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Title: A compendium of music by the illustrious Signor Georgio Carretto of the marquises of Saona, Doctor of law and Mantuan Senator.

Original title: Compendio della musica Dell' Illustre signor Georgio Carretto delli marchesi di Saona, dottor de leggi, et senator di Mantoua.

Editor: Massimo Redaelli

Source: Bologna, Museo Internazionale e Biblioteca della Musica, MS C.119, f.1r-23r

<[-f.1r-]> [283 in marg. alia manu] [numero 4. alia manu] A compendium of music by the illustrious Signor Georgio Carretto of the marquises of Saona, Doctor of law and Senator of Mantua.

[This book belongs to Pier Antonio Uitale Carleuari add. alia manu]

[-f.2r-] First Part.

Among the particular gifts bestowed by God on man, one is the ability to describe in words the feelings of the soul. Such practice, wonderful in itself, was subsequently ennobled by Harmony, Rhythms and Metres, all of which, besides their usefulness in expressing such concepts and feelings, bring with themselves the power to please the ear. This is the origin of hymns in praise of God, and of the heroic, tragic, comic, elegiac and lyrical verses through which one may move the soul to any emotion, reducing them to a state of contemplation and angel-like life, containing desires and persuading the listener to live a celestial life and to enjoy on earth that celestial Harmony. This most noble and elevated science was held once in great consideration but became neglected because of the barbarity of the times As Adriano Willaert chapel master in Saint Mark's, Venice restored it, while Reverend Don Giuseppe Zarlino codified its artistry, from the latter's work I intend to extract a clear manual by way of commentary, without overlooking anything that may be relevant to it, as one will be able to see in the continuation of this work. Of the human senses, touch and taste are, so to speak, most necessary to human existence, while the other ones, namely, sight, hearing and the sense of smell, are most necessary to the enjoyment of life. Of the latter three, albeit sight considered by itself [-f.2v-] may appear more valuable and necessary, hearing nevertheless is perchance more necessary and better for what involves the intellect (for which the senses have been created) because it is simpler. The science of music originated from the sense of hearing. It was invented before the deluge by the descendents of Cain through the sounds of hammers and anvil, and it was later restored by Mercury together with the progress of the stars, the harmony of the voice and numerical proportions, although others ascribe this invention to Pythagoras. This science is mathematical and absolutely certain, because it regards matters that have the true being as their nature and its principles are common sentences that cannot be contradicted. Therefore, Music is extremely ancient, most noble, founded on absolute certainty and holy, as the fabric of the universe demonstrates, as it is based on harmony. Also, the harmony of our soul embraces all the liberal sciences: through the correctness of grammar in speeches and poems; through the proportion of the syllogisms in dialectics; through the concordance and discordance of the planets and other stars in astrology; [-f.3r-] through the motion and order of the universe in philosophy, as everything has its position according to its natural weight; through the proportion of doses and knowledge of the pulses in medicine; through the division of the three hierarchies and angelic orders, of the four animal ones, seven candelabra and twenty-four old ones in theology. Such harmony is better understood and more realised in paradise than elsewhere, hence, in imitation of the triumphant church, our militant church employs singing and sounds in its rites to praise God. Similarly, sirens sing in the sea and birds in the sky. Since the whole of creation was put together with music, what shall we say of man? If the soul of the cosmos is imbued with harmony, the same must be said for the soul of man, who was created in the image of the cosmos. Hence man is defined as a microcosm. Therefore Music is necessary for a Christian, and it is a very good way to contemplative life as well as a mean to receive the good spirit, as it happened to Elisaeus. This is why Plato and Aristotle maintain that a cultured person should be knowledgeable about music [-f.3v-], which should be learned from the early years, not only as a pastime and as pleasure to the ear, but also in order to regulate the passions of the soul, to pass one's free time in a virtuous way and as an honest practice on the path of a moral life. Music is natural for mankind as it pleases even the infants, it restores to us our lost strength, it lightens our burdens, it makes us cheerful, excited and courageous, it moves us, it benefits us, it pleases us and it brings us honour and glory as it combines theoretical speculation and practice. Music is the science of concordant and discordant harmonies, hence the philosopher said that everything is realised through contrast. It is divided in animastic and instrumental. Animastic or cosmic music is the one derived by the combination of the elements of the cosmos. The other one is called human music, as it concerns the combinations of the parts of the human being and of its affections. Instrumental music is produced by instruments. Some instruments will be natural, such as the throat, the chest and the lips. In this case the music is defined as natural. Conversely, if it is produced by artificial instruments created by human artfulness and invention, it will be defined as artificial music. One kind of it is produced by brass and wind instruments, such as trumpets, [-f.4r-] piffari, recorders, cornets and trombones; another kind is produced by strings that are plucked with the fingers or with a plectrum, or rubbed with a bow, as in the case of the lyra or the viol; another kind is the sort produced by drums, castanets and cymbals. Natural instrumental music is plain or measured by rhythm and metre. Animastic or cosmic music is found in the distance between the spheres and skies. A Muse has been assigned to each sky: Thalia to the earth, which produces no harmony; Clio to the Moon, which produces the hypodorian harmony; Calliope to Mercury, which produces the hypophrygian harmony; Terpsichore to Venus, which produces the hypolydian harmony; Melpomene to the Sun, which produces the Dorian harmony; Erato to Mars, which produces the Phrygian harmony; Euterpe to Jupiter, which produces the Lydian harmony; Polyhymnia to Saturn, which produces the Mixolydian harmony; Urania to the firmament, which produces the hypermixolydian harmony;

The cosmic animastic harmony is found in the distance of the spheres. There are eight intervals from the third and six tones to the eighth sphere. These produce the diapason, according to Pythagoras. There are also five semitones, two tones and one and a half tones.
[-f.4v-] The distance from the earth to the moon is 15750 Italian miles, corresponding to one tone. The distance from the Moon to Mercury is 7875 miles or a semitone; from Mercury to Venus 7875 miles or a semitone; from Venus to the sun 23625 miles or a tone and a semitone; from Mars to Jupiter 7875 miles or a semitone; from Jupiter to Saturn 7875 miles or a semitone; from Saturn to the firmament 7875 miles or a semitone. Hence, between the earth to the sky of the stars lays the consonance of the diapason; from the earth to the sun the diapente; from the moon to the sun the diatessaron; from the sun to the sky of the stars the diatessaron.

