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Title: Notes on Monsieur Rameau
Original title: Annotazioni sopra Monsieur Rameau
Source: Udine, Biblioteca Civica, MS F.p. 1029/VI, 75-80

[-75-] Notes on Monsieur Rameau

Page 4. The division of he string into seven parts is avoided. Consequently, the number seven is banned from music, as it cannot produce any pleasant interval. All musical intervals can be measured on the basis of the sole prime number three through the repetition of the fifths. However, since the simplicity of the ratios is the essential base of music, the number five is allowed. For the same reason it is necessary to allow the number seven as well.
Page 6. A string struck with a degree of energy produces the vibration of the higher and lower octave. This does not occur in the case of the fifths, in which case only the higher one is made to vibrate, while the lower one remains still. This statement is found to be false through experience.
Page i2, e i3. Only the fourth and the two thirds are involved in the production of the natural or perfect chord. The fourth and the two sixths are determined as derived from the fifth and from the two thirds, since they are the intervals that, added to them, constitute the octave. I am inclined to consider the fourth as an interval derived from the fifth because it is produced from it through the division of the string into four arts. The same applies to the minor sixth. As it is produced by the division of the string into eight parts, it shall be a secondary interval compared to the major third, which is produced by the division of the string into five parts. However, this cannot be applied to the major sixth in relation to the minor third, because the major third originates from dividing the string into five parts, while the minor third is produced by dividing the string into six equal parts. The truth of the fact that only the fifth and the two thirds are involved in the perfect accompaniment must be deduced from other basic principles. Experience demonstrates that the following intervals are consonant: 3/1 3/2 4/3 5/3 5/4 6/5 8/5. A consonant accompaniment is an accompaniment in which, when we consider the established intervals between a note and another one, they correspond to each other in one of the aforementioned consonances. The lowest note [-76-] is the one that produces the strongest impression on the ear. Therefore, the harmony shall be all the more perfect, the simpler the ratios of the other notes with the bass. Therefore, the fifth will have to form and interval with the bass, while the fourth and the major sixth are excluded as the are dissonant with the fourth. The next interval in the sequence is the major third. If it is placed on the side of the bass, it produces the eminently perfect accompaniment expressed by the simplest series of numbers i 2 3 4 5 6, which conform to the requirements noted above. We can invert the division of the fifth by pairing the bass with a minor third. In this case we obtain a harmony that fulfils the ear, albeit it is not the best one. Consequently, note that the best accompaniment requires the octave and the fifth to be divided arithmetically, that we can deviate from the first condition when we require a principal accompaniment and that we can dispense with the second condition by substituting the harmonic division of the fifth instead of the arithmetic one. [Here is the reason why this is so. in marg.] Everyone knows that that the consonant accompaniments are exactly six. They are: major third, fifth and octave (first); minor third, minor sixth and octave (second); fourth, major sixth and octave (third); minor

third, fifth and octave (fourth); minor third, major third and octave (fifth); fourth minor sixth and octave (sixth). The second and the third one are derived from the first one and the last two from the fourth one. Now, the ear can be satisfied only by the first and by the fourth one that we have mentioned, because it hears that a better sequence of intervals is ascribed to a secondary note rather than to a principal one.
Page 23. Chapter five. The diatonic system is determined in reality by dividing the major third Vt Mi harmonically rather than arithmetically, in which case it follows that the fifth reduced by a comma, rather than a perfect fifth, corresponds to the very principal note Sol.
[-77-] Page 24. The idea of the dissonances originating by the union of consonances is false. Accordingly, those expected to determine the ratios of the imperfect consonances by adding together several fifths and by employing only the number three, thus expressing the major third 5/4 with the ratio 81/64 and the minor one 6/5 through the ratio 32/37 [31/27 ante corr.], would be mistaken. Similarly, those who profess to be able to express all the dissonances through the numbers 3 and 5, by employing, for instance, 30/25 as the ratio of the false fifth instead of the simpler one 10/7, and 216/125 as the ratio of the diminished seventh instead of the simple one 1/7. Should anyone out that in practice, for instance, a diminished seventh consists of three minor thirds that produce the assigned ratio when they are added together, I shall reply that the major sixth above the octave consists in practice of the addition of three fifths, hence it should be expressed by the ratio 6 27/8, but everyone disregards this rule and adopts the simpler ratio 19/13. For this same reason we must exclude the very complex ratio 216/215 and we must keep to the ratio 12/7, since we would be relying on the contribution of the number five in the former case and of the number seven in the latter. Note that three fifth added together differ from the true ratio of the major sixth above the octave by a comma 81/80, while three minor thirds differ from i2/7 only by approximately 7/11 of the comma. Therefore, it follows that if the number five can be combined with the number three through the best temperaments, the number seven can be combined all the more with each of them. In fact, if we determine the best participation in relation to the numbers three and five with the difference of one fourth of the comma in the fifth and in the minor third, consequently it occurs that many dissonant intervals are expressed by simple ratios, as the number seven lays at the distance of 1/8 of the comma from them. [-78-] The writer contradicts himself in this. In fact, after he has assigned to the diminished seven the fraction 2i6/215 [256/25 ante corr.] to the diminished seventh, at page 26 he places this interval in the category the altered ones and substitutes to it the [[simpler]] less complex ratio 128/75, which is reduced by a comma. Now, if for reasons of simplicity he embraces this last ratio which is reduced by one comma, with much greater reason he must accept the ratio 12/7, as it is smaller [[distant from]] than the ration 2i6/215 by -7/11 of the comma.
Page 25. The author prefers the minor seventh 9/5 consisting of a fifth and of a minor third to the other one 16/9 originating from the fourth of the fourth. Then, on the following page he rejects the first ratio as altered and places the second one among the perfect intervals. In the same passage one reads that every interval can have its comma except for the octave and for the major seventh that he calls augmented. I cannot understand how the author could have placed in the table of the natural and altered ratios at page twenty and how in its chromatic (page 28) two major semitones 16/15 and 27/25, which lay at the distance of one comma, should not receive consequently the interval that complement them to the octave, namely, the two major seventh 1/8 and 50/27.

