo]

[-987-] namely, A, which is softer in nature as it consists of three perfect accompaniments that are truly with the major third, and B, which is harsher, as two fundamental accompaniments with the semi-augmented third concur to its formation, Signor Marcello avoids the modulation to the tone A

in said versicle, but he employs frequently the modulation to the tone B. Moreover, at the end of the example that I presented he moves to the tone F #, which is entirely composed of semi-augmented thirds and is one of the subordinate tones of the tone B. In the continuation of our versicle this eminent composer moves solely to the tone with the minor third C #, which shares the scale with the tone E with the major third, and he allows the listener to hear the sixth and seventh artificial notes A # and B # as well as the second natural note D #. all of these three sounds are very apt to express the uncontrollable passions indicated by the words of the aforementioned versicle.

In the second example, transcribed from the recitative of the cantata entitled Il so begli occhi amati by Signor Benedetto Marcello, we note that the artificial accompaniments D # F # B and E # G # B C #, derived from the fundamental ones B D # F #, of the semi-augmented third and fifth, and C # E # G # B, of the semi-augmented third, fifth and minor seventh, are associated with the words a così fieri, e sì frequenti dardi. Said fundamental accompaniments, based on the fourth and fifth note B and C # of the tone F # with the minor third, belong artificially together with their derivative tones to the aforementioned tone, since they contain the artificial sixth and seventh note, namely D # and E #, which

[-988-] [Riccati, The laws of counterpoint, 988; text: a così fieri e sì frequenti dardi non ha l'anima mia tempre bastanti, et cetera]

the famous composer placed in the *basso continuo*.

[signum] 9. The sharps that we are discussing introduce an element of resentment in the intervals that are naturally apt to express weak emotions, which for this reason assume a mixed character that is very appropriate to express certain feelings that are both strong and weak. Such intervals are the thirds normally called minor thirds, but that, appropriately speaking, deserve the name semi-diminished, namely, D # F #, A # C # and E # G #, when they are considered in the fundamental accompaniments of the semi-diminished third and fifth D # F # A #, A # C # E # and E # G # B # and in their derivative ones, and also the semi-augmented thirds F # A #, C # E #, G # B #, which are placed above the semi-diminished thirds in the aforementioned fundamental accompaniments. Our accompaniments share their character with the tones in which they are used, especially with those called with the minor third, to whose formation they concur. The Reader can see them laid out in sequence herewith: G #, D #, A #, E # and B #.

The emotions of fright and horror are considered among the complex feelings that I mentioned just now. Therefore, in order to imitate the feeling of fright and horror felt by the nations that were enemies of the chosen people when they appear in front of God's tribunal and hear his just and bitter verdict of condemnation, Signor Marcello modulates in the ninth Psalm to the tone D # with the minor third. A few small alterations [-989-] that I employ in transcribing this passage do not alter at all the melody and the harmonies used by this most noble composer; on the contrary, they highlight them more clearly.

[Riccati, The laws of counterpoint, 989; text: onde sentan la loro giust'acerba condanna]

[signum] 10. If any of our sharps occurs in the intervals that have a vehement character, it has the effect of enhancing that particular quality. I hope that the Reader shall be satisfied with my presenting two examples, one in relation to the harmony and one in relation to the melody. They are borrowed from the very elegant duet by Monsignore Steffani entitled Pria ch'io faccia altrui palese. In the third bar of the first example, the composer employs the accompaniment F # A D # of the minor third and major sixth, derived from the fundamental one D # F # A of the minor third and minor fifth, based itself on the fourth note D # of the tone A and subordinate to the tone E with the major third with whom it shares the scale. [-990-] The tone A and the accompaniment D # F # A and those derived from it

[Riccati, The laws of counterpoint, 990; text: di mia man con giuste offese mi trarrò dal petto]

l'alma]

correspond respectively to the diatonic tone F and to the accompaniment B D F and to those derived from it, which I mentioned so many times. The accompaniment F # A D # would prove resentful because of the sound D # fourth altered note of the Tone A even if one presumes that its tuning were completely similar to the one of the corresponding accompaniment D F A. However, as we pointed out earlier ([signum] 7.), the note D # is raised considerably, so that it enhances the energy of the chord F # A D #, whose powerful note D # is placed in the part of the singer, who expresses the desperate sentiment of the words mi trarrò l'anima dal petto.

The second example resents us with three [-991-] melodic passages that express resentment. They are: F # G #, performed by the first soprano in the second bar; C # G #, performed by the second soprano in the third bar and G # A # performed by the first soprano at bar four. The vehemence of the mentioned passages depends on the fact that, since the composition modulates from one tone to another one thanks to them, they allow us to hear the sound, whether natural or artificial, that is raised by a minor semitone in the new tone. F # G # moves from the tone D with the major third to the similar tone A; C # D # from the tone A to the tone E, both with the major third; G # A # from the tone E with the major third to the tone B with the minor third. Now, there is no doubt that the last two passages, namely, C # D # and G # A #, of the three that we are discussing, F # G #, C # D #, G # A #,

[Riccati, The laws of counterpoint, 991; text: del mio desio, et cetera]

express more resentment than the first one. In fact, apart from the circumstance shared by all three that their second sounds G #, D # and A # are a sharp higher than the sounds that belong naturally to the preceding tones, in the second and in the third movement we find the property that the sharpened notes D # and A # are considerably higher in pitch.

perchè oltre la circostanza comune a tutti tre, che i suoni posteriori G #, D #, A # superano per un Diesis i suoni naturalmente propri dei Tuoni antecedenti, si verifica nei movimenti secondo, e terzo la proprietà, che i Diesis D #, A # considerabilemente ricrescono. In order to express the sentiment of the words

Vuò morire

[-992-] Pria che dire

La cagion del mio desio,

Monsignor Steffani employs the passages described above, which are extremely apt to express the violence of the passion that, were it to remain secret, might have led the lover to his death.

[signum] 11. The sharpened notes that we are discussing at present can be employed indirectly to depict weak passions when the ear judges other sounds as lowered, when they are perfect, or lowered more than they really are, by comparing with these sharpened notes. I refer the Readers to what I said earlier ([signum] 4.) on the sharps that are truly such, and I invite them to apply it with much stronger reason to the sharps that are presume to be such, while I present a few examples for their consideration.

The first example is suggested to me by the ending of the second Psalm by Signor Benedetto Marcello, which is written in the tone E with the major third and is eminently suitable to express the words

Quando, e in breve sarà, fia che di sdegno

Arda, ed [[avam]] avvampi.

While the singer expresses the words e in breve sarà, Signor Marcello employs the leap E A of the fourth upwards, which is very prominent because it moves from a weak beat to a strong beat. After such leap, the sung part returns to E by stepwise movements. I invite the Reader to remember that I noted in tenth paragraph of the previous fifth chapter that, in order to imitate in music strong passions, it is convenient to introduce into the composition long sequences of movements by step

and many leaps downwards and upwards, which must be all the more numerous and wider the more powerful the feeling that is being depicted. In our case Signor Marcello's sole scope is for the music to adopt a resolute character, therefore he employs the devices described above in moderation. Then, we encounter the words fia che di sdegno arda, ed avvampi. The repeated rising of the passage E, F # G # represents perfectly the lighting up and the progressive growing of God's disdain. I said earlier (chapter 6. [signum] 1.) that the movement from the low to the high register contributes to express a feeling of power. [-993-] Although the adduced consideration may appear alien in relation to the particular circumstances that we are discussing at the moment, I thought it inappropriate to overlook them, because I wanted to show that an accomplished depiction often depends by the application of several devices.

Having said this, I move on to illustrate how, in the example placed herewith,

[Riccati, The laws of counterpoint, 993; text: Largo, Quando (e in breve sarà) fia che di sdegno arda ed avvampi quel sarà beato ch'avrà nell'Unto del Signor sperato, et cetera]

our raised sharps are forced, against their natural inclination, to facilitate sentimental expressions. The aforementioned words, which predict the imminent ire of God, are followed by other words of a very different character promising forgiveness and happiness to those who place their hope in the Messiah. Here are the words:

[-994-] quel sarà beato,

Ch'avrà nell'Unto del Signor sperato,

So, the composer leaves behind the loudness of many voices, allows only one to sing and employs also all the devices described earlier (chapter 6. [signum] 7.) that enable the tones with the major third to express tender sentiments. I note, in relation to our present purpose, that, while the alto utters the word beato, signor Marcello adds the minor seventh A, sung by the alto himself, to the accompaniment B D # F # of the semi-augmented third and fifth and he places the sound D #, which is raised considerably, in the *basso continuo*, in order to highlight even more the emotional character of the minor seventh A. The alto itself sings the note A, minor fifth, or, to be more precise, semi-diminished fifth of the accompaniment D # F # A at the word Signor, which is very tender to the ears of those who are just and enjoy the compassion and love of God, in which case the note A appears very tender because of its juxtaposition with the sound D # played by the *basso continuo*.

The second example is taken from the conclusion of the twenty-fourth Psalm by Signor Marcello, whom I lauded so many times. It sets the following verses:

Deh ch'io possa cantar fra tuoi più cari

I pregi tuoi presso i sacri Altari.