[Carretto, Compendio, f.4v]

The music of the cosmos is found in the elements that are united with the same quality with the next element and divided through another quality. The contrary elements lay in the same proportion. For instance two adjacent cubic numbers [6 9 12 18. add in marg.] are connected by two intervening numbers that lay in the same proportion. For instance, the two cubic numbers 8 and 27 have two intervening numbers contained between them that lay in the same proportion. In fact, as 12 and 8 form a sesquialtera proportion, 18 and 12 and [-f.5r-] 27 and 18 do the same. Thus, heavy elements pull downwards and the light ones upwards, [Earth 8. Water 12. Air 18. Fire 27. add. in marg.] so that they balance each other as to their position. An element changes into the next one in ten-fold proportion, as in the case of the earth, whose proportion to fire is 1:1000. The tetrachord was derived from this cosmic harmony, as in the case of the four elements and of the four seasons of the year, while the eptachord was adopted for the seven planets. The former is accepted and employed more widely. Here the Hypate corresponds to the earth and assumes the Myxolydian mode; the Parhypate the water and the Dorian mode; the Paranete synemmenon the air and the Lydian mode; the Nete diezeugmenon and hyperboleon the fire and the Phrygian mode. The same occurs in the harmony of the microcosm. In the process of human procreation the seed is formed in the matrix as milk. In nine days, which number forms a sesquialtera proportion, it becomes blood; in twelve days, which constitute a sesquitertia proportion or diatessaron, it becomes unshaped flesh. Then, after eighteen days, it [-f.5v-] is formed in such a way that on the forty-fifth day the human being is created and God infuses into it the rational soul. Consequently, from the first to the second term lays the diapente; from the second to the third the diatessaron; from the third to the fourth the diapente; from the first to the third and to the third to the fourth the diapason. Similarly, the soul contains three faculties corresponding to the three consonances: the intellective and rational faculty is similar to the diapason, the sentient and irascible to the diapente and then vegetative and corruptible to the diatessaron, which corresponds to growth, stable state and decreasing state. As we have seen, the soul is linked to the body through harmony and number. When this number is fulfilled, the final knot is undone. This is called ‘the fatal journey’. However, according to poets and philosophers, if one dies before one’s time, such number is completed before the soul may rest in the place allotted to it. As Virgil says of Deiphobus, when he was killed: ‘I shall fulfil my number, and I shall be returned to the darkness’ Instrumental music, whether natural or artificial, is divided into plain music (or canto fermo) and measured music (or canto figurato). The former occurs when there are simple notes without intervals sung with the same pulse without a variety of note values or times. A note is a sign placed on a line or in a space [-f.6r-] indicating how much we must rise or lower the voice and how fast in conformity to its value. Rhythmic music is the harmony one perceives in a verse or in metric prose thanks to the quantity of the syllables and to the arrangement of the words according to particular metric clauses. This is what we call natural instrumental music. Metric music consists of verses and is born of the quantity of syllables and the number of feet that make up the verses. As it is possible to play such verses with artificial instruments, it can also be called artificial music. These two sorts of music belong to orators and poets. Instrumental music is a mathematical science, as it deals through the ear and the reason sounds, notes, numbers and harmonic proportions and organises low and high sounds. It is called music from the Muses, the daughters of Jupiter and Memory. This science is divided into theory and practice. Both are necessary to render a musician perfect. As they are separate, the most noble part is theory, just as the creator is more noble than the instrument through which one creates. [-f.6v-] Numbers and ratios are the subject of music. A number is nothing but the addition of more than one unit. The unit is the principle of the number of discreet quantity and it is the origin and common measure of every number, even though it is not a number. Similarly, the dot is the origin of continuous quantity, albeit it is not a quantity. Likewise, husband and wife are not related, but origins of relation. There are several species of numbers, but musicians concern themselves only with the following ten species: even numbers (those that can be divided into equal parts), odd numbers (the ones that cannot be divided into equal whole parts), evenly even numbers (those that can be subdivided several times into equal parts up to the unit, e.g. the number eight), prime numbers or true indivisible numbers (that can only divided by the unit, e.g. the number three, five and seven), compound numbers (those that can be divided by another number as well as by the unit, such as four, six, nine and ten), numbers that are prime with each other (those that cannot be divided both by the any number except for the unit, although they can be divided [-f.7r-] by different numbers individually, except for the unit, e.g. nine and ten. Every number separated from another one by a single unit will fall into this category. There are three species of these numbers: either they are both compound numbers, such as nine and ten, or both prime numbers, as three and five, or one is compound and the other one is prime, as twelve and thirteen). Communicating numbers are called the numbers that share a common divisor apart from the unit, e.g. four and six. There three species of these, since either both of them are even numbers, such as four and six, or they are both odd numbers, such as nine and fifteen, or they are one an odd number and one an even number, as in the case of six and nine. A square number is a number multiplied by itself, as, for instance, four times four gives sixteen, which is a square number. A cubic number is a square number multiplied by its root. For instance, four multiplied by sixteen gives sixty-four, a cubic number. Perfect number refers to a number whose divisors added together amount to the number itself, such as the number six, whose divisors one, two and three together add up to six. There is only one such number, six, among those below ten, one below one hundred, eighteen, and one below one thousand, which is 496. Such numbers always end in six or in eight. This perfect number six is important for musicians because it contains all of the harmonic proportions [-f.7v-] and all the various musical consonants, whose mother is the diapason. The diapason generates all of the consonances greater than it is and some of the smaller ones as well. The smaller ones derive from the division of the diapason itself. After Agrippa, Zarlino deals with the properties of this number six. It is called analogue, namely, proportionate, at chapter fourteen, because it consists of its divisors. It is also called circular, because multiplied by itself it produces the number six ad infinitum. There are six species of musical sounds (unisonant, aequisonant, consonant, dissonant, emmeles and ecmeles), six consonances (diapason, diapente, diatessaron, ditone, semiditone and unison) and six modes (Dorian, Phrygia, Lydian, Ionian, Aeolian and Mixolydian). Vitruvius deals with the number six and its perfection in the first chapter of the third book. The number six is the first perfect number and all perfect consonances are contained within it. All the eighteen numbers that relate to music are contained within its square number, thirty-six. Such property is not found in any other number. The consonances originate from the proportions here below.

