Author: Bottrigari, Ercole
Title: On the Commentary by Plutarch of Chaeronea on the Creation of the Soul described by Plato in his Timaeus, in that section which pertains to the Music of the Cosmos, translated into Italian by Knight Hercole Bottrigaro and illustrated with some considerations by him. At Bologna, 1610.
Source: Bologna, Museo Internazionale e Biblioteca della Musica, MS B44, 1-14

[-1-] ON THE COMMENTARY BY PLUTARCH OF CHERONEA **[He lived 122 years after Christ, under the emperor Trajan in marg.]** on the Creation of the Soul described by Plato in his Timaeus, in the section which pertains to THE MUSIC OF THE COSMOS, translated into Italian BY KNIGHT HERCOLE BOTRIGARO AND ILLUSTRATED BY HIM WITH SOME CONSIDERATIONS AND EXERCISES.

SOCRATES, in the Republic, when begins to describe the number in the Republic, which was called by some 'Coupling', said that the divine Nature is generated and converted, and this process is contained in the perfect number. Here, by the words 'Nature that is generated' he means nothing else but the Cosmos.

The first one is the one of the Monad [1. add. supra lin.] and of the couple [2. add. supra lin.], namely, of the number one [1. add. supra lin.] and of the number two [2. add. supra lin.]; The Second one consists of the Ternary [3. add. supra lin.] and of the [**1. 2. 3. 4. 5. 6. 7. 8. 36. in marg.] [When the two numbers of each of the four parts of that Tetractys, or Quaternio are multiplied with each other, their four products added together amount to 100, which is the Circular and perfect number
[[Bottrigari, On the Commentary by Plutarch, 1; text: 1. 2. 3.4. 5. 6. 7. 8. 12. 56. 30. 100.] in marg.]**

Plato mentions the Tetractys at page 5. 6. A and Macrobius talks about it in the Dream of Scipio, first book, Chapter 6. in marg.] Quaternary [4. add. supra lin.] or, as we say, of the number Three [3. add. supra lin.] and of the number Four. The third one of the Fifth and Sixth number, namely of the number five [5. add. supra lin.] and of the number Six [6. add. supra lin.] (each of these sounds by itself nor does it produce the perfect number by being added to the others). The Fourth consists of the number Seven [7. add. supra lin.] and of the Eighth number, or, as we say, the Seven [7. add. supra lin.] and the Eight [8. add. supra lin.], and this one, added to the two mentioned above amounts to thirty-six [36. add. supra lin.], which is a square number. That Tetractys (also called Quaternio) of the numbers reported by Plato has the most perfect Creation, as in it the even Intervals are multiplied with the Even ones, and the odd ones with the Odd ones. Now, the number one [[contains]] [is the corr. supra lin.] the common principle both of the Even and of the Odd ones. **[On these seven odd and even numbers which concur in the procreation of the soul according to Plato in the Timaeus, see Plutarch himself in his Commentary, or dialogue on Music, Macrobius in the Treatise on the Music of the cosmos on the Dream of Scipio by Marcus Tullius Cicero and the one by Marcus Tullius himself on the creation of the Universe in marg.]** Then the number Two [2. add. supra lin.] and the number Three [3. add. supra lin.] which are the origin [-2-] of those that are called plane and superficial. The number Four [4. add. supra lin.] and the number Nine follow, which are the first Square numbers, and finally the Eight [8. add. supra lin.] and the twenty-seven [27. add. supra lin.], which among the numbers (leaving aside the number one) are the first Cubic numbers. Therefore, it is also clear that he did not want that those numbers should be laid out in a straight line but in two series, one of

which contains the even numbers, and the other one the odd ones, as we have described them here beneath. **[Macrobius in the Dream of Scipio of the Music of the Cosmos, Marcus Tullius Cicero in the Creation of the Universe and Plutarch himself in his won Commentary or Dialogue on Music in marg.]**
[Bottrigari, On the Commentary by Plutarch, 2, 1; text: 1. 2. [piani add. supra lin.] 4. [quadri add. supra lin.] 8 [Cubi add. supra lin.] 3/5. 9/13. 27/35.]

And in this way, since the similar numbers will be added with the similar ones, the compounded with each other, and the multiplied ones with each other as well, they will produce even numbers. By composition, they will be added in this way: Two [2. add. supra lin.] and three [3. add. supra lin.] make five [5. add. supra lin.]; four [4. add. supra lin.] and nine [9. add. supra lin.] make thirteen; and eight [8. add. supra lin.] and twenty-seven [27. add. supra lin.] make thirty-five [35. add. supra lin.]
[[Bottrigari, On the Commentary by Plutarch, 2, 2; text 256, 81, 256, [236 ante corr.] 2048, 4i20736, 5184, 243, 64, 972, 1458, [1358 ante corr.], 3115552, [14552 ante corr.] 5184 Limma ditono dia[[.]]esi.] add in marg. alia manu]]

Of these, the Pythagoreans called the Fifth number [[[trophon] trofon]] [[[Trophon ò trofon corr. supra lin.] [Phtoggon, add. supra lin.], namely, [[sound]] [[pthongon, or sound corr. supra lin.]] [[phthoggon], Sound add. supra lin.], stating that, of the Intervals [[of the Sounds]] the five is the [first one add. supra lin.] [[that can be played]] [which creates a sound corr. supra lin.] **[The 13 is the difference of two numbers and represents the proportion of the Limma, or smaller natural Pythagorean Semitone, which is between 256 and 243 and it is the quantity that added to the proportion 81 to 64 of the two Sesquiottavi Tones, their sum is the proportion 4 to 3, Sesquiterza which represents the Diatessaron. The number 35 is called Harmony by the Pythagoreans because it contains within itself the Sum of the addition of the four numbers of the Enigma of the [mythical add. supra lin.] Pythagoras' Hammers. See my exposition and Explanation of that Enigma. The harmonic proportion is found between 12, 8 and 6, namely in the radical numbers 6, 4 and 3, which and it pertains to the dupla proportion, while the one pertaining to the tripla proportion is the 6, 3, 2 in marg.]** The number Thirteen [13. add. supra lin.] is called by them Limma, or, as they called it, [[remaining]] [remainder corr. supra lin.], since they had no hope to divide the Tone into two equal parts. Then, they called the number thirty-five [35. supra lin.] Harmony **[[8 27 35. 8 in marg.]]** and therefore It is the converging or discerning quantity, because it is composed of the first two Cubes **[8 27 25 8 in marg.]**, one of the even numbers and the other one of the odd ones. Similarly, out of these four numbers **[6 8 9 12 35. in marg.]** six [6. add. supra lin.], eight [8. add. supra lin.], nine [9. add. supra lin.], twelve [12. supra lin.] in which the Arithmetic proportion and the Harmonic one are contained. The strength of these will be rendered clearer with the following Demonstration. Draw the Rectangle [-3-]