The pious desire to sing the praises of the Lord together with the souls that are dearer to him requires handling full of emotion and tenderness. However, as such praises have to be sung in the temple, the great God's august residence, it was very appropriate to combine with the emotional element a degree of majesty and holy terror. The tone of E with the major third is very appropriate to such a mixture of emotional representations,

[Riccati, The laws of counterpoint, 994; text: Deh ch'io possa cantar fra tuoi più cari i pregi tuoi presso i sacri Altari, et cetera]

[-995-] when it coupled with a slow tempo and with the devices that enable the mode with the major third to express tender feelings. I invite the Reader to note the minor seventh A added to the accompaniment B D # F # of the semi-augmented third and fourth in the fifth bar. This note [-996-] is very prominent because it is sung by the alto, which is the highest part. It proves emotionally charged for three reasons, namely, because such is the nature of the minor third; because, as the perfect cadence F # 5 # 3 # B 5 # 3 # belonging to the tone B with the major third whose seventh note is A # leads immediately to our accompaniment B D # F # A, we hear the natural A straight away, which leads the composition from the tone B to the tone E both with the major third; and

finally because the note A is compared with the raised sound D # played by the *basso continuo*. We need to consider also the accompaniment F # A # C # E of the semi-augmented third, fifth and minor seventh that occupies the whole of the penultimate bar and belongs to the tone B normally called with the major third. The tenderness of the minor seventh sung by the alto is enhanced by the comparison with the raised sound A #, which makes a great impression on the ear since the modulation from the tone E to the tone B, both with the major third, occurs thanks to it.

[signum] 12. I shall conclude the present chapter by saying something about the five flats that are such merely by name, A b, D b, G b, C b and F b, in relation to the depiction of strong and weak emotions. The fact that our flats are lowered considerably enables them to express weak emotions admirably. If said flats are added to intervals that are naturally cheerful, they turn them into emotionally charged intervals, while, if they are applied to emotional intervals, they turn them into tearful intervals. If the intervals have a resentful character or they contain a mixture of sentimentality and resentment, the flats that we are discussing add to them either introduce an emotional note or heighten what sentiment they possess by turning it into sadness. Finally, the aforementioned flats can aid the imitation of the strong emotions in some way, by enabling certain perfect sounds and particularly those that are raised to be perceived as vehement by comparison with them. The aforementioned circumstances shall be illustrated somewhat by the examples.

In the fundamental accompaniments of the major third and fifth, which generate the tones with the major third, the aforementioned consonance of the fifth and major third, as well as the minor third located between the major third and the fifth, sound tranquil. Let us consider now the chords [-997-] commonly called of the major third and fifth that should be called more precisely of the semi-augmented third and fifth and that are made to be such by the flats. They reach the number of four, and they are: A b C E b, D b F A b, G b B b D b and C b E b G b. The first accompaniment contains only one flat of the sort that we are discussing, while two are involved in the following chords. The effect of mentioned the single flat in relation to the accompaniment A b C E b is to turn the fifth A b E b into a raised fifth and the third A b C into a semi-augmented third, both of which become emotionally charged as a consequence, since their being raised and semi-augmented is owed to the sound A b. the minor third C E b becomes only slightly more tender because of the small lowering of the sound E b, which is a true flat. The two considerably lower flats that occur in each of the three remaining chords constitute their base and the fifth and ensure that said fifth consists of two thirds that are both emotionally charged. The lower of them is semi-augmented, while the higher one is semi-diminished. I have listed diligently above (chapter 4., [signum] 27.) the tones that are called commonly with the major third and to which the aforementioned accompaniments belong. I only add at present that, apart from the tone E b, which composers use very often as principal tone, the other ones, A b, D b and G b, which are called with the major third and require a larger number of flats, need to be employed as subordinate tones. For instance, the tone D b, which originates from the three accompaniments A b C E b, D b F A b and G b B b G b, admits into its scale the three notes D b, G b and A b that are often employed in a function that is not their own, and four other notes, E b, F, B b and C, that retain their true name. It would appear on one hand that, as the four sounds that are employed as they are are more numerous, they would make the ear consider the other three, D b, G b and D b, as altered. However, if we reflect that these three sound form the system of the melody of the tone D b with the appropriate accuracy, the ear shall be prone to believe because of this that the alteration originates the four sounds E b, F, B b and C, which are no less principal despite being more numerous. [-998-] Such ambiguity is despatched when the tone D b is introduced into a musical composition as subordinate, for instance to the tone B b with the minor third or, more correctly, with the semi-diminished third. In this last tone, which is generated by the accompaniments F A b C, B b D b F and E b G b B b, the aequisonances and the perfect consonances are produced by the four sounds E b, F, B b and C, which retain their proper name, while only the imperfect consonances originate from the introduction of the sounds named with the improper names of A b, D b, G b. Consequently, the ear judges the aforementioned sounds as enharmonically altered in the principal tone B b with the semi-diminished third on the basis of evidence, and it continues to believe them such in the subordinate tone D b with the semi-

augmented third.

[signum] 13. I take the following example from a versicle of the Miserere by Signor Sassone and written in the tone of E b with the major third, so that we may note how much the choice of the tone contributes to render the composition sad and emotionally charged. After the words Quoniam si voluisses etcetera, the setting continues with the expression of a feeling of strong contrition and trust in God contained in the words Sacrificium Deo spiritus contribulatus, cor contritum et humiliatum Deus non despicias. We are presented immediately at the beginning of the example with the modulation to the subordinate tone A b with the major third. The second bar places in front of our eyes the accompaniment [[G B b]] G B b D b E b, which derives from the fundamental one E b G B b D b, in which the seventh E b D b is rendered semi-diminished by the lowered sound D b. While the minor seventh sounds emotionally charged, this one sounds tearful. The sadness of the note D b sung by the voice part is enhanced by the fact that it is reached through the mournful melodic passage C D b against the sound G played by the *basso continuo*, with which D b forms a semi-diminished fifth. Bar three and bar four are occupied entirely by the chord A C E b G b, which forms a minor third and minor fifth with the bass A of the raised note, with the addition of the seventh that becomes more-than-augmented because of the lowering of the sound G b. If the diminished seventh inspires compassion, there is no doubt that [-999-] such quality shall be evoked in a more lively fashion in our seventh, which, consequently, shall be eminently suited to the expression of the word contribulatus. The accompaniment A C E b C b is employed as belonging to the tone B b with the minor third, from which the composition moves to the tone B b with the major third in the fifth bar. The famous composer returns us to the principal tone E b with the major third around the middle of the same bar. Here the soprano sings the natural note G, which is a cheerful sound because it is the major third of the fundamental accompaniment E b 5 b 3. This accompaniment sounds

[Riccati, The laws of counterpoint, 999; text: Sacrificium Deo spiritus contribulatus cor contritum et humiliatum Deus non despicias, et cetera, Unisono, 6 5 b 3, 4 b 9, 3 8, 3 [sqb], 5 3[sqb], 6, 7, 4 2, 6 4, 5 3]

[-1000-] full of energy because of the memory still fresh of the sad sound G b, which the ear was made to hear throughout the two previous bars. The effect of the note G admirable, and it is also highlighted by the notes A and F # in the sixth and seventh bar, which are raised by a minor semitone. It is almost as if one could see the king sigh and take comfort by the consideration that God our Lord never despises a contrite and humble heart. At the beginning of bar eight the singer touches on the note D, major third of the perfect chord B b 5 3, which note acquires strength from the melodic and harmonic comparison with the lowered sound A b. D is compared melodically with A b through the leap of the minor fifth downwards, by which the soprano moves from the seventh to the eighth bar, while it is compared harmonically with it through the addition of the semi-diminished seventh A b to the aforementioned accompaniment B b 5 3, which seventh is placed in the *basso continuo*, thus allowing the soprano to sing the note D, forming an augmented fourth with A b so that the note D is made to appear much stronger because of such juxtaposition. This symbolises the unmovable trust of penitent David in the compassion of the Lord, who certainly does not refuse to pardon those who repent.

In the following example borrowed from the fiftieth Psalm by Signor Benedetto Marcello the melodic passages A b B and D b E of the more-than-augmented second upwards, [-1001-] which is produced by the enharmonic lowering of the lower notes A b and D b, deserve to be considered. All the passages mentioned above, except for one that I shall illustrate separately,

[Riccati, The laws of counterpoint, 1001; text: Abbastanza comprendo il grande eccesso del mino delitto, et cetera, 3 [sqb], 4, 3 b, 5 3, 6 3, 6 b 3, 6 [sqb] 3, 5 b, 5 4, 3 b, 5 3 [sqb]]

[-1002-] move from the same semi-diminished note to the artificial seventh of the tones C and F,

commonly called with the minor third. Even if the sixth notes A b and D b were to accept the participation that is assigned to the diatonic system on the keyboard instruments, nevertheless, as I stated earlier (chapter 6 [signum] 12. e 13.) when I discussed the seventh artificial note of the mode with the minor third, their juxtaposition with the seventh artificial notes B and E would ensure that the latter would sound resentful to the ear. Therefore, they shall prove more vehement in our case because of the enharmonic lowering of the sixth notes A b and D b, which shall be very useful to enable the aforementioned sounds B and A to represent the enormity of sin recognised clearly by David in his repentance.

The melodic movement that I promised to discuss separately is the one of the more-than-augmented second upwards through which the alto moves from the fourteenth to the fifteenth bar. The notes that constitute said passage are the semi-diminished seventh and the artificial fourth of the tone F with the minor third. The sound B in this case proves even more emphatic than it happens to be in the other cases mentioned above because the fourth note or the eleventh is more resentful than both of the artificial sevenths and because it is compared harmonically with the eighth note F and semi-diminished sixth D b with which it forms the major fourth F B and the more-than-augmented sixth D b B. The lowering of the sound D b contributes as well to highlight more vividly the strength of the sound B. [-1003-] I inform the Reader that I have omitted the two violas in the last example because the first one plays at the unison with the alto and the second one with the tenor.

