[-f.1r-] On the form of the Consonances 1668
I begin.

The fact that the hand of the Omnipotent enclosed the most inscrutable secrets and mysteries of the very noble science of music within numerical containers is the reason why one encounters every day greater difficulties and why there are always unresolved doubts which are extremely detrimental to its professors. Today I decided, as far as my small capacities will allow, to illustrate openly how it is created and whence it originates and to resolve a small difficulty which was put to me by a subtle and great mind not openly, but by private communication, and, since this same difficulty may arise in other subjects, I have decided to let it be known to the public together with its answer in order to dispel any doubt.

Therefore, firstly I state, to start from the first point, that music derives its form from the sounding number, and, this being true, every musical interval derives its being from an unequal proportion. In fact, Gaffurio in the second book of this Theory of music gives the following warning: "In fact, the Discipline of music considers only the proportion of inequality, while the Consonances are born of sounds which are unequal and different, which themselves it must be born of unequal numbers and proportions." [[But, since [-f.1v-] in this case we have to be better informed of the Authorities and reasons behind this matter, let us come to prove in practice how every musical interval receives its form from an unequal proportion.]] Today, it is my intention to come to the practical matter itself, leaving aside any sort of Authority and doctrine, so, I come straight away to the practical demonstration, which will provides in a short time with every certainty. I am aware that you know well that the unequal proportion is the comparison that one makes between two numbers, one larger and the other smaller, such as two and one, and similar ones. Franchino, in his book on the practice of music, chapter first on the Definition and Distinction of the proportion, says: "the proportion between unequal quantities is a reciprocal relationship, as in the case of two to one and of one to Two." If one takes two unequal numbers, for instance One and Two, which are the first two numbers, when they are compared together, they create the Dupla proportion, which is the first species of proportion of the multiplex Genus, as you heard being said in the last session of the Academy. In fact, the dupla proportion is created when, if one compares the larger number with the smaller one, the larger one contains [-f.2r-] the smaller one within itself twice. Gaffurio teaches us in the fourth book of his practice of music that "if the larger number contains the smaller twice, this is called dupla proportion, as in the case of the ratio of Two to One and One to Two. Presently, we shall see what great treasure is contained within this Dupla within this numerical container. Two and one. Open it. It is not possible. [[So, leave it to me]] Try harder, and you still cannot open it. Let us do it in this way. Let us take two weights, of which one weighs two pounds and the other only one pound, which will represent the number Two and One, and, consequently, the Dupla

proportion. Let us attach these two weights to two Gut Strings of the same thickness. Now they are attached. Now, do let us lay out the strings with the weights attached to them. Here they are, laid out. Pluck them one after the other. Can you not hear that they produce the consonance of the octave? O what a unique marvel! Therefore, the Consonance of the Octave is hidden within the container constituted by the numbers Two and One. Oh what a prodigy! Quickly, do let us not waste time. [[leave aside the weights of Two and One pound]]