The author adds at page 25 itself that the comma cannot be perceived, particularly in the intervals that belong to the harmony and in the melody. Were this true, those who have strived to divide the comma through different temperaments would have laboured in vain, as the system of twelve equal semitones practised by most tuners would be the best one, since the consonant intervals are not altered more than 8/11 of the comma.

[-79-] Page 26. It is redundant to waste time in examining the elementary quantities admitted by Monsieur Rameau. The difference between the two tones, the comma 81/80 and the enharmonic diesis of the ancients 128/125 can correspond to the difference between the major semitone 16/15 and the minor one 25/24. One cannot tell the advantage in subtracting the comma from the aforementioned enharmonic diesis to obtain the quantity 2048/2025, which is called diminished comma. The same should apply to the major diesis 250/243 and to the smallest semitone 648/625.

The fact that the minor semitone 135/128 = 25/24 + 81/80 is recorded in the aforementioned table contradicts the idea of his chromatic, because it does not and cannot belong to the chromatic scale as it is not an element that belongs to it, since only the minor semitone that is admitted is 25/24. However, it would occur in the chromatic scale by Signor Salvatore, who does not recognise any other major semitone except 16/15, apart from the one represented by the ratio 27/25.

Two ratios are ascribed to the diminished seventh, which is called improperly augmented tone, namely, the natural one 256/225 = 6/5 – 155/128 and the one that is augmented by a comma, namely, 144/125 = 6/5 – 25/24. It is worth noting that he has already placed the the minor semitone 135/128 among the altered ones, but then he wants to allow the diminished third derived from subtracting the aforementioned altered semitone from the minor third. The truth is that the two diminished seventh contained in his chromatic both equal 6/5 – 25/24 = 144/125.

[-80-] The ratio 256/225 [225/225 ante corr.] would belong of a chromatic system according to the idea of Signor Salvatore, as it is generated by the ratio of the major semitone at the power of two. However, the ratio of this interval is 8/7 = 256/225 + 225/224 = 144/125 – 126/125, which lays half-way between the two ratios assigned.

The author adopts 100/81 = 10/9 x 10/9 as the altered ratio of the major third. His chromatic system requires this because it contains two minor tones one after the other, namely, B b c, c d. Since the chromatic system according to Signor Salvatore contains two major consecutive tones, they produce 81/64 = 9/8 x 9/8, which is the tritone of the ancients. A good chromatic system must provide the altered major thirds mostly perfect, and some of them raised and others lowered, so that they may be reduced to be perfect by tempering them.

The altered ratio of the fifth called augmented, which is normally named major, which is the fraction 125/81, is considered in relation to the chromatic system of the author, while the Signor Salvatore's system would require one that is one comma higher than 15/16, namely 405/256. Note that the following become gradually lower [[higher]] and

[Riccati, Notes on Monsieur Rameau, 80; text; 405/256 [205/256 ante corr.], 11/7, 25/16, 14/9, i25/8i, 149/148, 4/19, 176/175, 225/224, 226/225, 81/80, 99/98]

deduce that the true ratio of the augmented fourth with regard [-81-] to the temperaments is the medium one 25/16, provided to us by the two chromatic ones by our author and by Signor

Salvatore, between too complex and too distant limits. Its true limits are 11/7 and 14/9 that are separated by a quantity that is slightly smaller than one comma.

The two ratios of the false fifth, 36/25 and 64/45, comprehend the simpler one 10/7. I note that Monsieur Rameau and Signor Salvatore consider respectively the ratios 36/25 and 64/45 as legitimate. Therefore, if we find the middle ground between these two different opinions, the best ratio shall be 10/7.

Page 28. The chromatic system relies on the division of the major tones into the ratios 27/25 and 25/24, which was adopted by Signor Henfling and leads to his tempered system of 50 parts.