Author: Bottrigari, Ercole
Title: Pythagoras’ Enigma
Source: Bologna, Museo Internazionale e Biblioteca della Musica, MS B44, ff. 22r-25v

[-f.22r-] PYTHAGORAS' ENIGMA REGARDING THE DISCOVERY THAT HE MADE OF THE PROPORTIONS OF THE MUSICAL CONSONANCES BY MEANS OF THE MYTHICAL HAMMERS, EXPLAINED AND DECLARED BY THE KNIGHT HERCOLE BOTTRIGARO WITH AN AMPLE RESOLUTION, AND WITH HIS OWN ADDITION OF HIS PRACTICAL DEMONSTRATION of the weight of the different Weights that have to be attached to the strings made of sinew or gut, according to the demonstration of Pythagoras' hammers related by Plutach, Macrobius, Boethius, Gaudentius and by other Writers. The exact measurement of the weight of those Weights, by which the true Sounds of the musical Consonances are produced, is also given, together with an important notice regarding the Strings both made of sinew or gut, and of Metals, such as brass and steel, et cetera.

AT BOLOGNA,

1609.

[-f.22v-]

```
Plutarch -- lived 122 anni after Christ -- under Adrian       }

Macrobius ----------------- 142 ------- under Antoninus Pius  } Emperors

Boethius -------------------500 ------- sotto Anastasius      }

Gaudentius ----------------------    -------
```

[-f.23r-] BRIEF AND COMPLETE EXPOSITION OF PYTHAGORAS' ENIGMA ON the Discovery of the Proportions of the musical Consonances made by him through the Sounds of the Hammers.

Many Writers of Musical matters - Gaudentius in the eighth Chapter of his Musical Institution, Macrobius in the first Chapter of the second book on the Dream of Scipio, and Boethius, who lived 360 years after Macrobius, in the tenth Chapter of the first book of his music – relate that, after Pythagoras, the very eminent philosopher, struggled in vain for a long time to find out how and in which manner He could ascertain with certainty the different size of each of the musical Consonances, with anxious and ardent study and varied experiments, because of the great inconsistency which he had found in the different Instruments which he had employed. Finally, as if by a certain divine inspiration, he managed to ascertain them through the Sound of four different hammers of those Blacksmiths, who were hitting a burning piece of metal on an anvil while they were creating one of their works. The larger and heaviest of those Hammers, which, according to his examination, he found to be weighing 12 pounds, when it hit that piece of iron, emitted the lowest sound. Equally, as to the smallest and lightest, he found that it weighed 6 pounds, and when it hit the same piece of burning iron on the same anvil, it produced

the highest Sound. One of the two intermediate Hammers, which was found to be weighing 9 pounds, produced, when hitting the same piece of iron on the same anvil, a sound which was considerably lower than the other one which weighed 8 pounds. Thus, between the sound of the lowest Hammer, and the Sound produced by the lightest one there was the dupla proportion from 12 to 6, which pertains to the Interval of the Diapason; between the sound produced by the hit of the lowest Hammer of 12 pounds and the sound produced from the hit of the middle one closest to it there was the Epitrita or sesquiterza proportion from 12 to 9, which is reduced to the smallest radical numbers from 4 to 3, and pertains to the Interval of the Diatessaron. Then, between the sound of the hit produced by that same Hammer of 12 pounds, which is the lowest, and the Sound of the hit produced by the other Hammer of medium weight of 8 pounds, there is the Emiolia or Sesquialtera proportion from 12 to 8, which is, in its radical numbers and terms from 3 to 2, and it pertains to the Interval of the Diapente. Beyond [-f.23v-] this, it is said that He found that, between the sound produced by the hit of the lower Hammer of the two middle one, which is of 9 pounds, and the sound of the lightest, which is of 6 pounds, there is the doubled sesquialtera proportion from 9 to 6, namely, from 3 to 2 in its smallest radical numbers, which corresponds to the Interval of the Diapente, just as it has been said that he had found it between the Sound of the lowest Hammer of 12 pounds and the Sound of the third middle one of 8 pounds. Similarly, it is said that He had found that, between the Sounds of the hits produced on the same piece of iron and on the same anvil by the one and the other middle Hammers of 9 and 8 pounds, there is the Epogdoa proportion, or, Sesquiottava, as we call it, which corresponds to the Tone, which is the difference between the size of the Diapente and of the Diatessaron, as it has been reported firstly by Gaudentius, by Macrobius, and Boethius in the passages mentioned above, and later by many other modern Writers on musical matters, as I said above. However, this is not the entire exposition, declaration and unveiling of this Enigma of Pythagoras which is so great, since it contains and encloses within itself many other mysterious matters besides the ones mentioned and of such nature that they pertain to musical Speculation, but which have not been explained or declared by any of those Writers (with the exception of Franchino, but only in passing and not very successfully at Chapter 8 of the first book of his Teorica). However, Martianus Capella deals with them at the end of the eighth Chapter of the seventh book of his Arithmetic, but with great obscurity. First of all, it is necessary to be aware that Pythagoras wanted to demonstrate four sounds by means of four proportional musical numbers, according to the numbers of the weights of those four mythical hammers, 12, 9, 8, 6, because, the two middle hammers of 9 and 8 pounds are median proportional numbers between the heaviest Hammer of 12 pounds and the lightest of 6 pounds, since those two numbers 9 and 8 multiplied together give 72, which is also the product of the two extreme numbers. This is demonstrated by Euclid in the nineteenth proposition of the seventh book of his Elements. Then, one must be aware that not only one finds the arithmetic proportion 12, 9, 6 (or 4, 3, 2 in the smallest radical numbers) among those numbers 12, 9, 8, 6, but also the harmonic one 12, 8, 6 (or 6, 4, 3 in the smallest radical numbers). Besides, one will notice that Pythagoras had wanted to signify, by the numbers 12, 9, 8 that the difference between the Diapente and the Diatessaron [-f.24r-] is not the [folium deesse videtur]