[signum] 14. Although I presented the first example of the preceding twelfth paragraph with the main aim to show that the flats that we are discussing introduce an emotional element in certain fundamental accompaniments commonly called of the major third and fifth that concur to the creation of the tones with the major tones E b, A b, D b and G b, nevertheless I also note in said first example, as well as in the second one, certain melodic and harmonic relationships that can be sufficient to demonstrate in practice the different sort of effects illustrated above, which are produced by our flats that are lowered considerably, in relation to weak and strong emotions. However, a very important circumstance comes to my mind, which deserves to be discussed in detail. The perfect accompaniments with the minor third are naturally soft and emotionally charged. However, if one introduces in them one of the aforementioned flats between the fundamental sound and the fifth, this sound divides the fifth into two thirds, the lower one is semi-diminished and the upper one semi-augmented, both of which inspire compassion and share their character with the whole accompaniments. These accompaniments, rigorously speaking, have to be called of the semi-diminished third and fifth. In the preceding paragraph twelve I happened to record the three fundamental accompaniments with the diminished third F A b c, B b D b F and E b G b B b that are found on the organ and on the harpsichord. I discussed them more deliberately earlier (chapter 4., [signum] 24.) when I dealt with the tuning of the keyboard instruments commonly used. I noted here that only the accompaniment with the semi-d third F A b C contributes to the formation of the tone C normally called with the minor third; that that two similar accompaniments F A b c and B b D b F are involved in the creation of the tone F also called with the minor third; and, finally, that the tone B b, which deserves to be called with the semi-diminished third consists entirely of the three chords F A b C, B b D b F and E b G b B b, which are based [-1004-] on the three sounds F, B b and E b, which constitute the system of the melody of the aforementioned tone. I also informed the Reader in the passage quoted above that a greater degree of sadness pervades the tones C, F and B b, as they accept in their formation a greater number of accompaniments of the semi-diminished third. The tone C, which employs the accompaniment of the diminished third and fifth only on its fourth note F, which is the less perfect among those that constitute the system of the melody, shall sound sweet and melancholic to the ear at the same time, with such a agreeable mixture of feelings that renders it very pleasant.

[signum] 15. A composer whom I do not know employs the aforementioned tone C in order to describe the state of mind of a passionate lover who derives his pleasure from the torments of love. I invite the Reader to enjoy the following example, with which the anonymous author concludes a duet of his. The tone is mostly C with the minor third, as the music modulates only

once and in passing to the subordinate tone E b with the major third. I limit my considerations to the tone C with the minor third and I inform the reader that the semi-diminished sound A b occurs in three fundamental accompaniments, of which two are consonant, F A b C and A b C E b, while the third one,

[Riccati, The laws of counterpoint, 1004; text: Adagio, Disciolto mi vorrei e pur fra le pene incatenato sol trovo il mio bene, 6, 5, 6 4, 6 5, 3 [sqb], 7, 5 4, 11 9, 10 8, 9 3 b, 8, 9, 6 3, 5 3 [sqb], 10 9]

D F A b, is employed as consonant by representation. The first accompaniment belongs strictly to the tone C, while the other two are borrowed from its derivative tones. In our example, the second and third chord are also employed, besides the first one, in such a way that they sound mournful. The fundamental passage F 5 3 b A b 5 b 3 of the minor second upwards, which moves from the second to the third bar (and, consequently, its [[derivative]] [dependent corr. supra lin.] tone B b 6 3 A b 5 b 3) belongs, strictly speaking, to the derivative tone D, in relation to which it moves from the fourth to the fifth note. In the system of the melody D, G, A b of the aforementioned tone, the first and the fourth notes are perfect, while the fifth is lowered. Were the fifth lowered only by a minor semitone, it would sound emotionally charged, but, since it is lowered by a [-1006-] semitone that is employed as a minor semitone, it becomes melancholic and it shares this property with the accompaniment A b C E b, which is based on it. The passage B b 6 3 A b 5 b 3 is followed by the three accompaniments G 6 b 4, derived from C 5 3 b, F 6 3 b, derived from D 5 b 3, and G 5 3 [sqb] which belongs to the dependent tone G, because they are based on the fourth note C, on the fifth one D and on the first one G, which constitute the system of the melody of the aforementioned tone. The perfection of said system ensures that the ear considers the sound A b of the accompaniment D F A b as lowered. As the sound A b is lowered by a major semitone, for this reason it become semi-diminished and it sounds very mournful. The accompaniments that are derived from our own are employed in three other places, and, since they are always based on the fifth of the derivative tone G, the same considerations as the ones made above apply to them. I inform the Reader that, when the feeling of the words did not require an enhanced expression of sadness, the accompaniment D 5 b 3 was employed twice in its derivative forms without any part touching on the sad sound A b. On two occasions the sound A b is heard repeatedly in the *basso continuo*, and it is also allotted to the singer, albeit only for the value of one quaver.

[signum] 16. The tone C with the minor third appears in the example that I add herewith in a more mournful form that is accompanied, however, by a feeling of resignation, of confidence and affection. These extract comes from the conclusion of the first versicle of the twenty-first Psalm for solo voice and violette by the most celebrated Signor Benedetto Marcello. There is hardly a bar in which the languid sound A b is not employed, and very often it appears several times within one bar. It occurs nine times as semi-diminished third of the fundamental accompaniment F A b C, which sometimes

[Riccati, The laws of counterpoint, 1006; text: Grave. deh mio Dio volgi un de' tuoi guardi perchè così mi lasci in abbandono mio Dio, 6, 9 3, 8 b 4, 7 6 b 4, 5 3 [sqb], 5, 6 5, 6 4 [sqb], 6 4 #, 3 [sqb], 5 b 3, 11 9, 10 b 8, 7 5, 10 8, 7 5, 5, 6, 6 5 3 b, 9, 6 5 b, 6 5 3, 6 [sqb] 3 b, 3 b, 5 4]

belongs to the principal accompaniment C, and sometimes to the subordinate tone F, both with the minor third, to the latter of which the composer moves frequently to adhere more closely to the compassionate sentiments expressed in the sacred poetry. The sound A b is the base of the chord A b C E b on four occasions. At bar five and [-1008-] t bar seven the proximity of the sounds B and F # renders it very emotional. Moreover, it is possible to note that, since the chord A b C E b appears twice within bar seven, it is employed the second time as belonging to the derivative tone D, as the passage G 5 3 [sqb] A b 5 b 3, which moves from the fourth to the fifth note of said tone, indicates manifestly. Now, even if we do not consider the fresh memory of the sounds B and F #, we learn

from our notes to the first example that the accompaniment A b C E b sounds mournful when it belongs to the derivative tone D. Our accompaniment is employed in the same role at bar five, because the passage A b 7 5 b 3 D 5 b 3 is a cadence from the fifth to the first note of the aforementioned tone. The accompaniment D F A b, in which A b forms the semi-diminished fifth, is employed twice as belonging to the derivative tone G in the second and eighth bar and once in the sixth bar as belonging to the derivative tone D. It sounds sad on both occasions because of the reason adduced in the notes mentioned just now. Apart from the accompaniments described above, which are consonant or are employed as such, there are also two dissonant ones that contain the sound A b as a dissonance. Consider the semi-diminished sixth at bar two, added to the consonant accompaniment C 5 3 [sqb] together with the other two dissonances of the fourth F and of the seventh B b. The sixth A b, especially when it is added to an artificial accompaniment with the major third, would sound sentimental even if it were simply minor rather than semi-diminished. Therefore, it is clear that the fact that it is semi-diminished shall render it mournful. Bar five contains the accompaniment B b D F, to which one sees added the diminished seventh A b. I consider it redundant to discuss any further the melancholic character of the seventh that becomes diminished by the enharmonic lowering of the upper sound, as I discussed it in several parts of this treatise. Signor Marcello employs at bar three and bar nine the accompaniment D b F A b, which contains two semi-diminished sounds D b and A b. If we consider the previous accompaniments, they persuade us that it belongs naturally to the tone F with the minor third. However, if we consider the following passages, D b 5 b 3 B 6 5 3 and B 6 5 3 C 5 3 b, which derive from the fundamental ones D b 5 b 3 G 5 3 [sqb], G 5 3 [sqb] C 5 3 b, they lead us to believe [-1009-] that the accompaniment D b F A b is based on the artificial second D b of the tone C with the minor third. Now I consider both the tone F and the tone C together and as one with their derivative ones. Even if our accompaniment is ascribed to the tone F, it sounds nevertheless very emotionally charged because of the double note enharmonically lowered that occurs in it. In fact, if said accompaniment is heard as belonging to the tone C, it makes an impression of great sadness on the ear, which impression is produced mainly from the second semi-diminished artificial note D b. The tenderness and the affection of the second artificial note of the mode with the minor third that would occur if the diatonic temperament were assigned to it turns into sadness when said note is turned into semi-diminished.

I move now from considering the harmonic accompaniments to discuss some of the most notable melodic juxtapositions containing the flats A b and D b. The movement A b D of the semi-diminished fifth downwards is used often. Said altered interval, which is lower than the fifth by a major semitone employed as a minor semitone, ensures that the said sound A b makes the ear believe that the sound D is furnished with a little energy and that the sound D enhances the sadness of the sound A b. The contralto moves from the first to the second bar with the passage A b D, which sets the two syllables of the word *perchè*, which needed to end on a rather active note, as it represents a question. It is worth observing in the *basso continuo* the melodic contrast, so to speak, of D b and B in the third and in the penultimate bar, and of A b and F # in the seventh bar, which sounds form in pairs the interval of the more-than-diminished third. We need to point out also the movement A b E of the more-than-diminished fourth downwards performed by the first violetta in the first bar. Because of all the contrasts described above, the preceding semi-diminished sounds become more mournful, while the following artificial sounds become more active. The strengthening of the fourth artificial note F # at bar seven due to its comparison with the sixth note A b enharmonically lowered suits the expression of the questioning tone with which the alto sings the end of the versicle. Similarly, Signor Marcello places in the singing part the following series of notes that ascend gradually by semitone because [-1010-] of a very fervent question. The accompaniments C 5 3 v, C 5 3 [sqb], F 5 3 b, D 5 3 # and G 5 3 [sqb] correspond to the above series of notes in relation to its fundamental harmony. The sound F #, major third of D, sounds very powerful to the ear after the listener has heard the note A b, diminished third of F, in the part of the violetta and of the *basso continuo*.