Remove the weight of one Pound and attach another weight instead [-f.2v-] of it which is of Three pounds without touching the weight of Two pounds. It looks as if the weight of three and Two pounds are already attached and disposed. Now, lay out the strings. Here they are, laid out. Next, strike the Strings one after the other. Oh what a sweet sound. Can you not hear that they sound the consonance of a fifth together? So, is the Consonance of a Fifth contained within the numbers three and two? There is no doubt at all. Quickly, do let us move further on to witness greater prodigies. Remove the weight of Two pounds and attach another one of four pounds leaving aside the one of three pounds, so the weights will be one of four pounds and another one of three pounds. Have you attached it? Yes. Lay out the Strings. Here they are laid out. Strike the strings. Oh what a beautiful sound, oh what a beautiful Harmony. Listen, listen! They constitute the Consonance of the fourth. Oh admirable secret. Hence, the Consonance of the Fourth is hidden and well contained in the container of the numbers four and three. No more dithering. Remove the weight of three pounds and attach another on e of five pounds, leaving aside the weight of four attached earlier, so that the weights will be one of five ponds and the other one of four pounds. Lay out [-4r-] the strings. Strike the strings. Oh what a sound! This is the Consonance of the major third! Therefore, the Consonance of the major third is contained within the container of the numbers five and four. Who would believe it? No more words. Remove the weight of four pounds and place another weight of six pounds instead of it, leaving the one of five pounds attached, so the weights will be one of six pounds and the other one of five pounds. Lay out the strings and strike them. O what sweetness. This is the Consonance of a minor third! O marvel of nature. Therefore, the Consonance of the minor third is hidden within the container of the numbers six and five. [[After this]] Conclude these beautiful experiments with this last one. Take the weight of five pounds and the one of three pounds. Attach them to the strings. Have you done it? Strike the Strings. What do I hear? This is the Consonance of the sixth. I am besides myself with astonishment. Therefore, the Consonance of the sixth is contained within the container of the number Five and Three? With this the experiments are completed. Do not be amazed. Here we have demonstrated in which numerical containers all of the perfect and imperfect Consonances are [-f.4v-] hidden. Therefore, one must conclude that the Octave derives its form from the Dupla proportion of Two to one; the Fifth derives its form from the Two and the Tree, which is called sesquialtera; the Consonance of the fourth from the number Three and four, which is called sesquiterza proportion; the major third derives its form from the four and the five, which are said to be in sesquiquarta proportion. The minor third derives its form from the numbers five and six, which are called sesquiquarta proportion, while the Consonance of the sixth derives its form from the numbers six and three, which form the proportion superbipartiente terza. What is cause of great marvel is that all the Consonances are contained and restricted within the number six, where there is no proportion which sours such great sweetness by providing the form to any of the

Dissonant intervals (of which there are so many in our own music) because of the perfection of this number. For this reason it is considered by musicians the most perfect number of all, not only because of the reasons which Arithmetic provides, but also for the aforesaid reasons. In fact, it contains bound up within itself not only all of the Consonances which are found as simple consonances throughout the ample science of music, but also those which derive from any [-f.5r-] Combination of the proportions which are contained within the number six. All of them provide the form to a Consonance and never to any dissonance. Therefore, because of its great perfection it is considered, as it is, by those who are experts in Arithmetic as the number most perfect among all others. I am already finding myself to be answering the second point, which is the answer to the question. Therefore, Academicians, do me the favour of your attention.

Gioseffo Zerlino in the first part of his Istitutioni Harmoniche at chapter 13 says that the number six is the most perfect number of any other, because if one takes its factors and adds them together they give six exactly. These are the number one, the number two and the number three, which they divide it entirely, the number one into six parts, the number Two into Three parts and the number Three into Two parts. These parts added together added together result completely in the number six. For this reason, Boethius says in his Arithmetic the following words, which are the same as Zarlino's ones quoted above. "In fact, the perfect Number is one, which does not extend itself by a redundant increase, nor does it diminish again by contraction, but reaches the limit of the right medium with its parts and neither it is too fat for its size, nor it is too small for its minuteness, as in the case of the number six. Its parts added together produce exactly the whole number. In fact [-f.5v-] they are the number three, which is its Half, the number Two, which is its third part, and the number One, which is its sixth part, but One, Two and Three added together produce exactly the number six, et cetera." As Zarlino says, it is not surprising that many elements of Nature and Art are contained within it. Starting with entities which are above us, of the Twelve signs of the Zodiac above us, we always see six rising above our Hemisphere, while the other sixth are hidden in the other Hemisphere below us. Six are the circles placed in the sky, namely, the Arctic, Antarctic, Two Tropics, namely the one of Cancer and the one of Capricorn, the equinoctial and the Ecliptic. Six are the Planets: Saturn, Jupiter, Mars, Venus, Mercury and the Moon. Here on Earth, six are the species of movement: Generation, Corruption, Increase, Decrease, Alteration, and change of place. Six are the variety of places according to Plato, namely, up, down, Forward, backward, Right and left. Six are the degrees of man: being, Life, movement, sense, memory and intellect. Six are the Ages of man: Infancy, childhood, Adolescence, Youth, Old age and Decrepit age. The shape of the Circle contains six major equilateral triangles, and this signifies its perfection, and the circumference of any circle measured in a straight line produces that measurement which is measured from the middle of the Circumference itself six times. Hence the instrument which we call compass is also called Sesto [Sixth]. The species of the musical notes are six, namely, Unison, Equisonant, Consonant, Emmeles, Dissonant and Ecmele. The Consonants are six, the Unison, as their origin, Third, fourth, Fifth, sixth and Octave. Six species of Harmony are found in the works of the Ancients, namely, the Dorian, the Phrygian, the Lydian, the Mixolydian, the Aeolian and the Iastian. The modes which are commonly called Tones are six, and six are the notes discovered by Guido: ut, re, mi, fa, sol, la. However, I have extended myself too much in illustrating the prerogatives of the number six, which demonstrate its perfection.