[[Bottrigari, On the Commentary by Plutarch, 3; text: Progressione, A N. D K M O B L C 6 8 9 12] in marg.]

[<per>taining to the tripla proportion is the 6. 3. 2. in marg.] whose side A B is of five small squares, while B C is of seven. Divide the side AB into two and three parts and mark the point K. Divide the side B C into three and four parts and mark the point L. Then draw the straight line L M N from the point L, which meet in the point M. Thus, the figure A K M N contains six [6. add. supra lin.] small Squares, the [N add. supra lin.] M

O D eight [8. add. supra lin.], the B K M L nine [9. add. supra lin.], and the M L C O twelve [12. add. supra lin.]. Hence, the whole of this Parallelogram, which contains thirty-five [35. add. supra lin.] small Squares, demonstrates the proportions of the [[first]] main Consonances. In fact, **[6/8 ¾ in marg.]** the number six [6. add. supra lin.] and the number eight [8. add. supra lin.] are in Sesquiterza proportion, which represents the Consonance called Diatessaron, or Fourth. **[6/9 2/3 in marg.]** The number six [6. add. supra lin.] and the number nine [9. add. supra lin.] **[6/12 ½. in marg.]** form the Sesquialtera proportion, which is particular to the Consonance called Diapente or Fifth. The numbers Six [6. add. supra lin.] and Twelve [12. add. supra lin.] contain the dupla proportion, which pertains to the Consonance called Diapason or Octave. Beyond this, there is the sesquiottava proportion, namely from Nine [9. add. supra lin.] to eight [8. add. supra lin.], and for this reason that number, that contains all of these was called Harmony. **[The number 35 was called harmony, because It contains all these harmonic proportions, as it has been noted [a little add. supra lin.] earlier in marg.] [35 : 6 210. in marg.]** The same number thirty-five [35. add. supra lin.] multiplied by six [6. add. supra lin.] gives two hundred and ten [210. add. supra lin.], in which, as they say, the gestation of children born within seventh months is complete. **[30:7 210 in marg.]** Then, starting again and multiplying **[2 3 6. 4 9 36. in marg.]** the number two [2. add. supra lin.] with the number three [3. add. supra lin.], their product is six [6. add. supra lin.]; the number four [4. add. supra lin.] multiplied by nine [9. add. supra lin.] makes thirty-six [36. add. supra lin.], and the number eight [8. add. supra lin.] **[8:27 216. in marg.]** multiplied with the number twenty-seven makes two hundred and sixteen [216. add. supra lin.]. Of these, the number Six [6. add. supra lin.] is perfect it is equivalent **[2 3 6. in marg.] [A perfect number is (as Euclid defines it in the twenty-fifth Definition of the seventh book of the Elements according to Zamberto and in the seventh of the ninth book of the elements according to Campano) one that is equivalent to all of its fractional parts added together. Of the perfect numbers, the six is the first one smaller than ten, and between ten and one hundred there is only the twenty-eight. See Fra Luca from Borgo in his Somma di Aritmetica and Boethius in his Arithmetic, noting that he gives as a strict Rule that that such perfect numbers end alternatively in six and eight. This is not true, however, because the number 209128 is a perfect number, and although it is the next one after 8128, which is the perfect one that precedes it, it does not end in six. Similarly, the perfect number 8589869056 follows the one that precedes it which also terminates in six. Note also that Rufus, the commentator of Boethius' Arithmetic, [after the number corr. supra lin.]] [[<..>]] 8128, puts the number 130816 as a perfect number, which is really a superabundant number. Equally, Faber Stapulensis, in his Platonic Diagramma, puts the number 5368528 as a perfect number, but it is also superabundant. See the Treatise on the perfect numbers by Cataldo. You must also know that the number six, besides being called perfect, and coupling, or knot, as Macrobius calls it some way further on, is also called circular and Triangular. It is very clear that the number thirty-six is the square of the number six, since the number six multiplied by itself equals thirty-six. But it is not so clear how it is Triangular in relation to the number eight. Therefore, you must know that, if one wants to describe and create the Triangle of said thirty-six, it is necessary that [[said Triangle]] [It add. supra lin.] is equilateral, or, with the sides equal as to quantity. So, starting from the number one in of the Angles of that Triangle, one must replenish all its Surface with the other numbers ordered in series, in this way. So in marg.]** to all of its parts. It is called also coupling because of the mixing of the even numbers with [-4-] the odd ones, and, therefore, of the first even and the first odd number. The number Thirty-six is the first number which

[Bottrigari, On the Commentary by Plutarch, 4, 1; text: 1 2 3 4 5 6 7 8 36.] in marg.]

[6 6 36. in marg.] is both Square and Triangular. It is the Square of the number Six [6. add. supra lin.], and Triangle of the number eight [8. add. supra lin.]. The same derives from the Multiplication of the first two Squares, one by the other, four [4. add. supra lin.] and nine [9. add. supra lin.] **[4 9 36. in marg.]**. The Sum of the first three Cubes, namely, one [1. add. supra lin.], eight [8. add. supra lin.] and twenty-seven [27. add. supra lin.] amounts to this number. Beyond this, **[1 8 27 36. in marg.]** if one takes the number Twelve three times [12. add. supra lin.] and the number nine four times, it creates a Parallelogram which is longer on one side
[[Bottrigari, On the Commentary by Plutarch, 4, 2; text: 12 .3. 4. 9.] in marg.]