The excellent passage with which the alto concludes its singing invites me to digress briefly.

The word abbandono cannot be expressed in a more vivid and better way. The descending leap of the voice from G to C through the note E b and the later descent from C to B represent perfectly the prostration in which a lost soul find itself. We have seen earlier (chapter 6. [signum] 1.) that the mere descent from the high to the low register contributes to the expression of weak emotions. The note C is held for a long stretch to indicate the duration of of the abandonment, while, at last, the composition concludes with a cadence from the fourth artificial note to the fifth one that leaves the ear suspended, which feels deprived of the perfect cadence that it desires so much. Here it is a vivid image of the thwarted hopes of a wretched abandoned man who is deprived of the succour that he desired so profoundly.

[signum] 17. The tone F with the minor third is heard as more mournful than the tone C not only because it admits two accompaniments with the semi-diminished third, F A b C, B b D b F, in its formation, which are based on the first and fourth note, while the tone C accepts only one, F A b C, based on the fourth note, but also because it borrows a greater number of accompaniments that are consonant or are employed as from its derivative tones. These accompaniments are consonant or are employed as such and contain the flats A b and B b either separately or at the same time. I place herewith the aforementioned accompaniments A b C E b, D b F A b and G B b D b. The five described accompaniments can be altered by the second, fourth, sixth and seventh artificial note, which are substituted to the corresponding natural ones. The tone F employs as second artificial note the semi-diminished sound G b. It is employed only in the accompaniment G b B b D b, which [-1011-] inspires great compassion. In the following accompaniments the artificially augmented notes B, fourth, D, sixth and E, seventh, are coupled with the flats in the following way: A b, D b, D F A b, A b C E, A D b F and G B D b. Such accompaniments contain a mixture of energy – thanks to the artificially raised sounds B, D and E – and of weakness – because of the enharmonically lowered sounds A b and D b. For this reason they are involved in complex emotions that partake of weak and strong feelings. Moreover, the sounds A b and D b can be added as dissonances to a great number of natural and artificial accompaniments borrowed by the tone F. A b forms a semi-diminished seventh with B b, a more-than-diminished seventh with B, a semi-diminished ninth with G, a ninth with G b, a lowered fourth with E b, a more-than-diminished fourth with E, and a semi-diminished sixth with C. D b forms a semi-diminished seventh with E b, a more-than-diminished seventh with E, a semi-diminished ninth with C, a fourth with A b and, finally, a semi-diminished sixth with F. While I illustrate the tone F with examples, I shall make the appropriate observation on the consonant and dissonant accompaniments and also on the melodic passages containing the notes A b, D b and G b, of which the first two are natural and the last one is altered.

[signum] 18. Signora Contessa Donna Teresa Agnesi starts with the extract quoted in the first example the second part of the first aria contained in the cantata for solo voice and instruments entitled Fra voi taciti boschi, which I mentioned several times. In this second part the sadness of the tone F with the minor third is very prominent because the first part is written in the tone F with the major third. We encounter immediately the tearful accompaniment F A b C, which is followed by another one, namely, F A b D b, employed as belonging to the dependent tone G, as it is indicated by the passage F 6 b 3 b F 6 [sqb] 4 [sqb] 2, which is derived from the cadence D b 5 b 3 G 5 [sqb] 3 [sqb] belonging to the said tone through which the music moves from the third to the fourth bar. The accompaniment D b F A b is sad because it is based on the fifth semi-d note D b of the derivative tone G. The enharmonically lowered sounds A b and D b contained in the mentioned accompaniment allow the artificial notes B, fourth, and D, sixth of the tone F with the minor third to sound, which constitute the major third and the fifth of the following accompaniment G B D, very energetic. [-1012-] The *basso continuo* moves from the sixth to the seventh bar in a way that evokes great compassion, by moving from D to D b, or, from the major third to the semi-diminished third in relation to the fundamental bass B b. It is clear that the immediate comparison with the major third highlights fully the weakness of the semi-diminished third. At bar seven the first violin moves from B b to B, which passage, already resentful by its own nature, is heard as even more powerful because of the harmonic contrast between B and D b, which is played by the *basso continuo*, which forms a more-than-augmented sixth with B. The movement of the more-rhan-diminished fourth

downwards A b E in the aforementioned *basso continuo*

[Riccati, The laws of counterpoint, 1012; text: Adagio, Col Basso, Non lasciarmi in abbandono senza te perduta io sono pace il cor trovar non sa, et cetera, 6 b 3, 5, 4 [sqb] 2, 6, 6 3, 6 [sqb], 3 [sqb], 6 5 3, 6 4, 5 3 [sqb], 6 4 [sqb] 3 b, 6 3 b]

[-1013-] from the eighth to the ninth bar deserves to be highlighted. The sadness of the previous sound A b, semi-diminished third of F, confers greater strength to the seventh artificial note E, major third of C. At the end of the mentioned bar nine, the semi-diminished sixth A b is prepared with the mournful accompaniment F A b C. A b together with the fourth F is added as a dissonance to the following accompaniment C E G with the major third. I have to repeat here what I said about the semi-diminished sixth in the previous paragraph in similar circumstances. The soprano sings the note D b at the beginning of bar eleven [ten ante corr.], which note sounds melancholic not only because of the memory of the sound B played a little earlier by the first violin, but also because it is added enharmonically to the sounds E and B b that concur to the formation of the consonant accompaniment by representation E G B b, with which it forms a more-than-diminished seventh and a semi-diminished third. The soprano then sings the passage D b E of the more-than-augmented second upwards and it ensures that the sixth note D b, which is enharmonically lowered, allows the energy of the artificial note to be even more prominent. Bar twelve consists of the two accompaniments A b C F, B b D b G. They are employed as belonging to the derivative tone C, in relation to which the fundamental passage F 5 3 b G 5 b 3 b moves from the fourth to the fifth note. The perfection of the system of the melody C, F, G of the aforementioned tone demonstrates clearly that the semi-diminished fifth in the accompaniment G B b D b sung by the soprano has to sound very tearful. The mixed character of the adduced example, now sad, now resentful, is very appropriate to the state of a lover who, afflicted and agitated, searches for the dear object of her love by whom she fears to have been abandoned.

[signum] 19. The poetry of the second example, taken from the duet Troppo dura è la mia sorte by Monsignore Steffani required the famous composer to adopt a very melancholic style.

Passo i giorni, e gli anni intanto

Fra dolori, affanni, e stenti.

The accompaniments with the semi-diminished third F A b C and B b D b F are employed frequently, the latter as belonging

[-1014-] [Riccati, The laws of counterpoint, 1014; text: Passo i giorni e gli anni intanto fra dolori affanni e stenti, et cetera, 3 b, 9 b 3 [sqb], 8, 5 3, 8 3 [sqb], 6 b 3, 6 3, 6 [sqb] 4 3, 9 3 [sqb], 4 9, 3 [sqb] 8, 4 8, 3 [sqn] 7, 6 [sqb] 3 b, 6 [sqb] 4 3, 9 b 4, 8 4, 7 3 [sqb], 6 b, 8 6 b, 7 5, 5 3 [sqb], 5 3, 6 4, 7 5 3 [sqb], 5 4, 3 [sqb]]

[-1015-] to the tone F, and the former one as belonging sometimes to the tone of F and sometimes to the tone of C, both of them with the minor third. All the melodic passages that move from one of the sounds of the preceding accompaniment to the semi-diminished third A b or D b of the aforementioned chords F A b C and B b D b F sound sad. The movements C D b in the first bar (from C fifth of F to D b semi-diminished third of B b) and C A b in the third bar (from C octave of the chord C 5 3 to A b semi-diminished third of F) both performed by the soprano, belong to that category. Monsignore Steffani employs twice, at bar ten and twelve, the accompaniment A b C E b as belonging to the tone D derived from C with the minor third, as one can see illustrated clearly in the passages A b 5 b 3 G 5 3 [sqb] and G 5 3 [sqb] A b 5 b 3, which move from the fifth note to the fourth and from the fourth to the fifth in relation to the aforementioned tone D. I invite the Reader to consider what I wrote in the previous paragraph on the subject of the sadness of the chord A b C E b, when it is employed in the aforementioned role. The passage C 5 3 [sqb] D b of the first bar is similar to the passage G 5 3 [sqb] A b. The first one belongs to the tone F and the second one to the tone C, both with the minor third and considered together and as one body with their derivative

ones. The melodic passages that move from one sound of the preceding accompaniment towards the base A b and D b of the chords A b 5 b 3 and D b 5 b 3 employed in the way described above sound tearful. The fundamental passages C D b, C A b and G A b of the *basso continuo* at bar one, ten and twelve enjoy this privilege. The accompaniment D b F A b is employed another three times in its derivative form F A b D b. The preceding chords allow it to be heard as naturally belonging to the tone F in the second and in the eighth bar, but the consequent accompaniments contradict this, so that it is considered based on the second artificial note of the tone C in relation to them. At bar nine the preceding and consequent chords demonstrate that the accompaniment F A b D b belongs artificially to the tone C with the minor third. The accompaniment G b B b D b, which evokes an extraordinary degree of compassion, is employed as based on the second artificial semi-diminished note G b of the tone F with the minor third in [-1016-] the fifth bar. It inspires such compassion because the composer modulates through it from the tone C to the tone F, both with the minor third, employing the passage C 5 3 b B b 6 b 3 b, which one encounters very rarely in music. If the natural flat D b sounds tearful, how much more mournful the artificial sound G b will sound, to which the contralto moves with the extremely emotional passage of the semi-diminished fifth upwards C G b.