Now, Signori Academici, be my witnesses at the appropriate time that the number six is more perfect than any other Number. In order to prove that the Number Four is also most perfect, I will quote what Guglielmo says in the third book at chapter 24 De Quaternario Numero et eius dignitate: "Pythagoras called the Number Four the most perfect of all and Divine in his Golden Verses. Just as that one reaches the number Ten, which embraces every number, equally in the same manner it contains every Consonance and, therefore, the whole of music." This Divine number is composed by the number One, by the number Two, by the number Three and by the number Four, which, added together constitute the number ten, aim and perfection of all numbers, since one plus Two is Three, plus three equals six, and plus four is ten. [[It is clear that one, Two, Three and Four added together produce the number Ten, which is the aim and the completion of all numbers]] If you multiply four by Ten, the result will be forty, namely, ten, twenty, thirty and forty, which, added together, produce the number One hundred, because ten and twenty equals thirty, plus thirty equals sixty, plus forty equals one hundred. Equally, if you multiply One hundred by four, it will equal four hundred. One hundred, two hundred, three hundred and four hundred added together equal one thousand, because One hundred and Two hundred equals three hundred, plus three hundred equals six hundred, and plus four hundred equals one thousand. Therefore, the number one thousand is a number produced by the Number Four, and it is extremely perfect, as the number four is. Please, Signori Academicians, consider the number six hundred and the number sixty. [-f.7r-] Who cannot see that these are numbers derived from the Number six and, consequently, most perfect? In fact, multiplying six by Ten, the result is sixty, which multiplied as before by ten produces six hundred. Then, the octave is the most perfect Consonance of any other, as it was demonstrated in the Last Academy. Let it be sufficient to remind you that Andrea Matteo, duke of Andria in the Commentary which he wrote to Plutarch, at chapter 22 De Diapason, which begins with the words: "Pythagoras extracted from the doubling of the weight the Diapason, the most sweet, beautiful and attractive of all the Consonances, as Aristotle says." I believe to have solved the question set at the beginning with my demonstrations. In fact, if one adds together the number one thousand, the number six hundred, the number sixty and the number eight, we will obtain as a result the number i668, and, since all those said numbers are the most perfect of all the others, it follows that the number 1668 is the more perfect Number than any other whatsoever. Consequently, the first year and the second is one. this beautiful number Two has produced this Harmonious octave, namely 1668, differently from all the other Years first and second of the past and of the future. Consequently, the praise which I gave to the first and second year in the <....> Academy does not suit any other. Therefore, if the Number Six is (and here the question is resolved) [-f.7v-] prodigious as you have heard, how will it be possible for Candia to fall, since its name is made up of six letter, since all the amplitude of Harmony is contained within six letters, as it has been said about the number six? Even less one should fear this in this Year i668, since it is all Harmony and all perfection, as you have heard in this unadorned speech. Therefore, to express our Cheerfulness let us start to play and let us stop talking.