[Carretto, Compendio, f.7v; text Diapente 1. 2.]

[-f.8r-] [Carretto, Compendio, 8r; text: Diapason. 1 2 10. 12. Semiditone. Diapente. 3 15. Ditone. Diatessaron. 4 6. Semiton maggiore. 18. Major Tone. Ditone. 5 20. Minor Tone. Semiditone. 6 24. Semiditone. Diatessaron. 8 25. Minor Semitone. Major Tone 9 30. Semiditone. Minor Tone 10. 36. Semiditone. 1 3 Major Sixth. 5 8 Minor Sixth]

The hexachord, or sixth, was admitted by composers despite being disallowed by theory writers because it belongs to the genus of superpartiente proportion, which cannot generate a simple compound one. In other words, other consonances are contained between the two numbers that generate the hexachord. In fact, the minor sixth is from five to eight, which contain five to six (the semiditone) and six to eight (the diatessaron). Hence one can say that the sixth consists of a diatessaron and a semiditone. Therefore, it is not a simple consonance. As to the major sixth from three to five, it contains the diatessaron [-f.8v-] (from three to four) and the ditone (from four to five). Hence it is not a simple consonance but a compound one. I shall deal now with the compound consonances that are possible, be they smaller than the diapason, such as the major sixth (consisting of diatessaron, ditone and semiditone) or larger than the diapason (consisting of the diapason and one of its smaller consonances), such as the thirteenth, which consists of the diapason and of the diapente and is represented by the ratio 1:3. It can consists of several diapason (with the proportion 1:4), but it is more correct to say that it is a compound consonance of the diapason. Therefore, the major and minor sixth are admitted among the simple consonances because their composition does not include the diapason. I conclude by saying that the number six contains all the proportions of simple and elementary consonances, namely, Diapason, Diapente, Diatessaron, Ditone and Semiditone. All of the other consonances contained potentially within the number six originate from these. Just as single figure numbers are repeated past the number ten [-f.9r-], as eleven and twelve follow, similarly the consonances reoccur past the diapason, which is represented by the ratio 2:1, since music employs finite rather than infinite quantities. Quantity is defined as continuous, when it has a common term (as in the case of the line, surface, body, location and time), or discreet (as in the concepts of number, discourse, flock and people). The notions of a lot and a little pertain to the discreet quantity. In arithmetic, some numbers are simple and self-standing, such as one, two and three. Others, such as the double or triple of a simple number, cannot be self-standing because they relate to another number. Such numbers are employed in music, as music deals with sounding bodies and proportional or relational numbers. Hence, music pertains to the sonorous number: pulse is connected to the concept of number and the pitches are related to sound. The sonorous number relates to pitches and sounds, [-f.9v-] it represents the harmonic consonances with its ratios and it is contained simply within the number six. High and low intervals are recognised by means of a sounding body, such as the human voice and strings made of metal or other material. The interval between two pitches is recognised through the division of a string. In fact, the diapason, represented by the proportion 2:1, originates from the division of the string into two parts. The smaller the divisions, the higher will be the pitches. Hence, pitches and sound are the matter of music, while numbers and ratios are its form. Moreover, since music borrows numbers and ratio from arithmetic and sonorous bodies from geometry, it is said to be a science subordinate to arithmetic.

On Proportions.