Let us follow the consonant accompaniments with the dissonant ones that contain the flats that we are discussing. The sound D b is added as a semi-diminished ninth to the accompaniment C E G, which belongs artificially to the tone F, once in the first bar and twice at bar seven.

Replicatamente nella Battuta quarta s'accoppia la Nona semidiminuita A b coll'accompagnamento artificiale G B D competente al Tuono C. Riuscendo patetica la Nona giusta, massimamente quando sia per Terza maggiore, ella è cosa chiara che nel nostro caso le Nona semidiminuite D b, A b si sentiranno lagrimevoli, andando congiunte cogli accompagnamenti per Terza maggiore C E G, G B D, le cui basi C, G, non patiscono alterazione.

[signum] 20. Innoltrandoci dal Tuono F al Tuono B b chiamati per Terza minore, incontriamo in maggior tristezza; e perchè trae la sua origine dai tre accompagnamenti per Terza semidiminuita F A b C, B b b D b F, E b G b B b; e perchè in tutti quattro gli accordi fondamentali A b C E b, D b F A b, G b B b D b, C E b G b, che prende in prestanza dai Tuoni derivati, c'entrano i B molli enarmonicamente calanti A b, D b, G b. Si acciungano a questi gli accompagnamenti artificiali, che ammettono i nostri B molli. Il Tuono B b accetta come sua seconda corda artificiale il suono semidiminuito C b, il quale ha luogo nel solo accompagnamento assai mesto C b E b G b. Le corde artificiali accresciute quarta E, sesta G, settima A s'uniscono coi B molli A b, D b, G b [-1017-] nei sottoposti accompagnamenti, che nel tempo stesso riescono veementi per motivo dei suoni artificiali aumentati, compassionevoli per motivo dei suoni decrescenti per un elemento enarmonico: G B b D b, D b F A, E G b B b, C E G b.

Oltre a ciò i B molli, di cui si tratta, A b, D b, G b s'accoppiano frequentemente in figura di dissonanze con varj dei nominati accompagnamenti parte naturali, e parte artificiali. A b corrisponde in Settima semidiminuita a B b, in Nona a G b, in Nona semidiminuita a G, in Quarta calante ed E b, in Quarta più che diminuita ad E, in Sesta diminuita a C, in Sesta maggiore a C b. Compone D b con E b Settima semidiminuita, con E Settima più che diminuita, con C nona semidiminuita, con C b Nona maggiore, con A b Quarta, con A Quarta più che diminuita, con F Sesta semidiminuita. Finalmente G b si riferisce ad A b in Settima minore, ad A in Settima più che diminuita, ad F in Nona semidiminuita, a D b in Quarta, a B b in Sesta semidiminuita. Le riflessioni, che anderò facendo sopra i seguenti esempj, riguarderanno principalmente gli accompagnamenti consonanti, e dissonanti, ed i passi di melodia, nei quali hanno posto le corde semidiminuite del Tuono B b, cioè a dire le naturali A b, D b, G b, e l'artificiale C b.

[signum] 21. Prendo i sue sottoposti esempj dalla Stabat Mater del Signor Giambatista Pergolese. Il primo è formato da due andamenti simili nei Tuoni F, B b chiamati per Terza minore. Tutti gli accompagnamenti adoprati sono di Terza semidiminuita e Quinta, cioè a dire compassionevoli, eccettuati due C 7 b 5 3 [sqb], F 7 b 5 3 [sqb] introdotti in grazia delle Cadenze, i quali per altro la Settima minore aggiunta fa divenire patetici. Le Appoggiature, D b che discende

in C, G b che discende in F, sono Seste diminuite dissonanti, che s'uniscono, quella coll'accompagnamento F 5 3 b, questa coll'accompagnamento B b 5 3 b. Meritano bene d'essere osservati relativamente al Tuono F i suoni G b E toccati nello stesso tempo cattivo di spezzatura dal Basso continuo, e dalla Violetta. I detti suoni, che non appartengono all'armonia fondamentale, assumono il titolo in riguardo al Tuono F [-1018-] di corde artificiali seconda, e settima. Ora la seconda corda artificiale G b, che si usa solamente nell'accompagnamento G b B b D b e ne; suoni derivati, non si unisce mai armonicamente colla settima corda artificiale E.

[Riccati, The laws of counterpoint, 1018; text: Fac me vere tecum flere Crucifixo condolere, et cetera, 5 3, 6 b 4, 7 5 3 [sqb], 5 3 b, 7 5 b, 3 [sqb], 6

[-1019-] Per salvare adunque il passo del nostro Autore, G b non va considerata come seconda corda artificiale armonica del Tuono F, ma piuttosto come una di quelle voci alterate, delle quali ho fatto (chapter 6. [signum] 20.) distintamente parola. S'introduce essa, acciò che il susseguente movimento G b F del basso continuo, imiti l'antecedente D b C del Soprano. In altra maniera altresì si può l'accennato passo giustificare, supponendo l'accompagnamento F 5 3 b, che immediatamente precede e seguita i suoni G b E, proprio de Tuono B b per Terza semidiminuita, ed attribuendo i predetti suoni al Tuono medesimo, relativamente a cui hanno il nome di sesta corda naturale, di quarta o sia undecima corda artificiale, le quali si congiungono insieme ne' due accompagnamenti E G b B b, C E G b, e nei loro derivati. Il suono G b s'ode sommamente lagrimevole pel confronto del suono E, che nello stesso tempo fa sentire la Violetta, il quale si riferisce a G b nell'intervallo alterato di Sesta più che superflua, ch'è divenuta tale per lo scemamento di G b. Delle consimili riflessioni si facciano intorno le voci C b A, che nel medesimo tempo cattivo di spezzatura si toccano dal Basso continuo e dalla Violetta nelle Battute quarta, e quinta. La voce C b dee considerarsi o come un suono alterato usato nel Tuono B b, o come sesta corda naturale del Tuono E b, a cui la modulazione si trasferisca. I molti B molli enarmonicamente minorati, i quali con dolente aspetto pervengono sempre all'orecchio, sono grandemente adattati alla espressione delle parole Fac me vere tecum flere,

Crucifixo condolere

Il secondo esempio svaga per varj Tuoni. Io farò delle avvertenze sopra que' luoghi, che appartengono al Tuono B b per Terza semidiminuita. La serie degli accompagnamenti D b 6 b 3 E b 6 3 b F 5 3 contenuti nella Battuta quarta, a cui corrisponde il basso fondamentale B b 5 3 b C 5 b 3 b F 5 3, dinota che l'accompagnamento C E b G b di Terza minore e Quinta semidiminuita s'usa come proprio del Tuono F derivato dal Tuono B b, di cui facciamo parola. Essendo giusto nel detto Tuono [-1020-] F il Sistema di melodia F, B b, C; egli è chiaro che G b Quinta semidiminuita di C ha da riuscir lamentevole. Nella battuta settima si fa transito dal Tuono A b al Tuono B b, 'uno detto per Terza maggiore, e l'altro per Terza

[Riccati, The laws of counterpoint, 1020; text: Violini unisoni, Largo assai, Quando corpus morietur fac ut animae donetur Paradisi gloria, et cetera, Violetta, e Basso, 7 5 4, 3 [sqb], 11 9, 7, 6 4, 5 3 [sqb], 6, 5, 3 b, 6 3 b, 7 3 [sqb], 3 b, 4 b 2, 7 b, 7 b 5 3, 7 5 3 [sqb], 6 b 4, 5 4]

[-1021-] minore, le scale dei quali differiscono in ciò, che quella del primo Tuono si serve di G, e duella del secondo di G b. Ora un tal suono appunto toccato dal basso continuo ci ammonisce della predetta mutazione dei Tuono, e per questo motivo s'ode più mesto del consueto. Il versetto, di cui ho trascritta una porzione, contiene in se stesso un misto di lugubre, e di supplichevole, ed è valevole a risvegliare negli animi la compunzione.

[signum] 22. Al Tuono B b mentovato nell'antecedente Paragrafo, continuando il cammino verso i B molli, succedono ordinatamente i Tuoni parimente detti per Terza minore E b, A b, D b et cetera, fra i quali solamente il primo è fondato sopra un tasto, che ritiene il suo vero nome. Non così gli altri, a cui servono di base i tasti, che propriamente si denominano G #, C # et cetera. Per un tale motivo io mi ristignerò a fare alquante riflessioni intorno al solo Tuono E b. Il Sistema di melodia del Tuono per Terza minore, che ha per base il tasto nero medio fra D ed E, si può esprimere in due differenti maniere, cioè E b A b B b, D # G # A #. La prima maniera fa comparire giuste le corde prima E b, quinta B b, e calante la quarta A b. La seconda maniera ci vorrebbe persuadere, essere

crescenti le corde prima D #, quinta A #, e giusta la quarta corda G #. L'orecchio frattanto crederà giuste le corde di maggiore importanza prima e quinta, e decrescente la quarta corda men principale; piuttosto che giudicare esatta questa, e quelle aumentate. Si cavi la conseguenza, che il nominato Sistema di melodia, come pure il Tuono fondato sulla prima corda d'esso Sistema amano d'essere dinotati con lettere dal B mollemodificate. Ascendendo per una Quarta, ci si presenta il tasto nero fra G ed A, al quale corrisponde un Sistema di melodia, che riceve la doppia espressione A b, D b, E b; G #, C #, D #. L'espressione per B molle dimostra calanti le corde prima e quarta, ed intatta la quinta corda. Tutto all'opposto l'espressione con Diesis indica giuste le corde prima e quarta, e crescente la quinta. Essendo le corde prima e quarta superiori alla quinta e per numero, e per dignità, atteso che la prima corda è più perfetta della quinta; predilige il sensorio l'espressione per Diesis: il che molto [-1022-] più accaderà seguitando a montare per Quarte, ed inoltrandoci alla considerazione dei Tuoni per Terza minore D b, C #; G b, F # et cetera, i quali congruamente si segnano coi Diesis, e forzatamente coi B molli.