an Instrument similar to the one sketched below, which is similar to the Monochord. However, instead of one of the two Bridges or Wood stops it should have a small brass or iron wheel, a small trestle in its Circumference to collect that string of brass thread or of steel inside it when one works it, and it should be easy to turn, without instability on any side or part on its Axis and Small Box. As to the height of that above the Table where it is

planted the foot of the small wheel, one must make sure that it is equal to the height of the wood stop, or bridge which comes before it, since the true length of that String of brass or steel threat becomes, in a certain way, the c. o. p. rather than the c d, as one can learn with certainty thanks to the seventeenth proposition of the third book of Euclid's Elements, and even better from the second Chapter of the third Book of Ptolemy's Harmonics. The proof of these numbers mentioned above squared in their proportions becomes readily available through the tenth proposition of the eighth book of Euclid himself, because, by adding together the dupla sesquiquarta proportion of the Diapente with the sepersettepartienteventinovecima proportion of the Diatessaron, the squared proportion of the Diapason is created exactly. Conversely, if one subtracts that supresettepartientenovecima proportion from the dupla sesquiquarta proportion, the remainder is the superdicesettesessantaquattrecima proportion of the Tone, in this way:

[Bottrigari, Enimma di Pitagora, f.24r; text: a, b. Tauola superiore, c, d, p, Corda di metallo, c, m, Ponticello <legi non potest> <mobile>, f, h, g, i, piè dello stromento, Base dello stromento, o, n. Girella di ottone, Asse della Girella. e, operationi, 9/4, 16/9, [[144/36]], 4/1, 81/64, [23.1 add. man. sec.]] [BOTENI 01GF]

And to what I said so far, which has to suffice for the entire and very clear exposition and elucidation of the harmonic Enigma of Pythagoras, it will be nothing but an extremely useful pleasure to add that, having Pythagoras under the veil of that Enigma enclosed and adumbrated the musical Proportions contained in those four numbers 12, 9, 8, 6, which are proportional with each other (as it has been demonstrated), those understood by other mathematical Philosophers were disguised and adumbrated under the veils of another Enigma. [[In fact, one reads in Suida that Diocles, almost as another Pythagoras, (if indeed Pythagoras was not another Diocles) passing by chance by the shop of a potter, where there were some empty vases, he hit them again and again at the same time and he observed that the larges, from which later he derived the 12 measures, produced the lowest sound, while the smallest which was <legi non potest> [-f.24v-] produced the highest sound, and was at the interval of a Diapason with the lowest one. Two other ones in the middle, one of which he found to be of 9 measures and the other of 8, one sounded a Diatessaron with those, and the other one a Dipente, while they produced the Sesquiottavo Tone when sounded together with each other. The numbers, therefore, of those musical Consonances were these: 12, 9. 8.]] In fact, others covered them under Mercury's false lyre or Cithara with four strings, as Boethius refers in the second Chapter of the first Book of his Music. Others produced their demonstration with the instrument called Elicona, which Ptolemy not only described in the second Chapter of the second book of his Harmonics, and enriched with those numbers 4 and 3, which are all proportional together with the other four 12, 9, 8 and 6, but which he has examined with every diligent care for detail. Others then produced its demonstration with the Cubic Body, and they called it 'The Greatest Harmony' because it contains 12 sides, 8 solid Angles and 6 flat Surfaces, but they did so very imperfectly, because only the three terms 12, 8 and 6 are contained within it, and those are not even proportional with each other, because one finds only the Harmonic proportionality within them out of the three, which are Geometric, Arithmetic and Harmonic, in their radical numbers. Others, together with Suida (Galileo writes this in his Dialogue of the ancient and Modern Music at page 127) maintain “that not Pythagoras, but Diocles researched this invention, and not from observation of the Blacksmiths, but they say that he hit by chance with a stick some Vases when he passed by the Shop of a Potter, and that he researched the musical proportions starting from the different size of the Sounds produced by those in relation to