Proper proportion occurs when two objects of the same kind are equal in a predicate. For instance, when Peter and Vincent are compared as to a kind of whiteness of the same genus or two lines are compared like for like. This does apply when a line is compared to a body, [-f.10r-] because a comparison occurs between two similar genera and consists of sameness and difference, as it belongs to the quantum and to what derives its name from it. Proper proportion is divided into rational (as the dupla) which has a medium common term, and for this reason is termed commensurable and communicating. An irrational proportion is one that cannot be described with a rational number, as in the comparison between a square to its diameter, which do not share any common measurement and thus are called incommensurable. Arithmetic proportion is always rational because it is always equal in the excess of the number. This does not occur in the case of geometric proportion. Music is concerned with unequal ratios. Such ratios are called ratios of greater inequality when the larger contains the smaller several times; they are called of smaller inequality when the smaller one is compared to the larger one and one refers to how the smaller one is contained in the larger one. The terms of the ratio relate to one another in five ways. The larger can contain the smaller entirely several times. In this case the ratio is called multiplex, as in 2:8. The larger one can contain the smaller one once [-f.10v-] and a portion of it (called multipliable) by which it can be divided, as in 2:3. This ratio is called superparticular. Alternatively, the larger can contain the smaller one once and a portion by which the smaller cannot be divided, called aggregative. This ratio is called superpartient, as in 3:5. If the larger terms contains several times the smaller one and a portion of it by which it can be divided, it is called multiplex superparticular ratio; if the remaining portion cannot divide it, it is called multiplex superpartient. There are another five instances of smaller inequality occurring when the smaller is compared to the larger. Such comparison occurs according to difference if the prefix sub- is added, as in submultiple, subsuperparticular, subsuperpartient, submultiplex, subsuperparticular, submultiplex and subsuperpartient. Albeit the aforesaid five genera of greater inequality are infinite as for the numbers, the musician considers only a small finite fraction of them closer to simplicity (the farther away from its origin, the less pure and intelligible it is) as the ear cannot recognise the consonances as promptly [-f.11r-] when they are far removed. For this reason they have established in advance a suitable number of notes from the low register to the high. In order to be able to read the number of the ratios and of the notes, consider how removed they are from the unit. Thus shall be named the multiplex proportion. For instance, from one to two it shall be called double, from one to three triple, and so on ad infinitum. They shall be greater according to the denominator of the multiple, namely, the quadruple is

greater than the treble and they are written as fractions, as, for instance 1/2 and 1/3. Conversely, in the case superparticular and superpartient ratios, the proportions shall be smaller as the denominators are larger, so that a sesquituarta shall be smaller than a sesquitertia. This occurs because in the multiplex genus the denominator indicates multiplication, but it indicates division in the other genera. Thus, the larger the number of parts into which something is divided, the smaller the size of each part.
In order to order to name the superparticular and superpartient ones, add the remainder above the whole number. For instance, I shall call the proportion from three to seven double sesquitertia or double superpartient, a third. Thus, it is easier to pronounce its name than to add the remainders before the word partient. Similarly, from, in the case of the ratio seven to ten, it will be easier to call it superpartient three sevenths [-f.11v-] than superpartient one seventh. One must note a few easy and most beautiful little rules goverining such superpartient proportional numbers.
[First add. in marg.] Find me a superpartient three fourths. According to the rule, as a fraction, the numerator shall be four, while the larger number will be obtained by adding the numerator of the fourths. The ratio shall be, therefore, four to seven. Find me a superpartient 8/7. One number shall be seven; the other one shall be fifteen.
[Second add. in marg.] In order to find express a multiplex superpartient ratio as a triple superpartient two-sevenths, first of all multiply the divided number by the multiplex proportion: three times seven is twenty-one. If you add two to this, the result will be twenty-three. Thus, the ratio shall be seven to twenty-three. Similarly, give me a quintupla superpartient three eights. The fraction shall be the smaller number of the proportion, namely, eight. Multiplied by the number five it equals forty. Added to it the numer three, numerator of the fraction, the result will be forty-three. Hence the proportion shall be 8 to forty-three.
[Third add. in marg.] In the case of every superparticular proportion, it shall occupy its place in the series according to the difference between the larger and the smaller number. For instance, 20 to 40 is a sesquitertia, and, since the difference is ten, I say that it shall be the tenth sesquitertia, preceded by another nine, which are: [-f.12r-] 4 3, 8 6, 12 9, and so on, multiplying the terms of the first sesquitertia 4 3 by any numbers of choice. If you multiply them by seven, they will give 21 and 28, as required. This is observed in every other proportion, by multiplying in this way the first terms of that proportion by a chosen number. In any case, if I want the ratio occupying the third place in the series of the quintupla superpartient [quarta add. in marg.] three eights, as the first terms are 8 and 43, if these are multiplied by three, the result shall be 24 129, as required.
[4. add. in marg.] In order to establish the magnitude of a proportion in the order and the space it occupies, divide the given numbers by the prime numbers of that proportion. For instance, as 24 29 are in quintupla superpartiente three eights, I divide it by 8, which equals three. Hence, I say that it is the third one in order of magnitude.
[5. add. in marg.] How to reduce three numbers placed in proportion to single term, namely, the smallest one. 8, 32 and 128 are three numbers in quadrupla proportion. [-f.13r-] If you want to reduce them all to the smallest term, which is 8, and to their arithmetic equality, subtract the smallest term, [quadrupla. 8. 32. 128. Tripla. 8. 24. 72. Dupla. 8. 16. 32. Equale. 8. 8. 8. add. in marg.] which is 8, from the median one, which is 32. The remainder is 24. Similarly, subtract 8 from the double of the median term, which is 64. The remainder is 56. Now, remove 56 from the largest term, 128, which will give 72. This shall be the one occupying the first place, as we can see in margin, and the proportion from a quadrupla becomes a tripla. Employ again the same operation and subtract 8 from 24, which will give 16; then subtract 8 from the double

of 24, which is 48, which will be 40; finally subtract 40 from 72, which is the largest number of the occupying the second place, and you will obtain the one in the third place, 8 16 32, which will be a dupla. If you apply again the rule written above you will reach the arithmetic equality, 8 8 8.

[Sesquialtera 4. 6. 9. dupla 4. 4. 6. dupla 4. 4. 4. add. in marg.] It is sufficient to subtract from the numbers the added part of the proportion. As sesquialtera means half, thus you will subtract the half of the median number from the largest. So, if you remove half of the first number from the middle one, you will have 4 4 6, where two numbers are already equal. Again now [-f.13r-] [First part add. supra lin.] remove half of this middle number from the largest number 6 and you will be left with the numbers 4 4 4, as required. You will do the same in the case of the others according to their proportion. [9. 12. 26. 9. 9. 12. 9. 9. 9. add. in marg.] As 9 12 16 are in sesquitertia proportion, reduce them to equality. Subtract one third of the median from the largest one and one third of the first one from the median one, you will have 9 9 12, where two are already equal. Subtract one third of the median from the largest one and you will obtain 9 9 9, three equal numbers as required.