Alle due corde esatte prima E b, quinta B b del Sistema di melodia del Tuono E b per Terza minore corrispondono i tristi accompagnamenti di Terza semidiminuita e Quinta. La quarta corda decrescente A b C b E b di Terza minore e quinta crescente. Riesce questo assai mesto; perchè l'orecchio capisce, che l'aumentazione della Quinta A b E b, e la semisuperfluità della Terza C b E b traggono l'origine dallo scemamento dei suoni A b, C b. Ai [A ante corr.] tre descritti principali accompagnamenti s'aggiungano i quattro sottoposti, che il nostro Tuono E b prende il prestantza dai Tuoni derivati, [[G b B b D b, C b E b G b]] D b F A b, G b B b D b, C b E b G b, F A b C b, in ognuno dei quali hanno luogo due B molli enarmonicamente diminuiti. Appartengono altresì ma in qualità di artificiali, al Tuono E b gli accompagnamenti F b A b C b, che ha per base la seconda corda artificiale F b, e consta di tre voci semitiminuite; D F A b, G b B b D, C E b G b, A b C E b, A C b E b, F A C b, nei quali le corde artificiali quarta A, sesta C, e settima D s'accoppiano con uno di que' B molli, di cui presentemente si tratta. I sei ultimi accompagnamenti hanno un carattere, che partecipa del risentito per cagione delle corde artificiali aumentate quarta, sesta, e settima, e del malinconico per cagione dei B molli decrescenti per un elemento enarmonico. Finalmente i suoni A b, D b, G b, C b possono venire ad uso in figura di dissonanze. A b è Settima semidiminuita di B b, Nona di G b, Nona semidiminuita di G, Quarta calante di E b, Sesta maggiore di C b, Sesta semidiminuita di C. D b forma Settima semidiminuita con E b, Nona con C b, Nona semidiminuita con C, Quarta con A b, Quarta più che diminuita con A, Sesta semidiminuita con F, Sesta maggiore con F b. Corrisponde G b in Settima minore ad A b, in Settima più che diminuita ad A, in Nona semidiminuita ad F, in Quarta a D b, in Quarta più che diminuita a D, in Sesta semidiminuita a B b. Per ultimo C b si riferisce a D b in Settima minore, a D [-1023-] in Settima più che diminuita, a B b in Nona semidiminuita, a G b in Quarta, ad E b in Sesta semidiminuita.

[signum] 23. La Cantata Al fin m'ucciderete o miei pensieri del Signore Scarlat<t>i il vecchio mi somministra per primo esempio una porzione di Recitativo sommamente patetica, la quale si raggira sempre per Tuoni che richiedono molti B molli. Vi scoprirà il Lettore i Tuoni A b per Terza maggiore, C, F, B b, E b per Terza minore. All'ultimo di questi Tuoni, cioè a dire ad E b, che cade presentemente sotto la nostra considerazione, [Riccati, The laws of counterpoint, 1023; text: da me lontana è Clori l'idono mio si rammentasse o Dio de' miei costanti amori con un sospiro almeno figlio del suo bel seno s'oncontrassero un giorno i sospiri miei a me che penso a lei e tante volte e tante ah se Clori pensasse in questo istante, et cetera, 7 b, 6 b, 6 [sqb], 6, 3 b, 6 4 [sqb] 2, 7 b 5 b, 5 [sqb] 3 b, 4 b 2 b, 5 3 [sqb], 6 [sqb] 4 3, 4 [sqb], 3 [sqb]]

[-1024-] appartiene la Battuta settima. Nell'atto che la cantilena si trasferisce dal Tuono B b all'E b, si pone immantenente inopera l'accompagnamento D F A b C b di Terza minore, Quinta [semidiminuita add. supra lin.], e Settima più che diminuita. Riesce molto flebile nel pincipio della settima Battuta il passo B b C b, che muove il Soprano da B b Ottava dell'accompagnamento B b 5 3 b a C b Settima più che diminuita dell'accompagnamento D 7 b 5 b 3. La voce C b perviene all'orecchio compassionevole; perchè produce cangiamento di Tuono, richiedendo il Tuono

antecedente B b per Terza minore la voce C; e perchè è Settima più che diminuita di D settima corda artificiale del Tuono E b per Terza minore. L'aumentazione della settima corda artificiale D serve a dar comparire vie più languida la sesta corda semidiminuita C b. Al passaggio B b 5 3 b D 7 b 5 b 3 segna l'altro D 7 b 5 b 3 E b 5 b 4 b, spezie di Cadenza che si trasferisce dalla settima corda artificiale all'ottava. Dell'accompagnamento E b 5 b 3 b canta il Soprano la Terza semidiminuita G b. Al detto accordo E b 5 b 3 b s'aggiunge poscia la Settima semidiminuita D b, che si colloca nel Basso continuo, e si tocca parimente dal Cantante nel principio della ottava Battuta, ascendendo ad essa col movimento malinconico B b D b. E qui finisce il mio esame, tornando il Signore Scarlatini al Tuono B b per Terza minore, conforme viene manifestamente indicato dal susseguente accompagnamento C 6 3 b al mentovato Tuono artificialmente spettante.

[signum] 24. mi viene suggerito il secondo esempio da un bellissimo versetto del Salmo undecimo del Signor Benedetto Marcello. Il Tuono E b per Terza minore non esprime solamente la miseria dei bisognosi, ed il pianto dei poveri, perchè mesto di sua natura; ma ancora perchè il confronto col Tuono E b per Terza maggiore, da cui viene preso in mezzo, rende più manifesta una tale mestizia. Si principia con un tempo allegro nel Tuono E b per Terza maggiore. Il tempo allegro ed il modo per Terza maggiore sono confacenti per esprimere le parole di Dio Io dic'egli sorgerò. Dopo ciò affine di rappresentare lo stato miserabile, ed il pianto dei poveri abbandonati, ed afflitti, si muta il tempo in adagio assai, e considerando l'accompagnamento B b 5 3 comune ai due Tuoni B b, E b per Terza maggiore come proprio di quest'ultimo [-1025-]

[Riccati, The laws of counterpoint, 1025; text: Allegro, Io dic'egli sorgerò mosso a pietà dello stato miserabile del pianto in cui si struggono i miei servi afflitti e poveri e soccorso a lor darò, 6, 5, 7, 6 b 4, 5 b 3, 6 b, 7 [sqb] 3 [sqb], 4 b, 3 b, 7 b 3 b, 6 b, 6 b 5 b 3 b, 5 b 4 b, 7 b 5 b 3 b]

[-1025-] Tuono, si fa da esso transito al Tuono E b per Terza minore. S'ode dolente assai il passo melodico B b G b di Sesta semidiminuita all'insù da B b corda fondamentale dell'accompagnamento B b 5 3 a G b Terza semidiminuita dell'accompagnamento E b 6 b 4 b, a cui G b corrisponde in Sesta semidiminuita; e quindi il nostro intervallo in doppia guisa si sottopone al giudizio del senso, e come melodia, e come armonia. All'accompagnamento [-1026-] B b 6 b 4 b succedono nella battuta sesta i due C b 5 b 3 b C b 6 b 3 b, quello preso in prestanza dai Tuoni derivati, e questo strettamente proprio del Tuono E b per Terza minore. nulla servendo i passi fondamentali di Terza all'ingiù E b 5 b 3 b C b 5 b 3 b, C b 5 b 3 b A b 5 b 3 b, da cui dipendono gli usati dal Basso continuo B b 6 b 4 b C b 5 b 3 b, C b 5 b 3 b C b 6 b 3 b, per determinare al qual Tuono derivato appartenga l'accompagnamento C b 5 b 3 b, seguita esso la natura delle corde sesta C b, terza o sia decima G b dal Tuono E b per Terza minore, che ne formano la base, e la Quinta. Nel Modo per Terza minore le corde terza, e sesta calano per un Semituono minore, e si sperimentano patetiche. Nel Tuono E b, di cui parliamo, patiscono in oltre una diminuzione enarmonica, e perciò riescono lamentevoli. Le cose dette c'insegnano, che dee sentirsi assai dolente il movimento derivato di melodia B b 6 b 4 b C b 5 b 4 b, col quale si trasferisce il Basso continuo dala quinta alla sesta battuta. Gittando l'occhio sulla battuta settima, si vede primieramente aggiunta all'accomagnamento artificiale per Terza maggiore B b 5 3 la Settima semidiminuita A b. Lo stesso suono diviene poscia Quarta calante di E b, che si risolve nella Terza semidiminuita G b. le nominate voci meste si pongono in bocca dell'Alto, affinchè il suo canto risvegli negli animi la compassione. I primi tre quarti della Battuta ottava spettano al Tuono subordinato C b detto per Terza maggiore. La scala del Tuono principale E b per Terza minore contiene il suono F, che nella scala del nominato Tuono C b riceve la modificazione d'uno de' nostri B molli. Questa voce modificata, che riesce assai aptetica, si fa tosto sentire risolvendo in essa la Nona G b unita all'accomagnamento F b 5 b 3 b, la cui Terza maggiore A b 6 b 3 b si colloca nel Basso continuo. Trasformasi poi F b prima in Settima minore di G b resa più trista dal paragone con B b toccato dal Basso continuo, a cui si riferisce in Quinta [[più che add. supra lin.]] semidiminuita; ed indi in Quarta di C b. Una tal dissonanza viene risolta dal Signor Marcello col discendere in E b Terza semisuperflua di C b. Il Tuono C b comunemente cchiamato per Terza maggiore si conta nel numero di quelli, che vogliono usarsi in figura di [-1027-] subordinati, e non di principali. Il tasto

bianco immediatamente inferiore a C si denomina propriamente B, ed impropriamente C b. Se il detto tasto s'assemblerà per fondamento ad un Tuono principale per Terza maggiore, verrà questo giudicato piuttosto B che C b; imperciocchè la scala di B contiene cinque suoni giusti, e fra dessi que' tre, che compongono il Sistema di melodia, e due soli suoni crescenti per un elemento enarmonico; laddove la scala di C b è formata da cinque scuoni scemati enarmonicamente comprendenti i tre, onde consta il Sistema di melodia, e da due soli suoni giusti. Termina il tempo Adagio assai coi due accompagnamenti A 7 b 5 b 3b di Quinta [[diminuita]] [minore add. supra lin.], Terza, e Settima più che diminuite, B b 5 3 di Terza maggior e Quinta, a cui s'aggiunge la Quarta, che in essa Terza risolvesi. Si fatti accompagnamenti convengono al Tuono E b per Terza minore, e formano quella Cadenza, che si muove dalla quarta corda artificiale verso la quinta. Il suono A toccato dal Basso continuo s'ode straordinariamente risentito, e pel passo fondamentale melodico C b A di Terza più che diminuita all'ingiù mosso dalla Parte stessa, e perchè è quarta corda artificiale, e perchè col suo mezzo si modula dal Tuono C b per Terza maggiore al Tuono E b per Terza minore. La veemenza della corda A infievolisce maggiormente la voce E b cantata dall'Alto, e la rende atta alla espressione della parola afflitti.