One must add that, by exposing the numbers of the sides, the one of the Square to the number six, and the one of the Triangle to the number Eight, and to one of the Parallelograms the number nine [9. add. supra lin.], and to the other one the number twelve [12. add. supra lin.] the proportions of the Consonances will be formed. In fact the proportion from Twelve [12. add. supra lin.] to nine [9. add. supra lin.] [[signifies]] [demonstrates add. supra lin.] the Diatessaron, such as it is the one contained within the low Note Hypate [namely, principal add. supra lin.] to the Mese, or Middle one; the proportion from the Twelve [12. add. supra lin.] to the eight [8. add. supra lin.] demonstrates the Diapente, such as the one contained within the Mese, or Middle one, to the highest one, or Nete. **[[the number 216. not only is the Cube of the number 6, but it is surface <.....> of the number 8. <....> 27. <tria verba desunt> in marg.]]** The proportion from Twelve [12. add. supra lin.] to [[eight]] six [6. add. supra lin.] [[8. add. supra lin.]] demonstrates the Diapason, as from the Low [principal add. supra lin.] Note to the [acute add. supra lin.] highest one. The number Two hundred and sixteen [216. add. supra lin.] **[6 6 36 6 216 8 27 216, as it was above 1 2 3 4 8 9 27. in marg.]** is the cube of Six [6. add. supra lin.], [[being equal to the perimeter of its Sides]] [which is a number equal to the sides that make up its perimeter add. supra lin.]. Now, since these are the proprieties of the mentioned number, the twenty-seven [27. add. supra lin.], which is the last one, has this particularity, namely, that It contains the Sum of [**As to the twenty-seven days that Plutarch says that are needed for the Moon to complete one of its revolutions, or, for it to return in the same place of the Zodiac, from which it departed, one can in [understand that the twenty-seven Mansions of the Moon itself, each of whom contains19. 20 of the 360. In which the Zodiac is divided. See Alkindo near the end of Chapter eight and of all his book on the Mutations of the Sky, which he entitled Epistle; Similarly see the ... towards the middle of the second Chapter and the beginning of the third of his book on the Mutations of Time. At the beginning of it there is an ample and detailed Table of those 27 Mansions of the Moon, since He establishes that there are some who want that the Mansions of the Moon are 28, and each**
[Bottrigari, On the Commentary by Plutarch, 5, 1; text: 1, 2, 3, 4, 10. 8, 15, 9, 27, 40. 13]

therefore, of
[Bottrigari, On the Commentary by Plutarch, 5, 2; text: 12. 51. 25. 43].

Therefore, it will be good to see what Firmino writes in the second Chapter of his Repertorio della Mutatione dell'aere, and also the Annotations B at page 306 which

Filippo Iollaino wrote on it. Plutarch then describes with very obscure brevity how the Pythagoreans marked with the number 27 the musical Tone, in the same way as He says afterwards that they called the number 13 Limma, or Reminder. The fact that He adds nevertheless that the number 13 was called Limma because it is removed from the half of the 27 itself (since I have not found, in my reading, any writer who mentions a word on this, let alone who discusses it) has shown me the way to know, after long speculation, why the 27 was called tone by those Pythagoreans. One has to know first of all that the number 13 is the difference between the numbers of the proportion of that limma, 256 and 243, which is the remainder or left over part between the sesquiterza proportion, which represents the Diatessaron, and the proportion of the two sesquiottavi Tones, which is between 81 and 64 to complete the Diatessaron or sesquiterza. Now, given that one wanted to add a sesquiottavo Tone to the proportion of the limma 256 243 adjacently towards the high register, one will suppose that the 243, the smaller number of that proportion, is (as it is, if one divides it) the number nine, and next to it one will put the number 216, which, it one divides it, gives the number 8 exactly. Therefore, it will be like this: 256 243 216, as one can see that Plutarch himself puts towards the end of this Commentary when he describes the Species of the Pythagorean Diatonic Tetrachords in numbers according to the proportions of the said Limma and the Tones. Therefore, given that this is such, as the difference of those two numbers of the proportion of the Limma is 13, thus the difference of the two numbers of the proportion of the sesquiottavo Tone 243 216 turns out to be 27, which is the one that was called by the Pythagoreans Tone by excellence for this reason. This is confirmed through the description of the second Species of the Diatonic Tetrachords themselves, which then has the Limma in the second degree towards the high register, while the Tone itself follows it adiacently as the third and last degree in that Diatessaron also in this way
[Bottrigari, On the Commentary by Plutarch, 5, 3; text: differentia 256 13. 243 27 216 Differentia, Limma, Tuono, prima, specie, differentia, Tuono]

in marg.] all the previous ones: and it is precisely the number of the days in which the Moon completes its Revolution. And the Pythagoreans marked the Tone between the intervals with pertain to Music: For this reason they called the number thirteen [13. add. supra lin.] Limma, which means [[lacking]] [remainder add. supra lin.] because it is one unit removed from the half. And it is clear,
[-5-] **[[Bottrigari, On the Commentary di Plutarch, 5, 2; text: 1 2 3 4 5 6 7 8]**