[Sixth add. in marg.] How to reduce to equality more numbers in multiplex superpartient proportion as in [52. 169. 48. 144. add. in marg.] the tripla sesquiquarta. 16 52 169. First remove the excess of the proportion by removing one fourth according to the rule. [Tripla 16. 48. 144. Dupla. 16. 32. 64. Simpla 16. 16. 16.add. in marg.] Then, you will multiply the number in the first place by three and also the resulting product, which will create the tripla proportion 16. 48. [144. add. in marg.] Finally, operate according to the fifth rule given above and you will reach the arithmetic equality. [Seventh add. in marg.] This shall be the rule to follow in order to find any numbers in any superparticular proportion, as I require four numbers in sesquitertia proporiton [-f.13v-] [Tripla. 3. 9. 27. add. in marg.] Find the fourth number in the multiplex proportion, namely, in tripla proportion, which is 27. This number [this number in sesquitertia proportion 27. 36. 48. 64. add. in marg.] shall be the smallest one among the ones required. Add a third of itself to it, which will make 36. Add a third of it to this one, which will make 48. Finally, add a third of 48 to 48 and it will make 64. They will be ordered thus, 27 36 48 64 in the required sesquitertia proportion, as one adds to the following number a third of the preceding one.

Musicians deal with proportions of greater and smaller inequality, as we said. There are five species of each of them, three simple species and three compound ones. Proportions are always written with two figures, in the manner of fractions, one above the line and the other one below. When the two numbers above and below the line are the same, the proportion is said to be proportion of equality, as in the case of 5/5 and 4/4. These are not considered in music. Conversely, when the number above the line is larger than the one below it, the proportion is called proportion of inequality, as in the case of 4/3 and 2/1. [-f.14r-] If the number written above is smaller than the one below the line and the number above contains the one before several times, it is called multiplex proportion, as in the case of 4/2 and 6/2 and takes its name from the number of times the number below is contained in the number above. When the number above the line contains a portion of the one below it, it is called superpartient. It is called superparticular when the remainder can divide perfectly the number below the line, as in the case of 4/3 (sesquitertia) and 7/3 (dupla sesquitertia). The same is said of the proportions of smaller inequality, namely, when the number below the line is larger than the one above. In which case, the prefix sub- is added, as in 1/3 (subtripla). The proportion of equality is a sort intermediate element between the proportions of larger

and smaller inequality, as they are reduced to the aforesaid proportion of equality. Thus, if a dupla is subtracted from a dupla or a subdupla from a subdupla, the result is the proportion of equality. Also, as it is not possible to subtract a larger number from a smaller one, but only from a larger one or equal one, the same goes for the proportions. So, the proportion of equality lays in the middle as substance, [-f.14v-] while the greater and smaller inequality are placed at its side as qualities and extremes, as the greater denotes excess above it and the smaller one lack and incompleteness. For this reason, the proportion of greater inequality can be called positive, while the one of smaller inequality can be called privative. The multiplication of proportions follows the same rules as the multiplication of fractions. The product of the multiplication of the numerators is written above a line. Under the same line one writes the product of the denominators. For instance, the multiplication of a quadrupla and of a quintupla (4/1 and 5/1) is 20/1 (a vicecupla). The same occurs in the case of multiplex proportions. However, as for the multiplication of suparpartient and superparticular proportions, Zarlino and other authors require that the proportions are placed according to the sequence placed here in margin.
[5/4 6/5 4/3 2/1 24. 20. 30. add. in marg.]. Firstly, he multiplies the denominator of the greater proportion by the numerator of the smaller one, which gives 24; then the denominator of the greater proportion with its numerator, which equals 20; finally, he multiplies both numerators of both proportions, which equal 30. The products will lay thus, 30 24 20, obtained in sesquiquarta and sesquiquinta proportion.
When there are several proportions, [-f.15r-] having multiplied the two as above, take the denominator of the other proportion and multiply the products already found, as in the above example with 4/3. [90. 72. 60. 120. add. in marg.] The products multiplied by 3 will give 90 72 60. If you then multiply the smaller of the first three by the numerator of this latest proportion, which is 4, the result will be 120. [120. 90. 72. 60. add. in marg.] The numbers shall lay thus 120 90 72 60 in sesquitertia, sesquiquarta and sesquiquinta proportion. If you want to multiply these with another one, for instance a sesquialtera 3/2, multiply all of the preceding ones by the denominator, which will equal 240 180 144 120. Then, multiply the smaller number, 120, by the numerator, which will give 360. By following this method, you shall have five numbers in the desired proportions, such as 360 240 180 144 120, the first and the last one of which form a tripla proportion. You shall be able to achieve this easily by multiplying the numerators with each other and placing them above a line and by multiplying in a similar way the denominators of the proposed proportions and placing their product under the [-f.15v-] line, as in the aforementioned example, which gave 360/120 (tripla proportion). However, although the authors accept this rule without discussion, as they follow each other, I state that it is clearly false. In fact, since the four proportions are larger than the equal and whole one, it is necessary for their multiplication to produce a proportion larger than a quadruple and also than a tripla. However, if you multiply the numerators placing them above the line and the denominators below (as 4/1 and 7/1, which equal 28/1; 2/1 and 4/3, which equal 8/3) the result is a dupla superpartient two thirds. This dupla is the third dupla in the sequence, as it is the dupla of the number three. Thus, you must not think it strange if a dupla multiplied with another dupla produces only a superpartient dupla, because the produced dupla is of a differente and higher degree, namely, of the third one, while its components are of the first degree. Although they share a name, they differ in their products and in their degree. This occurs when the proposed proportions to be multiplied are continuous [-f.16r-] as to their order, which is unbroken. Were they not to be continuous, one has to place the products beneath according to their specific