Posta in esecuzione la descritta Cadenza, si restituisce la cantilena al Tuono E b per Terza maggiore, etorna a camminare velocemente, così richiedendo il sentimento e soccorso a lor darò.

[signum] 25. Ed eccomi giunto al termine del mio lugno lavoro, che mi ha costato l'applicazione di molti e molti anni. Io certo ho procurato di non tralasciare cos'alcuna, che sia di qualche importanza; ingegnandomi di porla nel suo vero lume, e di renderne quelle ragioni, che mostrino il perfetto accordo fra la pratica, e la teorica. Come io ci sia riuscito, ne lascio ai dotti e discreti Lettori il giudizio. At bar four the semi-diminished ninth A b is added on two occasions to the artificial accompaniment G B D that belongs to the tone C. Since the perfect ninth sounds emotional, especially if it is with the major third, it is clear that in our case the semi-diminished ninth D b and A b shall sound tearful, as they are added to the accompaniments with the major third C E B and G B D, whose bases C and F are not altered.

[signum] 20. If we move on from the tone F to the tone B b with the minor third, we encounter even greater sadness because the tone B b with the minor third originates from the three accompaniments with the semi-diminished third F A b C, B b D b F and E b G b B b and because the enharmonically lowered flats A b D b and G b are contained in all four the fundamental chords A b C E b, D b F A b, G b B b D and C E b G b, which are borrowed from the derivative tones. Let us add to these the artificial accompaniments that admit our flats. The tone B b accepts as its second artificial note the semi-diminished sound C b, which occurs only in the very sad accompaniment C b E b G b. The artificially raised notes E, fourth, G, sixth, and A, seventh, are added to the flats A b, D b, G b [-1017-] in the following accompaniments, which prove vehement because of the raising of the artificial sounds and inspire compassion because of the sounds that are lowered by an enharmonic element. Such accompaniments are G B b D b, D b F A, E G b B b and C E G b.

Apart from this, the flats that are are discussing, namely, A b, D b and G b are added often as dissonances with several of the accompaniments named above, some of whom are natural and some artificial. A b forms a semi-diminished seventh with B b, a ninth with G b, a semi-diminished ninth with G, a lowered fourth with E b, a more-than-diminished with E, a diminished sixth with C and a major sixth with C b. D b forms with E b a semi-diminished seventh, with E a more-than-diminished seventh, with c a semi-diminished ninth, with C b a major ninth, with A b a fourth, with A a more-than-diminished fourth and with F a semi-diminished sixth. Finally, G b forms a minor seventh with A b, a more-than-diminished seventh with A, a semi-diminished ninth with F, a fourth with D b and a semi-diminished sixth with B b. The observations that I shall make on the following examples shall concern mainly the consonant and dissonant accompaniments as well as the melodic passages containing the semi-diminished notes of the tone B b, namely, the natural ones A b, D b, G b and the artificial one C b.

[signum] 21. I take the examples placed herewith from Signor Giambatista Pergolese's Stabat Mater. The first one consists of two similar movements in the tones F and B b with the minor third. All the accompaniments employed are of the semi-diminished third and fifth and inspire

compassion, except for the two accompaniments C 7 b 5 3 [sqb] and F 7 b 5 3 [sqb], introduced thanks to the cadences, which are rendered also emotionally charged because of the addition of the minor seventh. The appoggiaturas D b, descending onto C, and G b, descending onto F, are dissonant diminished sixths. The former is added to the accompaniment F 5 3 b, while the latter is added to the accompaniment B b 5 3. The sounds G b and E, played on a weak subdivision of the beat by the *basso continuo* and by the viola, need to be considered in relation to the tone F. Said sounds, which do not belong to the fundamental harmony, take the name of second and seventh artificial note in relation to the tone F. [-1018-] Now, the second artificial note G b, which occurs only in the accompaniment G b B b D b and in its derivative ones, is never added enharmonically to the seventh artificial note E.

[Riccati, The laws of counterpoint, 1018; text: Fac me vere tecum flere Crucifixo condolere, et cetera, 5 3, 6 b 4, 7 5 3 [sqb], 5 3 b, 7 5 b, 3 [sqb], 6

[-1019-] Therefore, in order to justify the passage of our author, G b must not be considered as the second note of the tone F enharmonically altered, but as one of those altered notes that I mentioned earlier (chapter 6., [signum] 20.) in detail. This note is introduced so that the following movement G b F of the *basso continuo* imitates the preceding one D b C of the soprano. The aforementioned passage can also be explained in a different way if imply that the accompaniment F 5 3 b, which belongs to the tone B b with the semi-d third, precedes and follows immediately the sounds G b E and if we ascribe the aforementioned sounds to the that same tone, in which they are called natural sixth note and artificial fourth or eleventh note and they unite in the two accompaniments E G b B b and C E G b, and in their derivative ones. The sound G b sounds profoundly tearful in comparison with the sound E played at the same time by the viola, which forms with G b the altered interval of the more-than-augmented sixth, itself produced by the lowering of the note G b. Similar considerations have to be made in relation to the notes C b and A, which are played by the *basso continuo* and by the viola at bar four and five. The note C b must be considered as an altered sound employed in the tone B b or as sixth natural note of the tone E b, to which the modulation moves. The great number of enharmonically lowered flats that evoke pain in imagination of the listener are very apt to depict the expression of the words

Fac me vere tecum flere,

Crucifixo condolere.

The second example wanders across various tones. I shall make some observation on the passages that belong to the tone B b with the semi-diminished third. The series of the accompaniments D b 6 b 3 E b 6 3 b F 5 3 contained within bar four to which the fundamental bass B b 5 3 b C 5 b 3 b F 5 3 corresponds indicates that the accompaniment C E b G b of the minor third and semi-diminished fifth is employed as belonging to the tone F, derived from the tone B b that we are discussing. Since the system of the melody F, B b, C in the aforementioned tone [-1020-] is perfect, it is clear that G b, semi-diminished fifth of C has to sound mournful. At bar seven the music modulates from the tone A b to the tone B b, the former with the major third and the other one with the

[Riccati, The laws of counterpoint, 1020; text: Violini unisoni, Largo assai, Quando corpus morietur fac ut animae donetur Paradisi gloria, et cetera, Violetta, e Basso, 7 5 4, 3 [sqb], 11 9, 7, 6 4, 5 3 [sqb], 6, 5, 3 b, 6 3 b, 7 3 [sqb], 3 b, 4 b 2, 7 b, 7 b 5 3, 7 5 3 [sqb], 6 b 4, 5 4]

[-1021-] minor third, whose scale differ in the fact that the one of the first tone employs the note G and the one of the second tone employs the note G b. Now, that sound played by the *basso continuo* warns us of the aforementioned modulation, and for this reason it sounds more sad than usual. The versicle from which the extract is taken contains a mixture of lugubrious and imploring, and it is capable to evoke in the listeners the feeling of compunction.

[signum] 22. The B b mentioned in the previous paragraph is followed in order, if we continue towards the flats, by the tones E b, A b and D b etcetera, also called with the minor third.

Only the first of these is based on a note or key that retains its true name. This does not occur in the case of the others, which are based on keys that are called G #, C # etcetera with their appropriate name. For this reason, I shall limit myself to make several observations about the sole tone E b. the system of the melody of the tone with the minor third base on the black key located between D and E can be expressed in two different ways, namely, as E b A b B b or D # G # A #. The first way presents the first note E b and the fifth one as perfect, while the fourth one A b is lowered. The second way would like to convince us that the first note D # and the fifth one A # are raised, while the fourth note G # is lowered. However, the ear shall believe perfect the notes of greater importance, namely, the first and the fifth one, while it shall believe that the fourth note is lowered, as it is a less important note instead of considering perfect the last one and raised the first two. Consequently, the aforementioned system of the melody, just as the tone based on the first note of said system, needs to be indicated with the letters altered with the flat sign. If we ascend by a fourth we encounter the black key between G and A. The system of the melody that corresponds to it can be expressed in two ways, as A b D b E b or as G # C # D #. The way of indicating it through the flats shows the first and fourth note as lowered, while retains perfect the fifth one. Conversely, the way of indicating it through the sharps indicates the first and fourth note as perfect and the fifth one as raised. Since the first and fourth note are superior to the mere fifth in number and importance, as the first note is more perfect than the fifth, the ear prefers them to be indicated through the sharps. This shall me much more [-1022-] important if we continue to ascend in fourths and if we venture further in considering the tones with the minor third D b, C #, G b, F # etcetera, which are written more appropriately with the sharps and more awkwardly with the flats.