In this Example one can see first of all that each of those Sides [[is created]] [is based on corr. supra lin.] the number 8, in equal fashion. Then, the second sequence is formed by the number seven, the third one by the number six, the Fourth one by the number five, the Fifth one by the number four, the Sixth one by the number three, and the seventh by the number two. The same happens in every part of it, since the number One is always at the top of the Angle, and these numbers in succession 8, 7, 6, 5, 4, 3, 2 and 1, add up to 36, which connects the first four couplings of the Tetractys, or Quaternio. From the particular multiplications of one number by another one, which are, 2, 12, 30 and 56, added together is produced the number one hundred, square of ten, which is also a perfect number. The number two is born from the multiplication between one and two, the number 12 from the multiplication between three and four, the number thirty from five timed by six, and the number fifty-six from seven timed by eight. Like thirty-six, said number one hundred derives

from the multiplication of four by twenty-five, the first one being the square of the fiftieth part of it, and the latter the square of five, which is the twentieth part of it. Thus, taken the second one four times, it makes 8, and, equally, four times three is twelve, four times four is thirty-six. Thus, those numbers 4, 8, 12 and 16 added together add up exactly to 40, just as four times ten, which is also called perfect number, is forty. It is called perfect not because it is the sum of his fractional parts, like the numbers six and twenty-eight, but because after it every number is repeated and added to it, and we say eleven, twelve, thirteen, fourteen and the others up to 100, which is the second Decenario, and this rule is followed always from one decenario to the next one ad infinitum. Hence, Aristotle in the first book of the Metaphysic said: "like when the number ten appears to be perfect and to comprehend all the Nature of the numbers." And in the third book of Physics (Tartaglia, Commentary 61) one finds "that the number does not go beyond ten, but is repeated in marg.] as to the odd numbers, the first three, namely 1, 2 and 9 added together make 13, and added to 27, which is the first Cube of the odd ones [[quarto, et ultimo]] add up to forty [40. add. supra lin.]. The proportions of the Consonances are also contained in these very numbers. In fact, the number two [2. add. supra lin.] is in dupla proportion with the number one [1. add. supra lin.], which represents the Diapason. The sesquialtera proportion is between three [3. add. supra lin.] and two [2. add. supra lin.], and it represents the Diapente. The numbers Four [4. add. supra lin.] and three [3. add. supra lin.] are in Sesquiterza proportion which signifies the Diatessaron. The numbers nine [9. add. supra lin.] and three are in tripla proportion, which represents the Diapason with the Diapente, or Fifteenth. Between the number nine [9. add. supra lin.] and the number eight [8. add supra lin.] is the Sesquiottava, which represents the Tone. Now, if the number one is added with the even and odd numbers, their entire Sum will be Ten [10. add. supra lin.],
because the first four numbers added together starting from the number one [1. add. supra lin.] amount to ten [10 add. supra lin,]. Moreover, these even numbers one [1. add. supra lin.], two [2. add. supra lin.], four [4. add. supra lin.], eight [8. add. supra lin.] add up to fifteen [15. add. supra lin.] which is the Fifth one in the sequence of the Triangular ones. And, with regard to the sequence of the Odd ones, added together the number thirteen [13. add. supra lin.] and the number twenty-seven [27. add. supra lin.] one obtains the number Forty, and with these two numbers the Mathematicians measure suitably the Intervals suitable for singing, by representing the Tone with this one, and the Diesis with that one. The number Forty itself [40. add. supra lin.] is arrived to through the strength of the already mentioned Tetractys, or Quaternio. In fact, if one takes each <of the first four numbers four times> they will produce, <multiplied by four, the numbers four, eight, twelve and sixteen, whose> sum is <forty, and in this number are contained all the proportions which form the Harmony.> In fact, [-6-] sixteen [16. add. supra lin.] forms the Sesquialtera proportion [Sesquiterza add. supra lin. manu rec.] **[4 8 12 16 40. in marg.]** with the number twelve [12. add. supra lin.], in dupla proportion with the number eight [8. add. supra lin.], and in quadrupla proportion with the number four [4. add. supra lin.]. The number Twelve [12. add. supra lin.] is Sesquialtera proportion with the number eight [8. add. supra lin.], and in Tripla proportion with the number four [4. add. supra lin.]. The Intervals of the Diatessaron, Diapente, Diapason, Diapasondiapente and Bisdiapason are contained within these proportions. The number Forty itself [40. add. supra lin.] is composed by the two first Squares **[1. 4 8 27 40 in marg.]**, one [1. add. supra lin.] and four [4. add. supra lin.] and of the first two Cubes, Eight [8. add. supra lin.] and twenty-seven [27. add. supra lin.] Therefore, it follows that Plato's Tetractys, namely, the Quaternio [[**Macrobius in the dream of Scipio book 1, Chapter 7 in marg.]]** is

much more abundant and perfect that the Pythagorean one [page 5. C. add. supra lin]. However, since these numbers presented above do not allow space to the halves, as they are called, which are interspersed in this matter, it has been necessary to take other larger numbers disposed according to the same proportions. We have still to say which ones these are, but first I will take about this Halving. One of those occurs when, laid out three numbers as terms, the middle one exceeds the one of the extremes by the same quantity by which the other extreme exceeds it. They called this one Arithmetic Halving. The Contrary halving is the one where, disposed the same number of terms, the proportion with the extremes is the same. The terms of the Arithmetic one
[[Bottrigari, On the Commentary di Plutarch, 6; text: 3. 6. 9. 12. 2. 4.] in marg.]

are six [6. add. supra lin.], nine [9. add. supra lin.] and twelve [12. add. supra lin.], because nine [9. add. supra lin.] exceeds [6. add. supra lin.] by the same quantity by which it is smaller than twelve [12. add. supra lin.]. The terms the Contrary one are

[[Bottrigari, On the Commentary by Plutarch, 6, 2; text: 2. 4. 6. 8. 12. 3. 6.] in marg.]

six [6. add. supra lin.], eight [8. add. supra lin.] and twelve [12. add. supra lin.]. Of these, the number eight [8. add. supra lin.] exceeds the number six [6. add. supra lin.] by the number two [2. add. supra lin.] and it is exceeded by the number Twelve [12. add. supra lin.] by the number four [4. add. supra lin.]; but the number two [2. add. supra lin.] is one third of the number six [6. add. supra lin.] as the number four [4. add. supra lin.] is a third of the number Twelve [12 add. supra lin.]. In the Arithmetic division the middle Term exceeds and is exceeded by the same quantity, but in the Contrary one [-7-] It exceeds the difference from one of the extremes, but it is exceeded by the other one. In that one the number three [3. add. supra lin.] is the third part. In this one, the number two [2. add. supra lin.] and the number four [4. add. supra lin.] are the third part from the extremes. Hence it derived the name of Contrary. It is also called Harmonic,