sequence, as in the following example: 2/1 5/4 6/5 8/7 10/9, where one finds the sesquialtera, the sesquitertia, the sesquisexta and the sesquioctava. If you proceed to multiply the numerators and the denominators, the product will be [3/1 superpartient 7/25 add. in marg.] 4800/1260, which would be a tripla superpartient twenty-seven fifths. However, this is false, as you will be able to know by proceeding according to the following rule, namely, by multiplying first two of those proportions and then the following one, and so on. When they are not continuous, as in this case 2/1 5/4 6/5 8/7 10/9, always begin to multiply the denominator of the larger one with the numerator of the smaller one starting from the last two proportions presented, namely, 8/7 10/9. 7 multiplied by 10 equals 70. Then, 7 multiplied by 9 equals 63, while, transversally, 8 multiplied by 9 equals 72. The proposed proportion will be ordered in this way: 72 8/7 63 10/9 70. Multiply them by the denominator of the following proportion 6/5. They will be: 432. 360. 315. 350. The last one is obtained always by [-f.16v-] multiplying the largest one by the numerator. In this case 6 multiplied by 72 amounts to 432. You will proceed with all the other proportions one by one until the end. These are the products in the proportions in the required sequence.
4320 2/1. 2160. 5/4 1728. 6/5. 1440. 8/7 1260. 10/9 1400. The required product and proportion will be 4320/1400, namely, tertia 3/35 tripla superpartient three thirty-fifths. However, if you started the multiplications from 7, 7 times 9 is 63 and so forth, the proportions would not occur at the required place in the series, but they would be ordered thus: 72 70 63, while the ones derived from them and the ones placed last would be ordered thus: 4320. 2/1 2160. 5/4 2728. 6/5 1440. 36/35 1400. 11/10 2260. Here, the two placed last are not the required ones because they are 36/35 and 11/10, where we need 8/7 10/9. However, if one proceeds from the beginning according to the advice I have given, all the required proportions as well as their product shall be found in their place. Conversely, if one follows the false procedure, the result will be 4320/1260, which is tripla [-f.17r-] superpartient one seventh. Division is the proof of multiplication. Also, the result is verified by multiplying the numerators of the produced proportions by the denominator of the simple proportions, and, in criss-cross fashion, by multiplying the smaller numbers that generate the simple proportions themselves. In fact, if the product is the same, you will have proceeded appropriately; if it is not, you will have not. As 1560/240 is in sesquialtera proportion, take the first sesquialtera, which is 3/2. Now multiply 3/2 x 360/240 in criss-cross fashion. 2 x 360 is 720. Equally 3 x 240 is 720, the same number. Therefore, you were correct. The third proof of the multiplication consists of dividing the numerator of the product by the numerator of the simple proportion itself and by dividing the denominator by the denominator. If the result is the same, you will have been correct, otherwise you will have not. For instance, in the case of 360/240 3/2, divide 360 by 3, which equals 120; then divide 240 by 2, which also equals 120. Therefore, it will follow that 360 240 is in sesquialtera proportion as it is [-f.17v-] 3 2.

On the division of proportions, or proportionality.