The mournful accompaniments of the semi-diminished third and fifth correspond to the two perfect notes E b, the first one, and B b, the fifth, of the system of the melody of the tone E b with the minor third. This tone proves very sad because the ear understands that the raising of the fifth A b E b and of the semi-augmented character of the third C b E b derive from the lowering of the sounds A b and C b. The following four accompaniments, [[G b B b D b, C b E b G b]] D b F A b, G b B b D b, C b E b G b and F A b C b, borrowed by our tone from its derivative ones and in each of whom we find two enharmonically lowered flats, are added to the three principal ones described earlier. The following accompaniments, F b A b C b (based on the second altered note F b and consists of three semi-diminished notes) and D F A b, G b B b D, C E b G b, A b C E b, A C b E b, F A C b (in which the three altered notes A, fourth, C, sixth, and D, seventh, are coupled with one of the flats that we are discussing at present) belong to the tone E b, but as altered accompaniments. The last sixth accompaniments have a somewhat resentful character because of the raised altered note (the fourth, the sixth and the seventh) and also a melancholic nature, because of the flats that are lowered by an enharmonic element. Lastly, the sounds A b, D b, G b, C b can be employed as dissonances. A b is the semi-diminished seventh of B b, ninth of G b, semi-diminished ninth of G, lowered fourth of E b, major sixth of C b and semi-diminished sixth of C. D b forms a semi-diminished seventh with E b, a ninth with C b, a semi-diminished ninth with C, a fourth with A b, a more-than-diminished fourth with A, a semi-diminished sixth with F and a major sixth with F b. G b forms a minor seventh with A b, a more-than-diminished seventh with A, a semi-diminished ninth with F, a fourth with D b, a more-than-diminished fourth with D and a semi-diminished sixth with B b. Lastly, C b forms a minor seventh with D b, a more-than-diminished seventh with D, [-1023-] a semi-diminished ninth with B b, a fourth with G b and a semi-diminished sixth with E b.

[signum] 23. The cantata beginning with the words Al fin m'ucciderete o miei pensieri by Signore Signore Scarlat<t>i senior provides me with the first example which consists in a section of extremely emotional recitative wondering continuously across tones that require several flats. The Reader shall find in it the tones A b with the major third, C, F, B b and E b with the minor third. Bar seven centres on the last of these tones, E b, which we are considering at present.

[Riccati, The laws of counterpoint, 1023; text: da me lontana è Clori l'idono mio si rammentasse o Dio de' miei costanti amori con un sospiro almeno figlio del suo bel seno s'oncontrassero un giorno

i sospiri miei a me che penso a lei e tante volte e tante ah se Clori pensasse in questo istante, et cetera, 7 b, 6 b, 6 [sqb], 6, 3 b, 6 4 [sqb] 2, 7 b 5 b, 5 [sqb] 3 b, 4 b 2 b, 5 3 [sqb], 6 [sqb] 4 3, 4 [sqb], 3 [sqb]]

[-1024-] When the composition modulates from the tone B b to the tone E b, the accompaniment D F A b C b of the minor third, semi-diminished fifth and more-than-diminished seventh, is employed immediately. At the beginning of bar seven the passage B b C b, through which the soprano moves from B b, octave of the accompaniment B b 5 3 b, to C b, more-than-diminished seventh of the accompaniment D 7 b 5 b 3, proves extremely tearful. The note C b evokes compassion in the listener because it produces a modulation, since the preceding tone, B b with the minor third, requires the note C, and because it is the more-than-d seventh of F, seventh artificial note of the tone E b with the minor third. The rising of the seventh artificial note D ensures that the sixth semi-diminished note C b sounds more languid. The passage B b 5 3 b D 7 b 5 b 3 is followed by this other one, D 7 b 5 b 3 E b 5 b 4 b, which is a sort of cadence that moves from the seventh altered note to the octave. The soprano sings the third semi-diminished G b of the accompaniment E b 5 b 3. Then the semi-diminished seventh D b is added to the chord E b 5 b 3 b. D b is placed in the *basso continuo* and it is also touched on by the singer at the beginning of the eighth bar, where the voice rises to it through the melancholic passage B b D b. Here ends my analysis, since Signore Scarlat<t>i returns to the tone B b with the minor third, as it is indicated manifestly by the following accompaniment C 6 3 b that belongs to the aforementioned tone.

[signum] 24. The second example is suggested to me by a very beautiful versicle of the eleventh Psalm by Signor Benedetto Marcello. The tone E b with the minor third expresses the misery of the needy and the tears of the poor not simply because it is naturally sad, but because the comparison with the tone E b with the major third with which it is interspersed renders more manifest that sadness. The music begins with an *allegro* in the tone E b with the major third. Such indication of the tempo and the major tone match God's words Io dic'egli sorgerò. After this section the tempo changes to *adagio assai* in order to represent the misery and the tears of the abandoned poor and of the afflicted. If we consider the accompaniment B b 5 3, common to the two tones B b and E b with the major third, as belonging to the latter one,

[-1025-] [Riccati, The laws of counterpoint, 1025; text: Allegro, Io dic'egli sorgerò mosso a pietà dello stato miserabile del pianto in cui si struggono i miei servi afflitti e poveri e soccorso a lor darò, 6, 5, 7, 6 b 4, 5 b 3, 6 b, 7 [sqb] 3 [sqb], 4 b, 3 b, 7 b 3 b, 6 b, 6 b 5 b 3 b, 5 b 4 b, 7 b 5 b 3 b]

[-1025-] the music modulates to the tone E b with the minor third. The melodic passage B b G b of the semi-diminished sixth upwards from B b, fundamental note of the accompaniment B b 5 3, to G b, semi-diminished third of the accompaniment E b 6 b 4 b forming a semi-diminished sixth with G b sounds extremely painful. Therefore, our interval is presented to the judgement of the ear in two forms, namely, harmonically and melodically. The accompaniment [-1026-] B b 6 b 4 b is followed at bar six by the two accompaniments C b 5 b 3 b and C b 6 b 3 b. The former is borrowed from the derivative tones, while the latter belongs strictly to the tone E b with the minor third. As the fundamental passages of the third downwards E b 5 b 3 b C b 5 b 3 b, C b 5 b 3 b and A b 5 b 3 b, on which the ones employed by the *basso continuo*, namely, B b 6 b 4 b C b 5 b 3 b and C b 5 b 3 b C b 6 b 3 b depend, are useless to determine to which derivative tone the accompaniment C b 5 b 3 b belongs, it follows the nature of the note C b, sixth and G b, third or tenth, of the tone E b with the minor third, which constitute its base and its fifth. In the mode with the minor third the third and the sixth note are lowered by a minor semitone and sound emotionally charged. In the tone E b, which we are discussing, they are lowered enharmonically, and for this reason they sound mournful. What I said teaches us that the derivative movement of the melody B b 6 b 4 b C b 5 b 4 b, with which the *basso continuo* moves from the fifth to the sixth bar, has to sound extremely painful. If we take a look at bar seven, we see first of all that the semi-diminished seventh A b has been added to the artificial accompaniment with the major third B b 5 3. The same sound later becomes the

lowered fourth of E b and it resolves onto the semi-diminished third G b. The aforementioned sad notes are sung by the alto with the intention that it may evoke the feeling of compassion in the listeners. The first three fourths of the eighth bar belong to the subordinate tone C b called with the major third. The scale of the principal tone E b with the minor third contains the sound F, which is altered by one of our flats in the scale of the aforementioned tone C b. This altered note, which proves very emotionally expressive, is heard soon, as the ninth G b added to the accompaniment F b 5 b 3 b, whose major third A b 6 b 3 b is placed in the *basso continuo*, resolves onto it. Then F b turns firstly into minor seventh G b, which is rendered more sad by the comparison with the note B b played by the *basso continuo*, with which it forms a semi-diminished fourth, and then it turns into the fourth of C b. Signor Marcello resolves such dissonance by descending onto E b, semi-augmented third of C b. The tone C b, commonly called with the major third, is counted among the tones that need to be employed as [-1027-] subordinate rather than as principal. The white key immediately below C is called B with its proper name and C b with its improper name. If said key is assigned as foundation to a principal tone with the major third, said tone will be considered B rather than C b because the scale of B contains five perfect sounds, three of whom are the ones that constitute the system of the melody, and only two that are raised by an enharmonic element. Conversely, the scale of C b consists of five enharmonically lowered sounds that contain the three on which the system of the melody is based and of only two perfect sounds. The section marked Adagio assai ends with the two accompaniments A 7 b 5 b 3b of the minor fifth, third and more-than-diminished seventh and B b 5 3 of the major third and fifth, to which is added the fourth resolves on the third itself. Such accompaniments belong to the tone E b with the minor third and form the cadence that moves from the fourth artificial note to the fifth one. The sound A, played by the *basso continuo*, sounds extraordinarily resentful not only because of the fundamental melodic passage C b A of the third more-than-diminished downwards in the part itself, but also because it is the fourth artificial note and because the music modulates through it from the tone C b with the major third to the tone E b with the minor third. The vehemence of the note A weakens further the note E b sung by the alto and renders it apt to express the meaning of the word afflitti.

After the cadence, the composition comes back to the tone E b with the major third and the fast tempo is reinstated, as required the meaning of the words e soccorso a lor darò.

[signum] 25. At last I have reached the end of my long work, which has cost me a continuous effort lasting many years. As far as I can ascertain, I have taken care not to omit anything of any importance, while I strived to place it in their true light and to explain the reasons that indicate the perfect agreement between practice and theory. I leave it to my learned and discerning Readers to decide to what extent I have been successful.