[[Bottrigari, On the Commentary by Plutarch, 7, 1; text: <Me>dietà, 6. 8. 12. 2. 4. 3.] in marg.]

because it contains in its extremities the first Consonances: from the largest to the smallest term it contains the Diapason; from the largest to the middle one, it contains the Diapente, and from the middle one to the smallest one it contains the Diatessaron, as if one sets the largest Term in the [[Nete]] [Hipate add. supra lin.], the Middle one in the Mese, so that this one sounds a Diapente with the larges one and a Diatessaron with the smallest one. Therefore, the number eight [8. add. supra lin.] will correspond to the Mese, the number six [6. add. supra lin.], [[il sei [6. add. supra lin.]]] to the Nete, the Twelve [12 add. supra lin .] to the Hypate. Eudorus produced a simple and clear method to find one and the other of these varied ways of Halving.

[[Bottrigari, On the , 7; text: 3. 6. 9. 1 ½ 3/1 ½ 2. 7. 12. 1. 7, 1. 3. 4, 18. 1/3.] in marg.]

Let us consider that one first in the field of Arithmetic. If the extremes are set out and their halves are added together, the sum will be the number in the middle, be the proportion of the Extreme a dupla or a triple one [, or an other one add. supra lin.]. But in the Contra-harmonic Halving, if [**After the two types of Halving, Arithmetic and Harmonic which Plutarch teaches us in this passage, which pertain to Music, there is**

the Contra-harmonic Halving, which is necessary in our times. To set up its structure one will have to take two thirds of the larger term, a third of the smaller one from the dupla proportion laid out. Their sum will be the middle term. For instance, take the two Terms of the dupla which form the proportion 6 and 3. We take two Thirds of the larger term 6, which is 4, and of the smaller one which is <3> in marg.] if the proportion of the Extremes is a dupla, we will obtain the Middle term by adding a third of the smaller term and half of the Larger one. On the contrary, in the Triple, the Middle term will be Half of the Smaller one together with a third of the Larger one. Let us establish the Terms of the Tripla proportion as six [6. add. supra lin.] and eighteen [18. add. supra lin.]. Therefore, three [3. add. supra lin.], half of the smallest one, and six [6. add. supra lin.], the third part of the Largest one, added together make nine [9. add. supra lin.], which is the Middle term. This one from the same [-8-] **[we take a third, which is one. Those added together make five, which is the middle Term which we were looking for. It will be laid out like this: 6, 5, 3, contra-harmonic proportion. Boethius, at Chapter 51 of the second book of his Arithmetic, was perhaps the first one to [[provide an account]] give evidence of knowing this, but without [[giving]] [teaching corr. supra lin.] the above Rule and Method to find said middle term. When the harmonic proportion is laid out in its three particular terms, there is not only one, but two and three Methods to find the middle term between the two extreme terms first and third. The first method, therefore, consists in taking the difference or remainder between the first larger terms and the second, and in adding this to the third smaller term. Their sum will be the quantity of the middle term that we were looking for. For instance, let us take the harmonic proportion 6, 4 and 3. The difference between the [second add infra lin.] term and the middle one is 2, which added with the third and smallest term makes 5, which is the term what we were looking for, hence the proportion will be this one 6 5 3. The middle term can be found also with a second Rule, namely, by taking the difference between the middle Term and the third smaller one, and by subtracting this difference from the major term. What is left will be the middle term. Let us take, for instance, said harmonic proportion 6 4 3. The excess of the middle term 4 on top of the middle term 3 is one, which, subtracted from the first largest term 6 will give as a result 5 add. supra lin.]** exceeds and is exceeded by each one of the Extremes. Therefore, the varieties of halving will be found following this method. Now, it would have been necessary certainly to fill it up with the above mentioned numbers, and to complete with those the Intervals of the double and the triple ones. Some of those numbers have no space and others have space, but not sufficiently [[large]] [wide add. supra lin.] Therefore, those are multiplied in such a way that they have enough space to receive the above-mentioned varieties of Halving. First, then, put the number Six [6. add. supra lin.] instead of the number one [1. add. supra lin.], because it has, as before, a Half and a third part. Then, put the other remaining ones, which follow, as sesquialteri, as it is one can see in this example

[Bottrigari, On the Commentary by Plutarch, 8; text: 6. 12. 2. 3. 18. 24. 4. 6. 9. 54. 48. 8. 27. 162.]

Where are contained the both the types of Halving with the Intervals doubled. Now, since plato said that, since the Intervals of these combinations are already Sesquialteri, Sesquiterzi and Sesquiottavi, he completed all the Sesquiterzi and the Sesquiottavi in the first Intervals leaving to each one a small particle which the space from a number to the next one represents, so that their terms are two hundred and fifty-six [256. add. supra lin.] and two hundred and forty-three [243. add. supra lin.]. So, because of these words by