As there are different sorts of proportions, each of them has its divisor, which is the median term between the two extremes. This divisor is called proportionality. Arithmetic considers numbers, and the excess must be the same. In order to find the divisor in the arithmetic proportion or proportionality, double the numerators and the denominators and add them together. I maintain that half of this sum is the divisor and the required proportionality. For instance, [3/2 multiplied by two is 6/4; the sum of numerator and denominator is 10; halved is 5, required diuisor add. in marg.] if I look for the divisor of 3/2, I double it and I obtain 6/4. The sum of doubled numerator and denominator is 10. Now I maintain that the divisor is 5, half of said sum. This is how to find the arithmetic proportionality. Given 6 5 4, 5 is proportionality of 6 and 4. [6. 5. 4. The excess is one and it is equal arithmetic. 3/2 4/5 5/6. add. in marg.] Moreover, the excess is the same arithmetic, which is one, as it was between 2 and 3. Nevertheless, said proportions are unequal because is a sesquiquarta and the other one is sesquiquinta, while the proposed 3/2 is [-f.18r-] sesquialtera. These two qualities are required in the division of the arithmetic proportions, namely, that the proportions hailing from the division be unequal and that the excess be equal and the same. The geometric proportion is divided by multiplying the denominator of the proposed proportion by the numerator. This product shall be one extreme and the root of this product shall be the divisor or geometric proportionality between those extremes. [4/1 moltiplied gives 4/4; the root is 2 add. in marg]. How to divide geometrically the quadrupla 4/1: I say that the denominator should be multiplied by the numerator. The product is 4 4. The root of 4 is 2, which is the required proportionality [1. 2. 4. dupla dupla add. in marg.] and divisor. It will be placed thus: 1 2 4. Here, the proportions are double and the excess is unequal, namely, from 1 to 2. I state that these qualities are accessory in the division of the geometric proportion, that the derived proportions have to be equal and the same, while the excess is unequal. When one cannot obtain the exact root of the multiplication, in that case one puts the unexpressed or 'deaf' root [3/2 multiplied is 6 add. in marg.] with the word root. For instance, to divide geometrically 3/2, the multiplication will produce 6, [-f.18v-] which has no precise root. Therefore, I shall leave it unexpressed or 'deaf'. To sum up, the root 6 is the geometric divisor of 3/2, and it shall be written thus: 2 root 6 3. However, when the multiplication has a precise number, in that case one must place the precise number. For instance, find me the divisor or geometric proportionality [16/4 multiplied produces 64, whose root is 8. Hence: 4. 8. 16. add in marg.] of 16/4. 16 multiplied by 4 is 64, whose root is precisely 8. It will be laid out thus: 4 8 16, as 8 is the divisor or geometric proportionality required. The same applies to the others. Musicians consider the harmonic proportions in order to calculate its proportionality. As this science is subordinated to arithmetic, we need to find the proportionality or arithmetic divisor by following the rule already given. Then, multiply the extremes by the arithmetic divisor that you have found. Finally, multiply the extremes themselves with each other, and you will obtain the harmonic proportionality, or harmonic divisor. [3/2 6 4 10 5 diuisor add. in marg.]. In order to calculate the harmonic proportionality of 3/2, first of all find the arithmetic proportionality, which will be [6. 5. 4. add. in marg] 6 5 4. Then, multiply the extremes 6 and 4 [30 24 20 harmonic proportionality add. in marg.] by the arithmetic divisor 5. The products will be 30 and 20. Finally, [-f.19r-] multiply the extremes themselves. Their product will be 24. Thus, the parts shall lay in harmonic proportion, in this way: 30 24 20 (harmonic proportionality). They are unequal, as one is a sesquiquarta and the other one a sesquiquinta, but the excess is not the same, as in the arithmetic proportions. Moreover, in these harmonic proportions the greater proportions have larger numbers. In fact, 30 to 24 is a sesquiquarta with 6 as the excess, while 24 to 20 is a sesquiquinta, which is a smaller proportion than the sesquiquarta, while the excess is 5. The opposite is observed in the arithmetic proportions, in which the larger proportions have a smaller numerical excess, while the smaller proportions have a larger one. The reason for this consists in the fact that arithmetic centres on multiplication, while music is based on division, as it considers the proportion that is born of the division of the sonorous string. The division is proven via multiplication. Although it may appear sometimes that smaller proportions are born of this multiplication, consider that they do not belong to the same genus and degree, as I said above.

[-f.19v-] On adding together the proportions.

According to Zarlino, in order to add together the proportions, add the numeators and place them above the line. Then, add the denominators and place them under the line. You will obtain the required sum is this way [4/3 6/5. 10/8 sesquiquarta of the second degree. add. in marg.]: 4/3 and 6/5 (sesquitertia and sesquiquinta) make 10/8 (superpartient one fourth). Do not think that the latter should be smaller than the two preceding ones, as the produced one is a sesquiquarta of the second degree (whose components are of the first simple degree) and this second order has greater arithmetic excess, albeit it has the same geometric one. If you then want to ascertain the degree of the proposed proportion, it is equivalent to the difference between the numerator and the denominator. In this case, 10 exceed 8 by 2. Therefore, it is of the second degree. We shall say, on this basis, that 4/3 6/5 produce the second sesquiquarta, which produced proportion can also derive by any other proportions whose sum of numerators and denominators is the same. For instance, 2/1 and 8/7 produce 10/8, as also 3/2 and 7/6 [-f.20r-] produce 10/8. They are all types, as it is the half of the denominator. Hence, there are four types in the proposed example, namely, 2/1 8/7 3/2 7/6 4/3 6/5 5/4 5/4. If you now add 10/9 and 7/6, the result will be 17/15, which is superpartient two fifteenths and the second proportion in the order of the superpartient fifteenth proportions. The same will derive from any other two proportions added together that produce 17 in the numerator and 15 in the denominator, such as 11/10 6/5 and 9/8 8/7. This can be done in seven ways, because the whole half of fifteen is seven, disregarding the decimal point. When you add two proportions, operate as above. Consider what sort of proportion has been produced and you will know if it is multiplied and superpartient. Everything said above refers to the proportions of greater inequality when the numerators are larger than the denominators. You will do the same in the case of the proportions of smaller inequality by adding the prefix sub- to the proportion produced by the sum, since it means ‘less’ and ‘defective’. In fact, if one adds smaller quantities, one obtains a smaller quantity; if one adds larger quantities, one obtains larger quantities. When you have to add more than two proportions, add them in pairs. When you sum proportions of greater inequality[-f.20v-] with proportions of smaller inequality, which is the same as to say that the plus and minus sign are added together, you will say that you counter the minus with the plus sign. For instance, if you add 14 with -3, it will produce 11 10, while the denominator remains the one of the proportion that has the larger number. For instance, if you add a sesquiquarta with a sesquiottava, you take away 1/8 from 1/4, which produces one eights and, therefore, a sesquiottava [with 1/8 it produces 7/8 add. in marg.]. I give you a general rule, brief, clear and infallible with regard to adding proportions of any sort, be they larger, smaller, multiplex, superparticular or mixed. I say that, given two proportions to be added together [5/4 7/6 26 40 produces 68/32, itself sesquiottava add. in marg.] multiply in criss-cross fashion numerators and denominators. Place the respective products above and below the line. Then consider how many times the numerator contains or is contained by the denominator. [<..> 3/2 5/4 produces 7/3 6/4 43/12 superpartient 7/12. add. in marg.] Thus, you will obtain the name of the proportion produced by the sum of the proposed proportions. If the denominator is larger, you will add, because the proportion remains of smaller inequality. [-f.22r-] [proportions to be added together 4/5 5/6 3/4 8/7 add. in marg.]