their highness or lowness." However, Suida (as to who the others are, it is not clear to me) refers something quite different in his historical matters. His words translated into Latin by Volfius, first of all under the entry DIOCLES, are precisely these: "Diocles of Athens or Phlisiasius - namely Diocles from Athens or from Phlius, and he continues – found this Harmony in the Oxibapha, namely in vases made of terracotta (or, he discovered this Harmony in the vases of terracotta) which he had hit with a small stick (namely, which he had hit with a small stick). At the word Oxybaphon, he says: "They say that Diocles from Athens found the Harmony in the Oxybapha, or the Vases of terracotta, or made of clay, which he had hit with a small stick." One must not omit to add this for sure, that [aliquae lineae desunt]

"to the Sound of the Nete, adding (one must be aware of this first of all) the number 6 to the first one, to the Second the number 8, to the third one the number 9, and to the fourth one and last one the number 12, so that one might know precisely from those numbers in which proportions it was with this other String." Some way further on from this, he adds: "The numbers, which Plutarch adds to his Strings, he cannot consider them in any way as such, or parts of the Monochord, but as weights <added by> some of his Subjects, and the others as <measures> of certain similar things. As weights one can consider principally Plutarch <aliqua folia desunt> [-f.25v-]

"attributing them to the Pythagorean Hammers, since the same proportions are found among them, because, when the one that weighed 12 pounds was compared with the one that weighed 6, they responded to each other at the distance of an Octave, but the one that weight 12 produced the high sound (secondly, take note of this) as this is the nature of these type of bodies which are not concave, and the one which weighs 6 sounds the low one. The Hypate is in the same relationship with the Nete, and the same proportion that is found between the other numbers also occurs in the various weights with the same order. And since the example validates this consideration of his, he adds: "What happens with the <low>, can be seen happening to the concave and subtle Bodies, which, when they are hit, produce a low sound, and if they were thick, albeit of the same capacity, they would produce a sound which is higher, or less low," however we want to say. Now, if Galileo had concluded with this discursive Consideration of his, that it could have been much less inconvenient not to the false Hammers of Pythagoras, but to the Strings made of <sinew> or gut, which Macrobius mainly, Gaudentius and Boethius describe, and he would have proved <legi non potest> to predict what, as we have said, he wrote later with greater wisdom in his Discourse on Zarlino's Works on the matter of attaching Weights to the Strings, to produce the low, middle, and high sounds with accuracy, conforming to those four proportional numbers of those Pythagorean hammers, namely, 12, 9, 8, 6. Leaving aside that Galileo, writing in this way, contradicts not only the such an account of Macrobius, Gaudentius and Boethius themselves and also the general consensus of all his other followers, ancient and <modern> who have relayed this story, but the daily and even hourly experience itself which one can have and the well polished sense of hearing itself, one will have to understand this at all differently from what has been said not only <.....> with regard to the weights of the Strings equivalent to those Pythagorean Hammers recited by Macrobius, Gaudentius and Boethius, and the repeated by many other modern Writers of Musical <matters>, but to the real certainty of those Weights, which (as it has been said above) will have to be attached to the Strings made both of gut and of Metal, so that, once hit, they <may produce> their <Sounds in real conformity> with the musical Consonances. But, oh what a great Marvel

[[at almost the time of just less than the sixth hour of the following night. The day 29 October 1609. <Hercole Bottrigaro>]]

[-f.24v-] Finished to expose and declare

at hours 22 of the day Wednesday 15 July 1609.

BY ME HERCOLE BOTTRIGARO

at Bologna.

This copy was completed

at 21 1/2 hours of the day

19 August

of the same year

1609

[Bottrigari, Enimma di Pitagora, f.24v; text: ELICONA. Corde. A, B, C, D, E, F, G, H, I, K, L, M, 3. 4. 6. 8. 9. 12. CVBO, MASSIMA ARMONIA. a, b, c, d, e, f, g, h, lati. angoli solidi. superficie <piane> lateralj. lati.]