Plato they were forced to increase the numbers again, because it was necessary to put two Sesquiottavi numbers straight after the number six [6. add. supra lin.] in sequence, and the number six [6. add. supra lin.] cannot be divided by eight. If one wanted to reduce the proportion to the minute terms of the [-9-] **[middle contra-harmonic term add. supra lin.] [[only one will be left, namely 3, for the middle contra-harmonic [namely, [[6 5 3]] 6 5 3] as before. and if this add. supra lin.] This itself will also happen if the excess or difference 1 between number 4 and 3 will be subtracted from the larger term 6, which, equally, will leave 5. Thus, the middle contra-harmonic Term is corrected again on the basis of this third Rule, as it had been according to the other two in marg.]] [it turns out in the multipla proportion, but in the triple one, whose harmonic proportion is this one 6 3 2, one has to take the difference between the term six [6. add. supra lin.], and the smaller third one, and subtract that one from the first larger term, and what is left will be the middle one. Between the number 3, middle term, and the number 2, minor and third term, the difference is 1, which subtracted from 6, which is the first and larger term, leaves 5, which is the middle term which we were looking for, thus, 6 5 3. It is called Contra-harmonic because it contains, between the first larger Term and the middle one, the smaller number of the [[above-mentioned]] proposed harmonic proportion, and, between the [middle term and the third and smaller one, the larger number of said [[preceding]] proposed harmonic [proportion add. supra lin.], which gives equally [as the difference add. supra lin.] between the first term larger and the Middle one the larger number of said proportion, and between the middle term of the [<.....>] smaller number of said proportion <aliqua verba desunt> add. infra lineas.] . 64 is the Cube of the first square cube of the number 2, because 4 multiplied by 4 is 16, which multiplied by 4 is 64, and thus it is the cube of the first Square. 64 is also the Square of the first Cube, namely 8, because 8 multiplied by itself, and therefore squared, is exactly 64, which trebled in marg.]** number one, the speculation would become very doubtful. Therefore, the matter itself taught the help to be offered by multiplication, namely, that in the harmonic mutation all the description should be enlarged according to the enlargement of the first number. Therefore, Eudorus, following Crantor put the first number as three hundred and eighty-four [384. add. supra lin.]; **[64 6. 384. in marg.]** which is arrived at by multiplying six [6. add. supra lin.] by Sixty-four [64. add. supra lin.]. He was induced to this by the fact that the number

[[Bottrigari, On the Commentary by Plutarch, 10, 1; text: 64. 72. 81. 9/8] in marg.]

sixty-four [64. add. supra lin.] has as its sesquiottavo the number seventy-two [72. add. supra lin.] and this one has the 81 [81. add. supra lin.] as its sesquiottavo. It is much more in agreement with Plato's words that one should halve these numbers. In fact, if in the first place **[192. 256. 243. in marg.]** one puts the number one hundred and ninety-two [192. add. supra lin.], the Limma or the remainder after the two

[[Bottrigari, On the Commentary by Plutarch, 10, 2; text: 4./3 192 384. 216. 432. 243. 286. 256. 512.] in marg.]

sesquiottavi after it will be contained between the numbers two hundred and forty-three [243. add. supra lin.] and two hundred and fifty-six [256. add. supra lin.]. However, if one puts three hundred and eighty-four [384. add. supra lin.] **[64x3 192 72 3 216 81 3 243**

add. in marg manu rec.] the Limma will be placed in the same proportion, but with the numbers doubled. Therefore, it is certain that between five hundred and twelve [512. add. supra lin.] and three hundred and eighty-four [384. add. supra lin.] and between two hundred and fifty-six [256. add. supra lin.] and one hundred and ninety [192. add. supra lin.] there is the same proportion and the reduction to this number is not inconvenient, but it adduces successfully [**4 4 16 2 4 8 64 3 192 in marg.]** the reasoning behind Crantor's opinion. **[Plato writes about Crantor at the beginning of the Timaeus on the procreation of the soul in marg.]** In fact, sixty-four [64. add. supra lin.] is the Cube of the first square, and the square of the first Cube, and, multiplied by three [3. add. supra lin.], which is the first one among the odd numbers, among the Triangular, the Sesquialteri [Sesquipli ante corr.] and the perfect ones, produces one hundred and ninety-two [192. add. supra lin.], a number that has an other sesquiottavo in relation to it, as we will demonstrate. First of all though, we will see much better what the Limma is [-10-] **[, namely, multiplied by the number three, which is really the first odd, Triangular, Sesquialtero to the number 2 [[which is the first even number]], and also perfect (but not perfect according to the way that Euclid describes it in the second definition of the seventh of his Elements according to Zamberto and in the seventh definition of the ninth according to Campano, but according to the second First Distinction of the second Treatise of Fra Luca of Borgo. However I believe that it has to be considered perfect, as also the number two and some other similar numbers which we call perfect, in the sense of complete, excellent and holding great authority and optimal. The Harmonics are understood by Plutarch as Aristoxenus' followers, as those attributed every musical matter to the Sense of Hearing, while he called the Pythagoreans mathematics, as those who wanted that arithmetic dominated everything. Hence the Harmonics, namely the followers of Aristoxenus, divided the Tone and all the other musical Intervals according to the quantity rather than the quality. Conversely, the Mathematics, or Pythagoreans, divided the musical intervals only on the basis of quality. For this reason the Tone, which was found to be the difference between the Diapente and the Diatessaron, as Aristoxenus writes at the end of the first book and around the middle of the second one of His Harmonics, was divided by him and his followers in two equal parts according to the quantity of that note, while it was divided by the mathematics, or Pythagoreans, according to the quality of the sound of that note and also by the proportion of that Diatessaron, namely, Sesquiterza, since they supposed that the Tone was always represented by the sesquiottava proportion. Therefore, in marg.]** and Plato's intention, if we recall briefly those two matters which are usually examined in the Pythagorean Schools. In Singing it is called an Interval every distance which is contained within two sounds of different Tone. Of these one is the above-mentioned Tone, which is the difference between the Diapente, or major Fifth, and the Diatessaron, which we call Fourth. The Harmonics believe that this tone can be divided into two equal parts, each of whom they call Semitone. The Pythagoreans, who believe that it is possible to divide it into two equal halves, they called the smaller part of that one Limma, because it is not the exact half of the Tone. Therefore, the former composed the Diatessaron with two Tones and a Semitone, the latter, similarly, of two Tones and a Limma. The Harmonics believe that the sense of hearing give evidence of this, while the Mathematics trust the arithmetic Demonstration. The matter is as follows. It has been ascertained through the Instruments that the Diapason derives from the dupla proportion, the Diapente from the Sesquialtera, the Diatessaron from the Sesquiterza, and the Tone from the Sesquiottava. We can find practical evidence of this. If two weights in dupla proportion are attached to two gut strings, the string weighed down by the weight which is double that the other one will