When you have several proportions to add, add the first two, add their sum to the third one, and finally the resulting sum with the fourth proportion, and so on. [4/5 x 7/6 24 35 equals 59/30 add. in marg.] Then, completed the operation, put in the last place the larger and smaller number and determine the denominator derived from the sum of all the proposed ones added together. [59/30 x 3/4 90 236 fa 326/ 320 add. in marg.] [326/120 with 8/7 960 2282 gives 3242/840, fifth proportion, which is tripla superpartient 722/840 add. in marg.] When the numerator and denominator are the same number, in that case the proportion shall be a proportion of equality, as in the examples shown at the side. You will note from this the very glaring mistake of those who add by multiplying them all the numerators and denominators, placing the first ones above the line and the others below it. In the case of 3/2 4/3 5/4 6/5, they produce 360/220, but, according to our method, the result is 634/120, which is quintupla superpartiente 17/60.

On subtracting proportions.

Zarlino and others practice the subtraction of proportions by multiplying in criss-cross fashion, by subtracting [-f.22v-] [de <..>x 4/3 <..> sesquiottaua. add. in marg.] the smaller from the larger one and by naming the proportion deriving from the subtraction from the excess, as one can see in margin. This is notoriously. In fact, if we subtract a sesquitertia from a sesquialtera, the remainder is not a whole proportion, but only the remainder of the fractions from 1/2 to 1/3, which is 1/6. Therefore, the proportion that originates is subsecupla and will be written thus: 1/6. You will proceed multiplying across and by subtracting the smaller from the larger number. You will place the difference above a line. Then you will multiply the [<..> x 4/3 1/6 3/5 x 5/6 18/25 7/30 <tre>decupla partient two <sev>enths num< bers ..> 3/5 <p>roduce 125/250. add. in marg.] denominators and you will place them under the line according to the example in margin. You will adopt this rule when you subtract from any sort of proportion. For instance, in order to subtract 3/5 from 5/6, when you multiply across, the result is 18/25. Their difference is 7. Then you multiply the denominators with each other, which will give you 7/30, subquadrupla superpartient two sevenths. To verify this, add the produced proportion with the one subtracted. For instance, if you add 7/30 with 3/5 according to the rule previously given, you will obtain 90/35, which is subsescupla. Therefore, the result will be always exactly accurate if you proceed according to our rule. [-f.22r-] Note that it does not follow that it would be possible to subtract a larger proportion from a smaller one of the same degree and order. For instance, one may not subtract 3/2 from 4/3, which is a smaller proportion, albeit each of them is of the first simple degree. However, were they to be of different degrees, it is possible for said proportion from which one subtracts to be smaller, because the excess of the number is greater. For instance, [9/3 subtracted from 16/8 72 48 produces 24 24 add. in marg.] if one wanted to subtract 9/3 from 16/8, namely, a tripla from a dupla, I say that it is possible because they are not of the same degree. In fact, the tripla is of the sixth degree (signalled by the difference between nine and three) while the dupla is of the eighth degree (signalled by the difference between sixteen and eight). [6/3 from x 12/8 36 48 sub dupla 12/24 add. in marg.] Therefore, if one follows the given rule, the result will be a proportion of equality. Thus, the subtraction of 6/3 from 22/8 [6/3 subtracted from 9/6 27 36 subdupla 9/16 add. in marg.] produces the subdupla 12/24; 6/3 subtracted from 9/6 is possible, because the dupla and the sesquialtera are both of the third degree and they produce a subdupla. Finally, if one subtracts 7/1 from 16/8, namely, a sepcupla of the first degree from a dupla of the eighth, the result [26 56 produces 40/8 add. in marg.] produces 40/8, which is the quintupla of the thirtieth of the second degree. If we add 40/8 with 7/1, we obtain 96/8. If we subtract 56, the result is 40/8; if we subtract 40, we obtain 16/8.

[-f.22v-] On extracting the root of the proportions.

If the numbers of the compound proportions are compound themselves, seek out the largest number that may express both proportions. If you divide the proportions by this number, you will have the prime radical numbers, namely, the required root. Now, in order to find the largest number that divides them, use this method. Firstly, divide the larger number of the proposed proportion by its smaller number. Then, divide the smaller number by the excess of the first division. Finally, divide this excess by the second excess. Carry on until you find a number that divides them neatly without excess. This shall be the required number. You will divide the numerator and denominator of the proposed proportion and you will find the required root. [45/40 is 1; the excess is 5. Divide 40 by 5. The result is 8, exact number without excess. Hence, 5 is the required number. Divide 45 and 40 by 5. The result is 9/8, the required root. add. in marg.] For instance, if you want to find the rood of 45/40, divide 45 by 40. The excess is 5. Now divide 40 by 5. The result is 8 without excess. This 5 is the largest required number by which 45 and 40 are divided. Divide both of them by 5. The result will be 9/8, sesquiottaua, [-f.23r-] the required root. This is the procedure to be followed when we look for the root of a single proportion, but when there are several proportion whose root is required, I advise you to use the same method as before. Divide the first two larges numbers by the smallest one. For instance, given 360 240 180 144, find me the largest [360
240 120 60 120 60 144 60 12 add. in marg.] number that may divide the two largest ones given, namely, 360 and 240. You will find it to be 120. Then, find a common divisor of this number and of the one coming afterwards, 180. You will find it to be 60. This number divides the proportions 360 240 180 120. Then, find the largest number of 144 (the fourth portion) which is also a shared divisor of 60. You will find it to be 12. This number is the divisor of all the proposed proportions and you will have found the required root. Let us now divide the parts 360 240 180 144 and 120 by 12 as its largest divisor: 30 3/2 20 4/3 15 4/5 12 6/5 10.
The end of the first book.