produce a lower sound, at a distance which is the one between the Hypate and the Nete, namely, a Diapason. If two flutes with the same bore are one twice the length of the other one, the longest will produce the lower Sound, which will be distant to the other one by the distance that separates the Nete from the Hypate. Equally, [-11-] **[the followers of Aristoxenus divided the quantity of the Diatessaron into thirty particles equal to each other. They created the Tone with twelve of those, the Semitone with six, and, consequently, the Diapente with 42 of those particles and, finally, the Diapason with 72, which added up to six Tones of equal quantity. The Mathematics, or Pythagoreans, built the Diatessaron with two sesquiottavi Tones and with the Limma, under the sesquiterza proportion. Consequently, the Diapason was composed by that Diatessaron and that Diapente, of five sesquiottavi Tones and two Limmas super tredicipartienteduecentoquarantarte 256/243, and represented by the dupla proportion. On this see Ptolemy at Chapter 15 of the first book, and at Chapter 14 of the second of his Harmonics. A little further on Plutarch himself talks about it. If one attaches two weights in dupla proportion to two Strings made of gut or sinews or equal thickness and length, it is true that the String weighed down by the largest weight, which acts as a peg , will produce a higher sound than the other one, which is weighed down by the lower weight, will produce. But it is not true that one [d.....] sounds with the other one their Diapason, nor the Diapente and the Diatessaron those two in which in marg.]** if one takes weights which are in the proportion from three [3. add. supra lin.] to two [2. add. supra lin.], which is the sesquialtera proportion, they will sound the Diapente, while if they are in the proportion from [4. add. supra lin.] to three [3. add. supra lin.], which is the sesquiterza, they will sound the Diatessaron. If the difference between the weights is from nine [9. add. supra lin.] to eight [8. add. supra lin.], the resulting Interval will be a Tone, and the sounds will not be consonant, but it has some musical quality nevertheless; since, every time that the sounds played separately are sweet and pleasing, but played together are harsh and unpleasant. In the Consonances however, if those notes are played in sequence or at the same time, the Combination of the Sounds is perceived always as pleasant. Moreover, they prove this also with the proportions, since in Music the Diapason is composed ot the Diapente and of the Diatessaron, thus in the numbers the dupla proportion is created from the Sesquialtera and the Sesquiterza. Certainly Twelve [12. add. supra lin.] **[12. 9. 4. 3 12. 8. 3. 2. 12. 6. 6. 3. 3/2 4/3 12/6 2/1 4/3 3/2 9/8 in marg.]** is in Sesquiterza proportion with nine [9. add. supra lin.], in sesquialtera proportion with eight [8. add. supra lin.] and in dupla proportion with six [6. add. supra lin.]. Therefore, the Dupla proportion is composed of the Sesquialtera and Sesquiterza, just as the Diapason is made up of the Diapente and Diatessaron. Just as the Diapente exceeds the Diatessaron by a Tone, thus in that case the Sesquialtera exceeds the Sesquiterza by a Sesquiottavo. So, one can see clearly that the Diapason is meant to be in dupla proportion, the Diapente in sesquialtera, the Diatessaron in Sesquiterza, and the tone in Sesquiottava. Now that this has been demonstrated, see now if the sesquiottava proportion [-12-] **[will be attached two weights in sesquipla or Sesquiterza proportion, since those weights in sesquiottava proportion represent the Tone. Hence Plutarch himself, together with Macrobius in his Treatise on the Music of the Cosmos in the Dream of Scipio by Marcus Tullius Cicero, and also Boethius with Gaudentius and all the others who reported this, have been greatly mistaken. See my exposition and declaration of the Enigma of the mythical Hammers of Pythagoras in marg.]** can be divided into two equal parts. If it cannot, it will not be possible to divide the Tone either. Firstly, the numbers which contain the sesquiottava proportion, which are nine [9. add. supra lin.] and eight [8. add. supra lin.] do not admit any number between them, and if both the numbers are doubled, the

number that occurs between them produces two intervals. This number is seventeen [17. add. supra lin.], which occurs between sixteen [16. add. supra lin.] and eighteen [18. add. supra lin.], which are the doubled numbers. It is clear that, if the numbers are equal, the proportion sesquiottava can be divided into two equal parts. However, they are different one from the other, since one is the number sixteen and the other one is the number seventeen. Therefore, the [[Octave]] **[As to the fact that between two numbers in superparticular proportion one can place one or more numbers which are proportional halves, see the third Theorem of the Sound of the Harmonic Rule attributed to Euclid, and also the tenth Theorem. [<....>] or the impossible division of the sesquiottavo Tone into two or more Equal parts. See also Boethius in the first chapter of the third book of his Music. in marg.]** Sesquiottava is divided unequally. Therefore, the Semitone is neither this or that part of the Tone divided in this way. This is what Plato says:

[[Bottrigari, On Plutarch's Commentary, 12; text: 9/8, 81/64, 4/3, 256/243, 192, 64, 256. 3, 8. 13. 27. 34, 256. 243. 216. 192. Limma, Tuono] in marg.]

" God filled the Sesquiterza and left over the Limma, whose proportion lays two hundred and fifty-six [256. add. supra lin.] and two hundred and forty-three [243. add. supra lin.]." Take the Diatessaron within two numbers, one of which contains the other one and a third of it, as two hundred and fifty-six [256. add. supra lin.] and one hundred and ninety-two are [192. add. supra lin.]. Assign the larger of these to the lowest Sound of the tetrachord and the smaller one to the highest. It will be shown transposed the two Sesquiottavi **[The product of the two sesquiottavi Tones is eigthy-one sixty-fourths 81/64, which, subtracted from the sesquiterza proportion of the Diatessaron, leaves the proportion supertrediciduecentoquarantetre, namely, 256/243, which represents the Limma itself in marg.]** that Limma, or Interval, which is between the numbers two hundred and fifty-six [256. add. supra lin.] and two hundred and forty-three [243. add. supra lin.]. Lowered the highest sound by a Tone will give two hundred and sixteen [216. add. supra lin.] and the other one, lowered by a Tone, will be two hundred and forty-three [243. add. supra lin.]. This one will exceed [-13-]

[[Bottrigari, On Plutarch's Commentary, 13, 1; text: Operatione. Tuono, 9/8, Ditono, 81/64, 4/3, Diatessaron, Restante, 256/243, Limma]] in marg.]

the number two hundred and sixteen [216. add. supra lin.] by twenty-seven [27. add. supra lin.], which is larger than one hundred and ninety-two [192. add. supra lin.] by uentiquattro [24. add. supra lin.]. However, the number twenty-seven [27. add. supra lin.] **[27 8 216. 24 8 192. in marg.]** is the eighth part of the one hundred and ninety-two [192. add. supra lin.], so the [[largest]] [larger corr. supra lin.] of these three numbers is in sesquiottava proportion with the middle one and the middle one with the smallest one. The Interval which is between the [[largest]] [larger corr. supra lin.] and the smallest is of two Tones, which are made of two eighth parts one added after the other one. Once these are subtracted, nothing remains of all the sesquiterza proportion between the extremes but the Interval between two hundred and fifty-six [256. add. supra lin.] and two hundred and forty-three [243. add. supra lin.], which is thirteen [13. add. supra lin.]. Therefore, this number is called Limma, which means remainder. I consider certain that Plato's intention is very clearly declared in this numbers. Others, demonstrating the terms of these consonances, assign the low sound to the number two hundred and eighty-eight [288. add. supra lin.] and the high one to the two hundred and sixteen [216. add. supra lin.]. The

remaining ones proceed in the same way, except for the fact that they put the Limma between two Tones. From the higher sound

[Bottrigari, On Plutarch's Commentary, 13, 2; text: 32. 13. 27. 288. 256. 243. 216. Tuono limma Tuono in marg.]

Lowered by a Tone comes the number two hundred and forty-three [243. add. supra lin.] and from the lowered or deepened by a Tone the Two hundred and fifty-six [256. add. supra lin.]. It is certain that Two hundred and forty-three [243. add. supra lin.] and Two hundred and sixteen [216. add. supra lin.] are in sesquiottava proportion, just as two hundred and eighty-eight [288. add. supra lin.] and two hundred and fifty-six [256. add. supra lin.], so that the Interval of a Tone occurs in each part, and the one which is contained between Two hundred and fifty-six [256. add. supra lin.] **[216 27 243 256 32 288 in marg.]** and Two hundred and forty-three [243. add. supra lin.] is not a Semitone, but it is considerably smaller than a Semitone. In fact, Two hundred and eighty-eight [288. add. supra lin.] is larger than Two hundred and fifty-six [256. add. supra lin.] by thirty-two [32. add. supra lin.], and Two hundred and forty-three [243. add. supra lin.] exceeds [-14-] **[These two Tetrachords described by Plutarch are the two different species of the diatonic Pythagorean Diatessaron. The first one has the Limma in its lower part, while the second one has it in the middle. The third one, which has it in the high register, is lacking. The Description of its Tetrachord is this one.**

[[Bottrigari, On the Commentary by Plutarch, 14, 1; text: prima 256. 243. 216. 192 limma, Tuono, seconda 288. 256. 216. terza 324.] in marg.]

The first one <exceeds> Two hundred and sixteen [216. add. supra lin.] by twenty-seven [27. add. supra lin.]. Two hundred and fifty-six [256. add. supra lin.] exceeds two hundred and forty-three [243. add. supra lin.] by thirteen [13. add. supra lin.]. However, thirteen [13. add. supra lin.] is certainly smaller of each one of the two differences, which are thirty-two [32. add. supra lin.] and twenty-seven [27. add. supra lin.]. Therefore, it has been ascertained that the Diatessaron is not constituted by two Tones and a half, but of two Tones and the Limma. Here is the Demonstration of this.
[Bottrigari, On the Commentary by Plutarch, 14, 2; text: 288. differentia 32. 256. Differentia 13. 243. 27. 216. 9/8, Limma, 4/3]

[[Bottrigari, On the Commentary by Plutarch, 14, 4; text: 13. 32. 36. 243. 256. 288. 324. 9/8] in marg.]

[thus in the Hypaton Tetrachord. The first is Hypate hypaton, Parhypate hypaton, Licanos Hypaton, Hypate Meson. The second is Proslambanomenos Hypate hypaton, Parhypate hypaton, Licanos hypaton. The third is Hypoproslambanomenos, Proslambanomenos, Hypate hypaton, Parhypate hypaton. Alternatively this is the order of the Meson nel Meson: the first is Hypate meson, parhypate Meson, licanos Meson, mese. The second is Licanos hypaton Hypate Meson, Parhypate Meson, Licanos Meson. The third is Parhypate hypaton, Licanos hypaton, Hypate [[Hypaton]] [Meson add. infra lin.], Parhypate Meson. They correspond to the following, in our musical notation: The first one is Mi, fà, sol, là [sbq] mi, c fa ut, D sol re, or, E la, mi: F fa ut; G sol, re; a a, la. The second one is: Rè, mi fà, sol, A re, [sbq] mj, C fa, ut, D sol re, or, D sol, re; E la, mj; F fa, ut; [sbq]

sol, re, ut. The third one is Ut, re mi, fà. [Gamma] ut, A re, [sbq] mj, C f, ut, or, C fa, ut; D sol, re; E la, mi; F Fa, ut. As to their application, see

[Bottrigari, On the Commentary by Plutarch, 14, 3; text: prima specie, seconda, terza]

the beginning of Euclid's Musical Introduction and the end of the third book of Aristoxenus' Harmonics, which also translated into Italian <……> [of the Music]] in marg.] Therefore, one can have clear evidence from what it has been said why Plato, when he said that the Sesquialteri, Sesquiterzi and sesquiottavi Intervals are created when the Sesquiterzi are completed adding the sesquiottavi, did not mention at all the Sesquialteri, but he left them aside, because the Sescquiterza and the Sesquiottava added together produce the Sesquialteri. Thus, once we have demonstrated this matters ……..

This Italian translation was completed just before the two of the evening following the day 12 March 1610. The present copy was completed on the day 18 of the same month of March. It is an autograph copy.